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# A Study of Current Redistribution in an Aluminum Stabilized Superconductor

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## ABSTRACT

When superconductor switches from a superconducting to a normal state, the current will move out of the superconductor and redistribute itself in the stabilizer. To study this current diffusion phenomenon in the conductor, a separation of variables method is used to directly solve the current diffusion equation for a rectangular composite conductor. The average power generation over the conductor cross section is obtained based on calculated the current density profile. An excess power generation term is found during the transition period; it decays exponentially with the time t. Finally, a comparison made between this approach and Devred solution shows a good agreement .

**Keywords:** superconductors; stability; mathematical models

## INTRODUCTION

When a superconductor switches from a superconducting to a normal state, the current will move out of the superconductor and redistribute itself in the stabilizer. An excess Joule heat will be produced during this transition period of current diffusion. Devred[1] has studied this problem by using the magnetic field diffusion equation to derive the current density profile for a rectangular composite conductor. Luongo, Loya and Chang[2] did a similar analysis by first solving the magnetic field equation, and then taking the derivative of B with respect to the direction of diffusion to get the current density. In this study, the cross-sectional area of conductor is assumed to be only aluminum by neglecting the copper-superconducting composite because the area ratio between these two materials is about 15. A separation of variables method is used to directly solve for the current diffusion equation with the boundary conditions appropriate for a rectangular composite conductor. The average power generation per unit volume as function of time has been calculated by integrating $J^2(x,t)\,\rho$ over the cross-sectional area of the conductor. The energy generated per unit volume by the current is obtained by integrating the power curve with respect to time.

## COMPUTATIONAL METHOD

### Determination of the Current Density Profile

The conductor used in this analysis is shown in Figure 1. It is assumed that the current is diffusing only along x axis. At time t=0, the current density is known since all the current flows in the very small area As, which is equal to the area of the superconducting composite. The integration of the current density over the cross-sectional area of conductor should be equal to the current I at any time t. A symmetry condition also gives a zero value of the first derivative of the current density with respected to x. The current density diffusion is therefore

$$\frac{\partial^2 J(x,t)}{\partial x^2} = \frac{1}{D_m} \cdot \frac{\partial J(x.t)}{\partial t} \tag{1}$$

with the boundary conditions

$$\frac{\partial J}{\partial x}(x=0,t) = 0 \tag{2}$$

$$\int_A J \cdot dA = I \tag{3}$$

and the initial condition

$$J(x,t=0)=\begin{cases} J_0 & 0 \le x \le d \\ 0 & d < x < L \end{cases} \qquad (4)$$

where , $D_m$ is the magnetic diffusivity which is equal to $\rho/\mu_o$, and $J_o = I/As$.

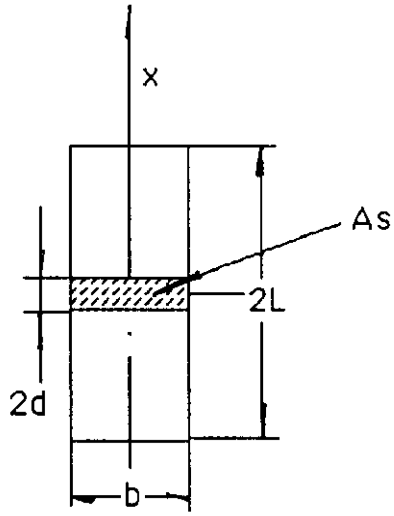


Figure 1 Assumed conductor cross section

Equation 1 with the boundary conditions given by Eq. 2 and 3 can not be immediately solved by a separation of variables because of the nonhomogeneous boundary condition given in Eq. 3. Therefore, Eq. 1 was separated into a partial differential equation with a homogeneous boundary condition, and an ordinary differential equation with a nonhomogeneous boundary condition. A solution of Eq. 1 of the form

$$J(x,t) = v(x,t) + F(x) \qquad (5)$$

was assumed. Substituting Eq. 5 into the Eq. 1 yields

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} = \frac{1}{D_m} \cdot \frac{\partial v}{\partial t} \qquad (6)$$

