



The Marinari-Parisi Model and Collective Field Theory

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Abstract

We derive the supersymmetric collective field theory for the Marinari-Parisi model. For a specific choice of the superpotential, to leading order we find a one parameter family of ground states which can be connected via instantons. At this level of analysis the instanton size implied by the underlying matrix model does not appear.

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1. Introduction

Nonperturbative effects are not yet understood in string theory. Since much important string physics relies upon these [1], it is important to understand any known examples. In the matrix models descriptions of non-critical strings [2], a source of both supersymmetry breaking and other nonperturbative effects is one eigenvalue tunneling processes [3,4,5,6]. One string theory which exhibits supersymmetry breaking nonperturbatively is the Marinari-Parisi model [4]. In an attempt to understand better the nature of the nonperturbative physics found there, in this paper we transform the model to collective fields. This transformation led to much insight about the spacetime interpretation of the $d = 1$ bosonic model [7]. We then consider the supersymmetry breaking seen previously in the matrix description [4,8,6,9] and close with some comments about the current status of the spacetime identification of the model.

2. The Marinari-Parisi Supermatrix Model

The one-dimensional string is described by a two-dimensional worldsheet embedded in one spacetime dimension. This may be approximated by a triangulated surface with an additional degree of freedom on the faces of the triangulation which describes its position in the one-dimensional space. This leads to the matrix model description of the $d=1$ string which has a single matrix function of one spacetime variable [10,11].

The one-dimensional superstring has worldsheet supersymmetry, which leads to supersymmetry in the spacetime spectrum of the superstring. We do not know how to build a matrix model which describes a theory with worldsheet supersymmetry, but we can impose spacetime supersymmetry by describing surfaces imbedded in one-dimensional superspace. This is the Marinari-Parisi [4] model for one-dimensional superstrings. The action of this model is:

$$S = N \int dt d\bar{\theta} d\theta \text{Tr} \left[\frac{1}{2} \bar{D}\Phi D\Phi + W(\Phi) \right], \quad (2.1)$$

where D is the differential operator on superspace and Φ is a hermitian $N \times N$ matrix-valued superfield. In components, the expansion of Φ is

$$\Phi = M + \bar{\theta}\Psi + \bar{\Psi}\theta + \bar{\theta}\theta F \quad (2.2)$$

The matrix superfield Φ cannot be diagonalized by a unitary rotation. However, there exists a consistent truncation to a supersymmetric subsector of the Hilbert space, where M is diagonalized [9]. To define this truncation let U be the unitary matrix such that $UMU^\dagger = \text{diag}(\lambda_i)$. Then we restrict our theory to only those states generated by the diagonal elements $\psi_i = (U\Psi U^\dagger)_{ii}$ acting on the vacuum, whose wavefunctions depend only on the eigenvalues λ_i . This theory is described by an action with N superfields X_i and an effective superpotential which incorporates the Jacobian for this change of variables.

$$S = N \int dt d\bar{\theta} d\theta \left(\sum_i \left[\frac{1}{2} \bar{D}X_i DX_i \right] + W_{\text{eff}}(X) \right) \quad (2.3)$$

$$W_{\text{eff}}(X) = \sum_i W(X_i) - \frac{1}{N} \sum_{i < j} \ln(X_i - X_j) \quad (2.4)$$

The component field expression for the superfields is $X_i = \lambda_i + \bar{\theta}\psi_i + \bar{\psi}_i\theta + \bar{\theta}\theta f_i$. In terms of the components the supercharge of this theory is

$$Q = -\frac{i}{N} \sum_i \left(\frac{\partial}{\partial \lambda_i} - N \frac{\partial W_{\text{eff}}(\lambda)}{\partial \lambda_i} \right) \psi_i \quad (2.5)$$

and the Hamiltonian,

$$H = \frac{1}{2} \sum_i \left(-\frac{1}{N^2} \frac{\partial^2}{\partial \lambda_i^2} + \left| \frac{\partial W_{\text{eff}}}{\partial \lambda_i} \right|^2 - \frac{1}{N} \frac{\partial^2 W_{\text{eff}}}{\partial \lambda_i^2} \right) + \frac{1}{N} \sum_{i,j} \psi_i^* \frac{\partial^2 W_{\text{eff}}}{\partial \lambda_i \partial \lambda_j} \psi_j. \quad (2.6)$$

This theory was considered at length in the eigenvalue description in ref. [9](see also [12] for a study of N particle supersymmetric quantum mechanics).