with a boundary and initial condition given as

$$\frac{\partial J}{\partial x}(x=0) = (\frac{\partial v}{\partial x} + \frac{\partial F}{\partial x})\bigg|_{x=0} = 0 \tag{7}$$

$$\int_A J \cdot dA = \int_A v(x,t) \cdot dA + \int_A F(x) \cdot dA = I \tag{8}$$

$$J(x,t=0) = v(x,t=0) + F(x) \tag{9}$$

The solution of F(x) is given by

$$F(x)=Ax+B \tag{10}$$

where the constant A and B are determined from

$$(\frac{\partial F}{\partial x})\bigg|_{x=0} = A = 0 \tag{11}$$

$$\int_A F(x) \cdot dA = 2\int_0^L B \cdot bdx = 2BbL = I \tag{12}$$

Therefore, Eq. 10 may be written

$$F=I/(2bL) \tag{13}$$

To solve for v(x,t), assuming that v(x,t) can be written as the product of by two functions X(x) and T(t),

$$v(x,t)=X(x)T(t) \tag{14}$$

The boundary condition can be obtained from Eq. 7 and 8,

$$(\frac{\partial v}{\partial x})\bigg|_{x=0} = X'(0) \cdot T(t) = 0 \tag{15}$$

$$\int_A \mathrm{v(x,t) \cdot dA} = 2\int_0^L \mathrm{X(x) \cdot T(t) \cdot bdx} = 0 \tag{16}$$

However,

$$T(t) \neq 0 \tag{17}$$

and

$$2 \cdot T(t) \cdot b \neq 0 \tag{18}$$

for all values of t. Therefore,

$$X'(0) = 0 \tag{19}$$

$$\int_0^L X(x) \cdot dx = 0 \tag{20}$$

Now, the general solution for Eq.14 for the space variable x is

$$X = C1 \cdot \sin\beta x + C2 \cdot \cos\beta x \tag{21}$$

where C1 and C2 are constants. From Eq. 19 and 20, it gives C1=0, and

$$\int_0^L X(x) \cdot dx = \int_0^L C2 \cdot \cos\beta x dx = \frac{C2}{\beta}\sin\beta x\Big|_0^L = \frac{C2}{\beta}\sin\beta L = 0 \tag{22}$$

For a nontrivial solution, C2≠0, and

$$\sin\beta \cdot L = 0 \tag{23}$$

This equation is satisfied when

$$\beta_n = \frac{n \cdot \pi}{L} \qquad for \text{ n} = 1,2,3 \;.... \tag{24}$$

Substituting these eigenvalues into Eq. 21,

$$X_n = C_n \cdot \cos\beta_n X \tag{25}$$

Next, solution of Eq.14 for time variable t is

$$T_n(t) = F_n \cdot e^{-D_m\beta_n t} \tag{26}$$

Finally, v(x,t) may be written as the product of Eqs.25 and 26

$$v(x,t) = \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi}{L}x \cdot e^{-D_m\beta_n^2 t} \tag{27}$$

where $a_n = C_n F_n$ is an arbitrary constant, which is determined by applying the initial condition given by Eq.9

$$v(x,t=0) = \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi}{L}x = J(x,t=0) - F(x) \tag{28}$$

Multiplying both sides of Eq.28 by cos(mπ/L) and integrating from 0 to L, $a_n$ is given

$$a_n = \frac{2J_0}{n\pi} \cdot \sin\frac{n\pi}{L} \cdot d \tag{29}$$

Substituting Eq. 29 into Eq. 27, v(x,t) is found to be

$$v(x,t) = \sum_{n=1}^{\infty} \frac{2J_0}{n\pi} \cdot \sin(\frac{n\pi}{L} d) \cdot \cos\frac{n\pi}{L} x \cdot e^{-D_m \beta_n^2 t} \tag{30}$$

and therefore, the complete solution of Eq. 1 is

$$J(x,t) = \frac{I}{b \cdot 2d} \cdot \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \sin(\frac{n\pi}{L} d) \cdot \cos\frac{n\pi}{L} x \cdot e^{-D_m \beta_n^2 t} + \frac{I}{b \cdot 2L} \tag{31}$$

The current density J(x,t), shown above, consists of two terms. The first term decays exponentially with time t, which characterizes the current diffusion transition period. The second term is the current density at the end of current diffusion. Integrating Eq.31 over the cross-sectional area of the conductor is the current I at any time t.

Taking L=22 mm, b=4.37 mm, d=1.47 mm, and $\rho=2.51\times10^{-8}$ Ω-mm, $\mu_0=4\pi10^{-7}$ Hm, the current density as a function of position for different times can be calculated, Figure 2. At time t=0, the current density is nonzero over a very small area as expected. At time t > 0, the current diffuses into the stabilizer. After approximately 6 seconds, a steady state is reached and the current density is almost constant across the stabilizer. To verify this method, the conductor geometry used by Devred was used to calculate the current profile form Eq.31. The result of the two methods are shown in Figure 3; the agreement is very good.

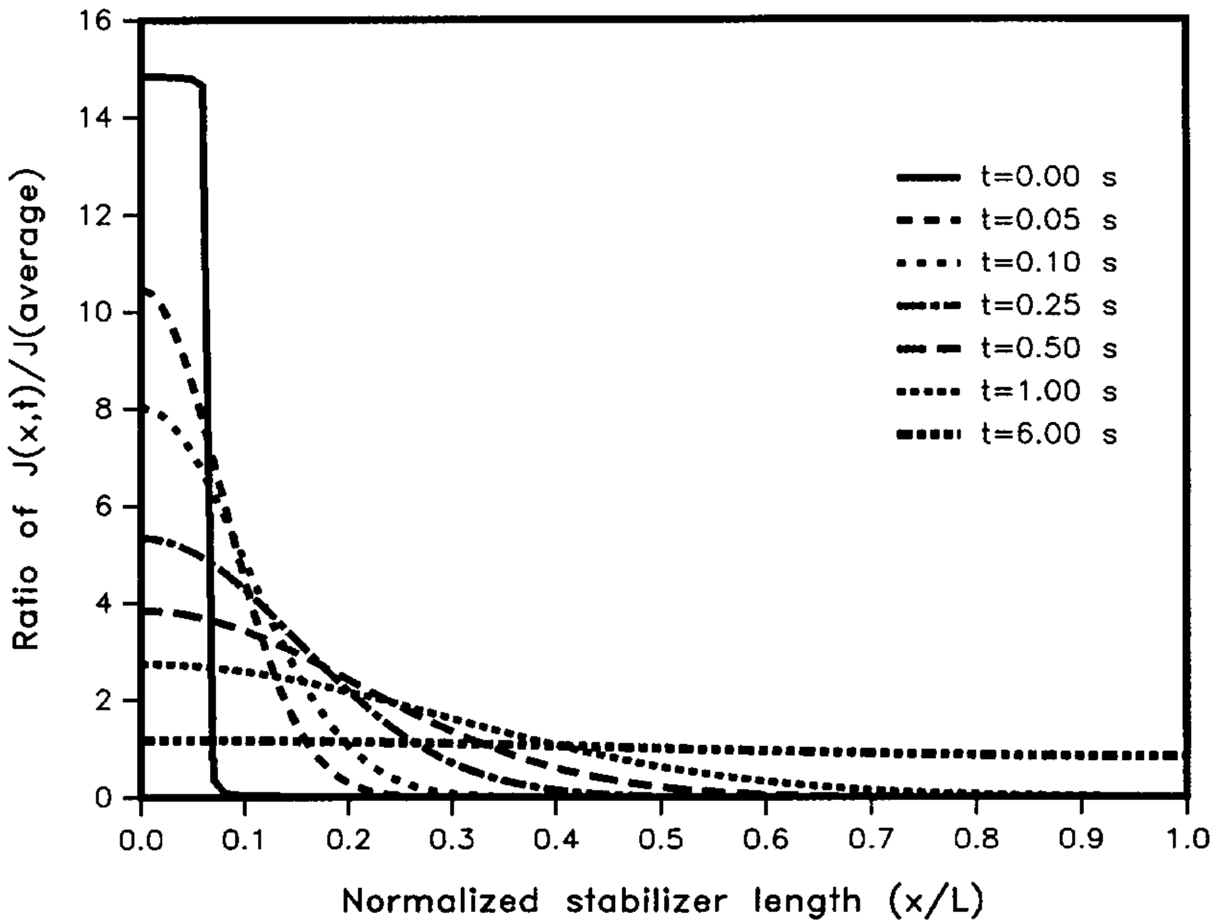


Figure 2 Relative current density profile. J(steady state)=I/(2Lb)

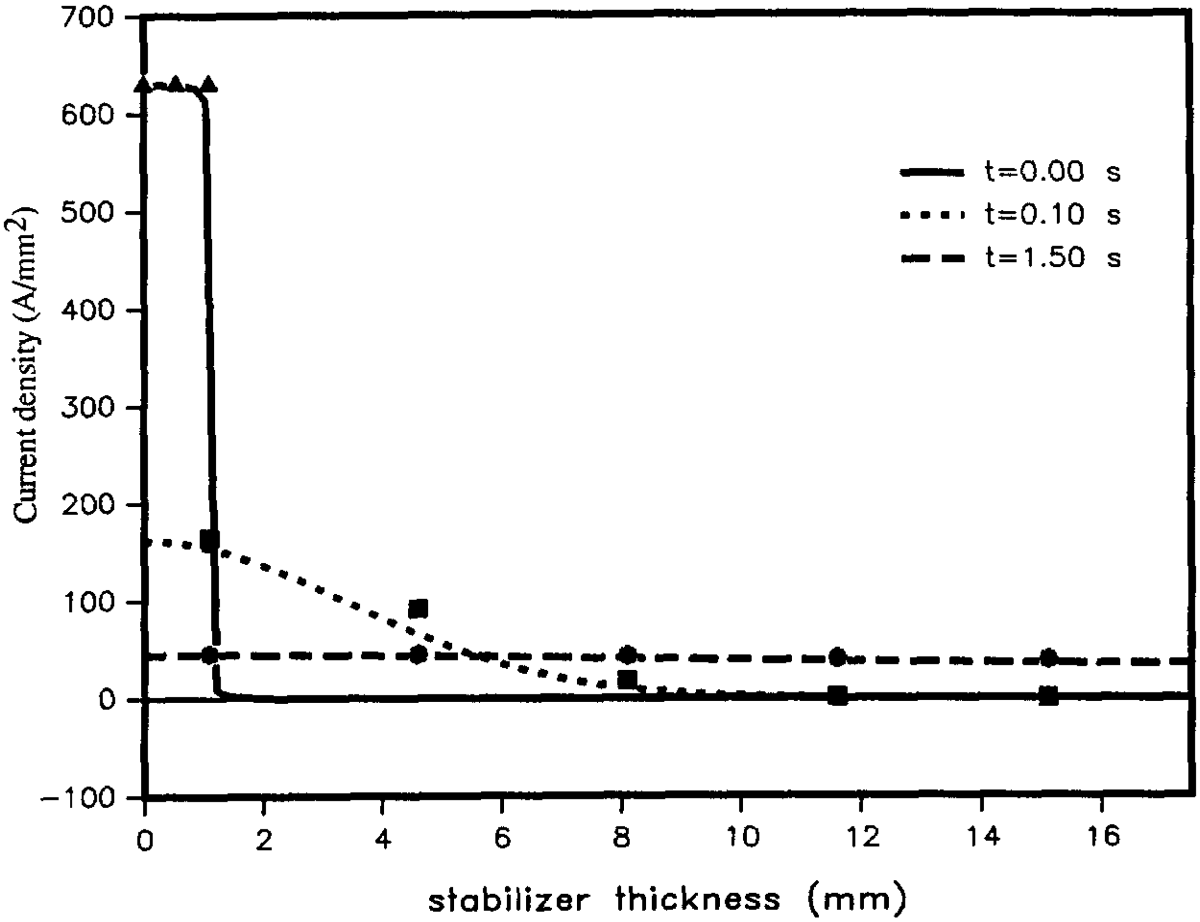


Figure 3 Comparison between the results obtained with Eq. 31 and Devred's method;

▲ at t=0 s, ■ t=0.1 s, ● t=1.5 s.