What is the interpretation of this truncation? In the bosonic $c=1$ matrix model the dynamics of the eigenvalues describes the singlet sector of the theory, that is, operators such as $\text{Tr}M^n$ which do not depend on the angular variables U_{ij} . These operators may be generalized by replacing the matrix M by the superfield Φ . The components of these operators such as $\text{Tr}(\Psi M^n)$ or $\text{Tr}(\bar{\Psi} M^n \Psi M^n)$ act within the diagonal sector (of M) of the theory. So this truncation is a consistent supersymmetric counterpart to the truncation to the eigenvalue variables in the bosonic case. Since the supercharge for the full theory does not take states out of the truncated sector, the calculation of quantities such as $\langle \text{anything} | Q | \text{state in truncated sector} \rangle$, the trademark of supersymmetry breaking when $|\text{state in the truncated sector}\rangle$ is the vacuum, are valid for the theory as a whole.

3. Supersymmetric Collective Field Theory

We would like to treat this theory using the collective field method [7,13]. To begin introduce the density variables for the eigenvalues λ_i :

$$\phi_k = \sum_i e^{ik\lambda_i}. \quad (3.1)$$

Only N of these variables are independent. In the $N \rightarrow \infty$ limit these become the Fourier modes of the density $\phi(x) = \sum_i \delta(x - \lambda_i)$, with the constraint $\int \phi = N$. This is the usual collective field for the bosonic $d=1$ theory. To complete the field content of this theory, introduce fermionic fields:

$$\begin{aligned} \psi_k &= \sum_i \psi_i e^{ik\lambda_i} \\ \bar{\psi}_k &= \sum_i \bar{\psi}_i e^{ik\lambda_i} \end{aligned} \quad (3.2)$$

In the large N limit these variables become the Fourier components of fermionic partners to the bosonic collective field.

To quantize this theory we introduce canonical momenta $p_i, \pi_i, \bar{\pi}_i$ for the eigenvalue variables $\lambda_i, \psi_i, \bar{\psi}_i$ and similarly $p_k, \Pi_k, \bar{\Pi}_k$ conjugate to $\phi_k, \psi_k, \bar{\psi}_k$, with Poisson brackets

$$\begin{aligned} \{p_k, \phi_q\} &= \delta(k+q), \\ \{\Pi_k, \psi_q\} &= \delta(k+q), \\ \{\bar{\Pi}_k, \bar{\psi}_q\} &= \delta(k+q), \\ &(\text{all others zero}). \end{aligned} \quad (3.3)$$

In addition there are constraints corresponding to the fermionic momenta. In the eigenvalue variables these are determined by varying the action (2.3) with respect to $\dot{\psi}_i$:

$$\begin{aligned} \chi_i &= \pi_i - \frac{i}{2} \bar{\psi}_i = 0, \\ \bar{\chi}_i &= \bar{\pi}_i + \frac{i}{2} \psi_i = 0. \end{aligned} \quad (3.4)$$

By using the canonical change of variables these may be rewritten in terms of the density variables:

$$\begin{aligned} \chi_k &= \sum_i e^{ik\lambda_i} \chi_i = \phi_{k+q} \Pi_{-q} - \frac{i}{2} \bar{\psi}_k \\ \bar{\chi}_k &= \sum_i e^{ik\lambda_i} \bar{\chi}_i = \phi_{k+q} \bar{\Pi}_{-q} + \frac{i}{2} \psi_k \end{aligned} \quad (3.5)$$

These constraints can be formally solved to give $\Pi_k = \frac{i}{2}(\bar{\psi}/\phi)_k$. With the constraints we use the Dirac quantization procedure to find the commutation relations of the density variables:

$$\begin{aligned} [p_k, \phi_q] &= -i\delta(k+q) \\ \{\bar{\psi}_k, \psi_q\} &= \phi_{k+q} \\ [p_k, \psi_q] &= \bar{\Pi}_{k+q}. \end{aligned} \quad (3.6)$$

In the large N limit these become the commutators of continuous fields $\phi(x), \psi(x)$. These commutators agree with those found by Jevicki and Rodrigues [14] by supersymmetrizing the bosonic $d=1$ collective field theory.

With the quantization complete we rewrite the Hamiltonian in terms of the new variables. By the canonical change of variables we have an expression for p_i :

$$p_i = \frac{\partial \phi_k}{\partial \lambda_i} p_{-k} + \frac{\partial \bar{\psi}_k}{\partial \lambda_i} \bar{\Pi}_{-k} + \Pi_{-k} \frac{\partial \psi_k}{\partial \lambda_i}. \quad (3.7)$$

This is a classical expression for the relationship between the canonical variables. After quantization the variables become operators with non-trivial commutation relations. The classical expression does not fix the ordering of these operators, but the expression given for the fermionic part is the only one consistent with the requirements that p_i be a Hermitian operator and $p_i |0\rangle = 0$.

Inserting this expression into (2.5) gives the collective field supercharge and the anti-commutator $H = \frac{1}{2}\{Q, \bar{Q}\}$ is the Hamiltonian. We need the quantities $\sum_i p_i \psi_i$ (for Q) and $\sum_i \bar{\psi}_i p_i$ (for \bar{Q}); using (3.7) it may be seen that Q and \bar{Q} are not naively Hermitian conjugates of each other in collective field variables, but Q has an additional term:

$$\sum_k ik[e^{ik\lambda_i}, p_{-k}] \psi_i \quad (3.8)$$

which can be traced back to the fact that $p_k^\dagger \neq p_{-k}$ because of the Jacobian for the change to collective variables. However, the commutator can be calculated in the purely bosonic theory, where it is already known that the similarity transform which restores the naive Hermiticity properties is [13]:

$$\partial p \rightarrow \partial p - \frac{1}{2}i \frac{\partial \phi}{\phi}. \quad (3.9)$$

All other fields remain unchanged under this transformation. This can be understood from the fact that the change of variables is linear in the fermionic degrees of freedom, hence the Jacobian depends only on ϕ , which commutes with all fields except p .

The transformed supercharge in collective field variables is:

$$Q = \int dx \frac{1}{N} \partial\sigma(x)\psi(x) + i \left(W'(x) - \frac{1}{N} \int \frac{dy \phi(y)}{x-y} + \frac{1}{2N} \frac{\phi'(x)}{\phi(x)} \right) \psi(x), \quad (3.10)$$

and \bar{Q} is the naive conjugate. The field σ has been defined by

$$\partial\sigma = \partial p - \frac{i}{2} \frac{\bar{\psi}}{\phi} \partial \left(\frac{\psi}{\phi} \right) + \frac{i}{2} \partial \left(\frac{\bar{\psi}}{\phi} \right) \frac{\psi}{\phi} \quad (3.11)$$

We note that in effect the superpotential has picked up a new term from the collective field Jacobian[15]. The final result for the Hamiltonian is

$$H = \int dx \left[\frac{1}{2N^2} \partial\sigma(x)\phi(x)\partial\sigma(x) + \frac{1}{2}\phi(x) \left(W'(x) - \frac{1}{N} \int dy \frac{\phi(y)}{x-y} + \frac{1}{2N} \frac{\phi'(x)}{\phi(x)} \right)^2 + \frac{1}{N} W''(x) \frac{\bar{\psi}(x)\psi(x)}{\phi(x)} + \frac{1}{N^2} \bar{\psi}(x) \partial_x \int dy \frac{\psi(y)}{(x-y)} \right]. \quad (3.12)$$

A term proportional to $\int dx [\partial\sigma(x), \psi(x)]$ has been set to zero (its value in one regularization scheme) and the constraint $\int dx \phi(x) = N$ is implicit. For a generic choice of superpotential this Hamiltonian is apparently nonlocal due to the instantaneous interaction between the eigenvalues. However, for the special case of a cubic superpotential the nonlocality vanishes to leading order in $1/N$. In particular we may choose $W(x) = \frac{1}{2}(gx - \frac{1}{3}x^3)$ as considered in [9]. Then the bosonic potential terms of the theory can be seen to be equivalent to (dropping the subleading ϕ'/ϕ term):

$$V = \frac{1}{2} \int dx \left[W'(x)^2 \phi(x) + \frac{\pi^2}{3N^2} \phi(x)^3 - \frac{1}{N} \int dy \frac{W'(x) - W'(y)}{x-y} \phi(x)\phi(y) \right] \quad (3.13)$$

$$(\text{for specific } x^3 \text{ potential}) = \frac{1}{2} \int dx \left[(W'(x)^2 - x)\phi(x) + \frac{\pi^2}{3N^2} \phi(x)^3 \right],$$

making use of the constraint $\int \phi = N$ and using the assumption that the support of ϕ is nonsingular to obtain $(\pi^2/3) \int dx \phi^3(x)$ from $\int dy \phi(y) (\int dx \phi(x)/(x-y))^2$ [13,16].