**Determination of the Power Generation**

The power generated per unit volume can be calculated as

$$q(x,t) = J^2(x,t)\cdot\rho \tag{32}$$

The q(x,t) averaged over the conductor

$$q_a(t) = \frac{1}{A}\cdot\int_A q\cdot dA = \frac{1}{L}\cdot\int_0^L J^2(x,t)\cdot\rho\cdot dx \tag{33}$$

By substituting the current density from Eq.31 into Eq.33,

$$q_a(t) = \frac{\rho}{L}\left\{\int_0^L v^2(x,t)\cdot dx + \int_0^L 2v(x,t)\frac{I}{2bL}\cdot dx + \int_0^L (\frac{I}{2bL})^2\cdot dx\right\} \tag{34}$$

The first term in parenthesis is calculated from,

$$\int_0^L v^2(x,t)\cdot dx = \int_0^L \{(v_1+v_2+\cdots+v_n)\cdot(v_1+v_2+\cdots+v_n)\}\cdot dx = \sum_{n=1}^{\infty}(\int_0^L v_n^2\cdot dx) \tag{35}$$

since the integration of $v_m v_n$ from 0 to L is zero for $m \neq n$. Equation 35 can now be easily integrated term by term

$$\sum_{n=1}^{\infty}(\int_0^L v_n^2\cdot dx) = \left(\frac{I}{b\cdot 2d}\right)^2\cdot\left(\frac{L}{2}\right)\cdot\sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\cdot\sin(\frac{n\pi}{L}d)\cdot e^{-D_m\beta_n^2 t}\right)^2 \tag{36}$$

The second term in parenthesis in Eq.34 is equal to 0 and last term is equal to $(I/2bL)^2L$

The average power generated over the cross-sectional area of conductor becomes

$$q_a(t) = \frac{\rho\cdot I^2}{2\cdot(b\cdot 2d)^2}\cdot\sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\cdot\sin(\frac{n\pi}{L}d)\cdot e^{-D_m\beta_n^2 t}\right)^2 + \frac{\rho\cdot I^2}{(b\cdot 2L)^2} \tag{37}$$

Equation 37 also has two terms. The first term is an excess power generated during the transition period, which exponentially decays with time. The second term is the value when steady state is achieved, or when the current is uniformly diffused across the stabilizer. By using the same dimensions as before ( L=22 mm, b=4.37 mm, d=1.47 mm ), the power generation as a function of time t was calculated and is shown in Figure 4. It can been seen that the power generation is initially very high compared with the steady state value. As the current diffuses into the adjacent stabilizer, the power generation decreases. After approximately 3 seconds, the excess power term has nearly decayed to zero and a steady state value has been reached.

### Determination of the Excess Energy

It is of great interest to estimate the amount of excess energy produced during the transition period; this is done by integrating the power over the transition time interval. Integrating the first term of Eq.37 from t=0 to t=3 s, and comparing it to the second term of same equation integrated over the same time interval, it is found that the excess energy is approximately 50% higher than the steady state energy for this specific conductor geometry. The excess energy may have a big effect on the hot spot temperature and the normal zone propagation velocity. A similar calculation has been performed by using the Devred's geometry; the comparison is shown in Table 1 and Figure 5.