4. Ground States of the Collective Field Theory

To investigate this theory further we consider the ground state. As usual in a supersymmetric theory, this is expected to be a static field configuration with zero potential

energy in the zero fermion number sector. A look at eqn. (3.12) shows us that we must solve the equation

$$W'(x) - \frac{1}{N} \int dy \frac{\phi(y)}{x-y} + \frac{1}{2N} \frac{\phi'(x)}{\phi(x)} = 0 \quad (4.1)$$

This equation may be formally integrated to give $\phi \propto \exp(-2NW_{\text{eff}}(\phi; x))$. However, this result may be misleading for several reasons. In the particular case of the cubic superpotential above this is inconsistent with the normalization constraint on ϕ . That the ground state is not normalizable might be interpreted as a signal of supersymmetry breaking, but there is another difficulty as well. At large values of x we expect $\phi(x)$ to be exponentially small; but the term in (4.1) arising from the collective field Jacobian is just the first term in a series which is particularly badly behaved when ϕ is small [17]. So the equation can only be trusted in the region where ϕ is large. In this case the Jacobian term is down by $1/N$ and can be neglected at the sphere level.

Thus we are led to consider an equation which is formally identical to the BIPZ [18] method for bosonic matrix models, where now the superpotential plays the role of the potential. We may expect that the critical points of these models may be classified by the critical behavior of this equation; i.e., eigenvalues spilling over the barriers in the *superpotential*. However, the interpretation will be different, since both minima and maxima of the superpotential correspond to local minima of the potential. Thus, for instance, in the cubic superpotential criticality occurs when the eigenvalue density reaches the top of a quadratic maximum of the superpotential; in spacetime we see instead a coalescing of a second potential well with the endpoint of the eigenvalue density to form a *cubic* critical point. There does not appear to be a critical point which is quadratic in the spacetime potential (as for the bosonic $c=1$ theories). As long as supersymmetry is unbroken, solutions to an equation of the form (4.1) corresponding to a one matrix model configuration appear to rule out $-x^2$ critical behavior in the actual potential. One way to see this is to start with the $-x^2$ potential and work backwards to the corresponding superpotential. The eigenvalue density on two sides of a $-x^2$ critical point in the potential corresponds to eigenvalue density in the minimum and maximum of the corresponding superpotential, with one of the densities negative. Higher order potentials will also induce explicit nonlocal interactions between the eigenvalues in the bosonic sector; in some cases this may become local for symmetry reasons [14] or disappear in the double scaling limit.

For the cubic superpotential $W = gx - \frac{1}{3}x^3$ this equation has been solved in [9]:

$$\begin{aligned}\phi_0 &= -\frac{N}{2\pi}(x+a+b)\sqrt{(2a-x)(x-2b)}, \quad 2b \leq x \leq 2a, \\ g - (a+b)^2 - \frac{1}{2}(a-b)^2 &= 0, \\ 2 + (a-b)^2(a+b) &= 0.\end{aligned}\tag{4.2}$$

The potential for fluctuations around this background, away from the support of ϕ_0 , is linear in the fluctuation (since here the fluctuation is constrained $\delta\phi \geq 0$). The lowest order term is

$$\frac{1}{8}(x+a+b)^2(x-2a)(x-2b)\delta\phi(x), \quad x < 2b, x > 2a.\tag{4.3}$$

The potential is zero at $x = -(a+b)$. In [19,9] this was interpreted as an alternate classical ground state for the highest eigenvalue. In supersymmetric quantum mechanics the presence of two ground states signals the possibility of supersymmetry breaking by instanton tunneling from one well to another.

In the collective field theory this extra ground state is represented by a singular field configuration with δ -function support at $x \sim -(a+b)$:

$$\phi \sim \phi_c + \alpha\delta(x+a+b).\tag{4.4}$$

In this equation ϕ_c represents a continuous distribution, equal to ϕ_0 in (4.2) to leading order in $1/N$, and the parameter α in the matrix model picture counts the number of eigenvalues sitting in the second well. In the collective field theory a solution of this form may be found perturbatively in α/N by a simple modification of the BIPZ procedure. The double scaling limit of [9] requires α to be finite (or zero) as $N \rightarrow \infty$.

In the eigenvalue picture of [9], it is clear that there are precisely two ground states, corresponding to eqn. (4.2) and eqn. (4.4) with $\alpha=1$, that is, one eigenvalue sitting near $x = -(a+b)$. The repulsion of eigenvalues prevents more than one eigenvalue from living at this point. It is easily verified that with an ansatz of the form (4.4) there are no further zeros of the potential for real values of the eigenvalues, so there are no additional ground states. The existence of two degenerate ground states with zero energy perturbatively implies that supersymmetry is broken, and nonperturbative contributions give a non-zero ground state energy as calculated in [9].

By contrast, the field theory seems to allow a continuum of ground states assuming the perturbative expansion in α/N has a finite radius of convergence. These involve

singular field configurations which cannot be simply regarded as a limit of smooth field configurations. (The self interactions of a ‘ δ -function’ of finite width are self-repulsive. As a result, configurations energetically prefer to spread out rather than approach a singular δ -function, unless the principal value prescription is interpreted more broadly.) Further, these singular field configurations give singular contributions from the collective field Jacobian which is naively down by $1/N$ (although it is expected that the singular higher order in $1/N$ terms in the Jacobian will be important in this case). Since the neglected terms from the Jacobian are non-linear in ϕ , they could either fix α or destroy the solution entirely. Neglecting the superpotential in (3.12), the bosonic sector is the theory considered by Jevicki [20]. He found a soliton solution with fixed coefficient corresponding to single eigenvalue motions. From the underlying matrix model we might speculate this is true in our case also. We have not found a transformation analogous to the one in [21] which takes the potential ($-x^2$ in that case) to zero and it is not clear how to consistently treat the subleading terms in the action if the potential is nonzero.

5. Supersymmetry Breaking

In the eigenvalue description we are doing the quantum mechanics of N discrete degrees of freedom. In refs. [19,9], the distribution of $N - 1$ eigenvalues was used as a background to find the action for the N th eigenvalue. In quantum mechanics, the presence of two ground states for the last eigenvalue implies the existence of an instanton tunneling from one minimum to the other, and the instanton effects give rise to a nonperturbative lifting of the ground state energy.

In the field theory we have a number of candidates for the classical vacuum field configuration in eq. (4.4) for varying values of α , and in eq. (4.2). Are there instantons in this theory which connect any of these putative ground states? One may study this question by expanding the action for the bosonic sector of the theory around the background field configuration ϕ_0 in (4.2). Since all the configurations described by eq. (4.4) match ϕ_0 to leading order in $1/N$ this background is a useful way to study ground states and instantons.

The instanton of [9] is described in our language by separating out a δ -function from ϕ as in (4.4) where now the position of the δ -function is time dependent. The leading order Lagrangian for the bosonic part of the theory is:

$$L = \frac{1}{2} \frac{(\partial^{-1} \dot{\phi})^2}{\phi} - \frac{1}{2} \left(W'(x) - \frac{1}{N} \int \frac{dy \phi(y)}{x-y} \right)^2 \phi. \quad (5.1)$$

This Lagrangian may have instantons connecting different ground states. The instanton equation, satisfied by a minimum of the action in Euclidean time, is:

$$\pm \frac{(\partial^{-1} \dot{\phi})}{\phi} = W'(x) - \frac{1}{N} \int \frac{dy \phi(y)}{x-y}, \quad (5.2)$$

and the instanton action is

$$S_{\text{inst}} = \int dx \phi(x) \left(W(x) - \frac{1}{2N} \int dy \phi(y) \ln |x-y| \right) \Bigg|_{t=-\infty}^{t=+\infty}. \quad (5.3)$$

In the approximation of neglecting back reaction, this gives rise to exactly the instanton of [9], but weighted by the factor α . To understand the effect of this field configuration on supersymmetry breaking in the collective field theory we have to understand how to treat this parameter.