Table 1. Comparison of Excess Energy

| Method | Excess Energy (J) | Difference |
|---|---|---|
| This paper, Eq.37 | 88661 | - |
| Devred | 89233 | +0.6% |

## CONCLUSION

By using a separation of variables technique, the current diffusion equation is directly solved to estimate the excess energy during the diffusion period rather than solving the B diffusion equation. It is found that the difference between the two approaches is negligible. It is justified that the assumption of neglecting copper-supercoducting composite is a reasonable assumption for the case of the conductor with a large aluminum stabilizer. In such case, the method presented in this paper avoids the calculation of the inverse Laplace transform, as well as the numerical solution for the eigenvalues. It is simple and easy to use.

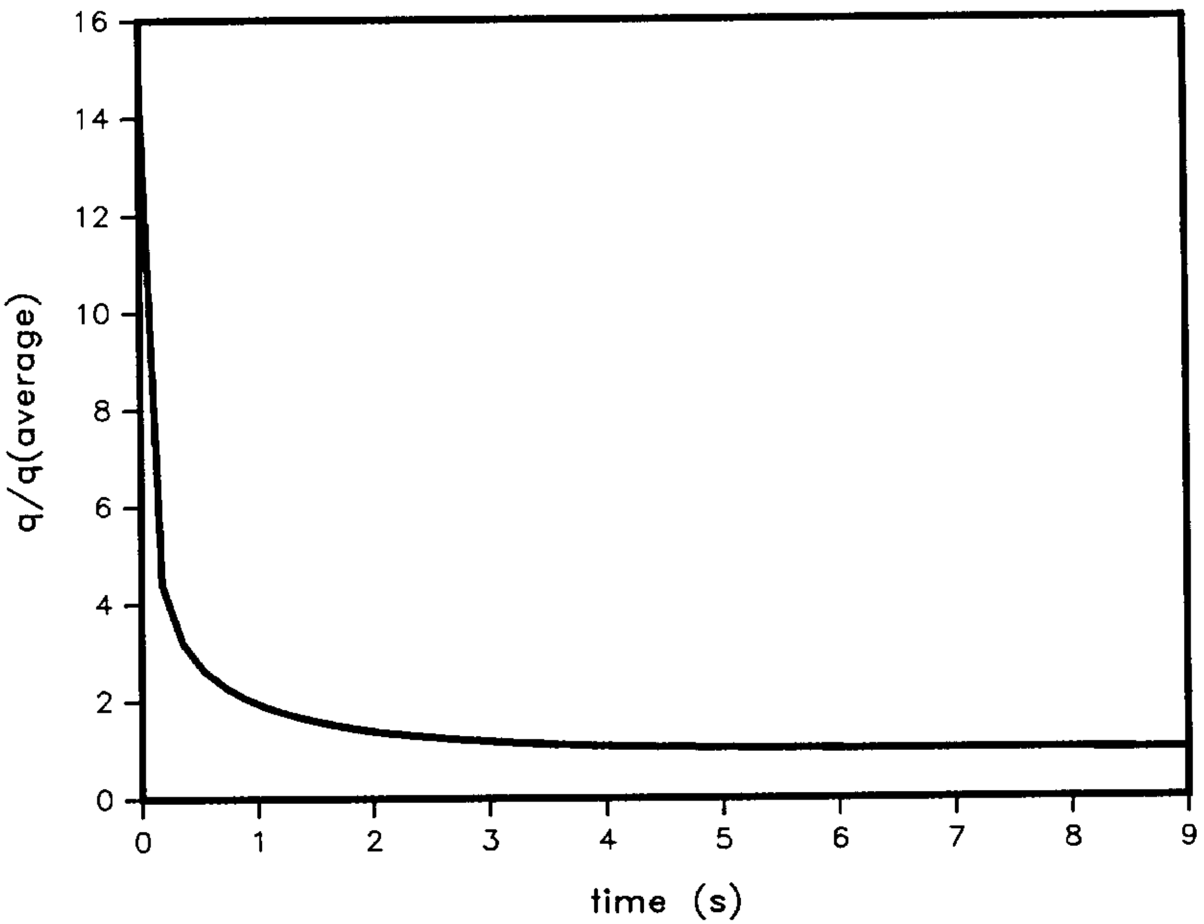


Figure 4 Relative power generation as a function of the time for a L=22 mm, b=4.37 mm, d=1.47 mm aluminum stabilized conductor; q(steady state)=$J^2$(steady state)$\rho$.

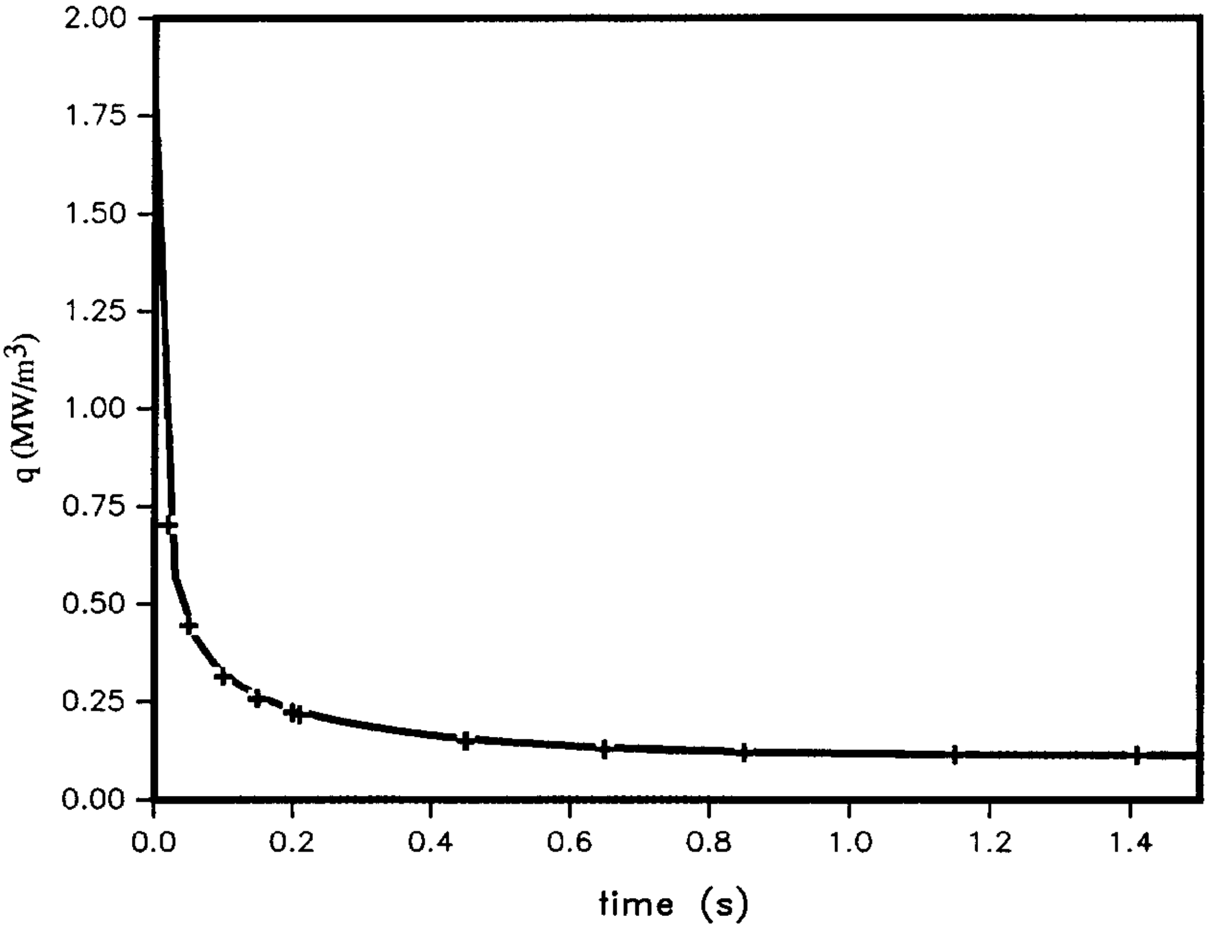


Figure 5 Comparison of power generation calculated using Eq. 37 ——, and Devred's method, +.

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