6. Discussion

We have thus found the supersymmetry collective field theory description of the truncated Marinari Parisi model. It coincides to leading order in N with that discussed in [14]; the derivation here makes clear the link to the surface interpretation and also shows how the next order term in N from the Jacobian appears. The coefficient of the instanton at this level appears unfixed, although it may be determined by subleading terms in the superpotential, as happens when the potential vanishes. The larger instanton effects in the matrix model description of string theory [5] than that expected in field theory has been associated with the nonlocality and lack of translation invariance in the collective field action[20,22].

It would be interesting to make a connection to the spacetime description of this theory. There are many things known about the Marinari-Parisi model, but its spacetime interpretation is not one of them. Some observations were made in [9], mostly about the full supersymmetric model. Even the bosonic sector alone, corresponding to the first $c = 1$ higher multicritical theory, with an x^3 potential, has not been identified. The nonlocality in the bosonic sector of the Marinari-Parisi models with higher order superpotentials means they do not naively correspond to $c = 1$ multicritical points. In the $d = 1$ model, the fluctuations around the static ground configuration ϕ_0 to leading order in N describe a massless particle related to the massless tachyon of the theory [23,7]. For the cubic potential considered in this paper, the ‘tachyon’ is also massless, for higher order potentials

the nonlocality in (3.13) gives the fluctuations an effective mass at this order. An effective mass appears in subleading order in $1/N$ for all cases. Writing $\phi(x, t) = \phi_0(x) + \partial_x \eta(x, t)$ and keeping only second order in the fluctuations and leading order in $1/N$:

$$\begin{aligned}
L_{\text{fluctuations}} = \int dx \left[\frac{1}{2N^2} \frac{(\dot{\eta})^2}{\phi_0} - \frac{\pi^2}{2N^2} \phi_0(x) (\partial_x \eta(x))^2 \right. \\
+ \frac{1}{2N} \int dx dy \frac{W'(x) - W'(y)}{x - y} \partial_x \eta(x) \partial_y \eta(y) + \frac{i}{2} \int \frac{dx}{\phi_0} (\bar{\psi} \dot{\psi} - \dot{\bar{\psi}} \psi) \\
\left. - \frac{1}{N} W''(x) \frac{\bar{\psi}(x) \psi(x)}{\phi_0(x)} - \frac{1}{N^2} \bar{\psi}(x) \partial_x \int dy \frac{\psi(y)}{(x - y)} \right]
\end{aligned} \tag{6.1}$$

when $\phi_0(x) \neq 0$.

Much has been calculated for the bosonic higher multicritical local potentials, generalizing from $d = 1$, which can be compared with any suggested spacetime interpretation. The Hamiltonian can be rewritten in terms of fermi sea momenta [24], and again, most of the scattering takes place at the boundaries of x space. An infinite number of symmetry generators analogous to those found at $c = 1$ (before double scaling) [21,25] are present and presumably linked to the ground ring [26] structure in the matter plus gravity theory. The large order behavior in perturbation theory and some scaling exponents[11], the correlators of the operators M^n and the related analogues of the $c = 1$ discrete states [27,28,29] have all been calculated, in part using the Virasoro constraints [27,28] present.

One could try to deduce directly how the modifications of the $-x^2$ potential appear as modifications of $c = 1$. In the one matrix model, as n in the potential x^n increases, the value of $c_{\text{matter}} \rightarrow -\infty$ for the matter sector. For the two matrix model [30], taking the same criticality in the potential x^n for both matrices, c_{matter} increases as n increases. One suggestion [28] for these multicritical potentials, based on their Wheeler-DeWitt equation, is that the matter sector remains unchanged (leaving open the possibility of altering the standard ghost-Liouville mixing as in [31,32]).

The understanding of what matrix model instantons are in either the Liouville or spacetime background picture would give clues to how they appear in more general string backgrounds. For instance, the effective operator for the $c = 1$ instanton found in [20] should induce the phase shifts found in [33]. Representing the string instanton processes as effective operators in the theory defined around the usual ground state would allow a better study of consequences of these nonperturbative effects in string theory.

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Note added: After we submitted this paper for publication, we learned of the final version of [34], which uses a variable much like the collective field in the $d = 0$ matrix model. An important difference is that the analogue of the subleading term in the Jacobian is under more control and so can be used more reliably. It may be that the results for instantons found there carry over naturally to the case discussed in this paper.

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