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FRACTALS AND COSMOLOGICAL LARGE SCALE STRUCTURE *

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ABSTRACT

It is known that observations of galaxy-galaxy and cluster-cluster correlations as well as other large scale structure observations can be fit with a "limited" fractal, with dimension D~1.2. This should not be over interpreted since it is not a "pure" fractal out to the horizon, the distribution shifts from power law to random behavior at some large scale. In fact the fractal correlation must break down beyond $\sim$ 200 Mpc to be consistent with the cosmic microwave background isotropy limit. In this paper it is shown that if the patterns and structures are formed through an aggregration growth process, the fractal dimension D can serve as an interesting constraint on the properties of the stochastic motion responsible for the limited fractal structure. In particular it's found that the observed fractal should have grown from 2-dimensional sheet like objects. This result is generic and doesn't depend on the details of the growth process. These fractal arguments can be related to seed formation processes. Topological remnants such as soft walls from late-time phase transitions or wakes of strings, or to early pancaking or to shells from explosions, all work well but the argument appears more awkward for some traditional hierarchical models starting with random gaussian seeds.

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The establishment of the galaxy-galaxy correlation function $\xi_{gg} \sim (r)^{-1.8}$, for r up to $\sim$10 Mpc, marked the beginning of quantitative attempts to understand the large scale structure of universe *(1)*. Later on, the correlation function for clusters of galaxies appeared to have a similar behavior but with higher amplitude *(2, 3)*. Initial worries about projection effects biasing the results have been minimized somewhat by the result of West and van den Bergh *(4)* for cD galaxies and Lahav *et. al (5)* for X-ray clusters which shows the same behavior as the clusters. (cD's are associated with the core of rich clusters.) Several years ago, Szalay and Schramm *(6)* showed that the correlation functions could be written in a unified way by using a dimensionless variable r/L, where L is the average seperation of objects in the catalogue being examined, $\xi(r) = \beta(L)(r/L)^{-1.8}$, they found $\beta \approx 0.35$ is a constant for all clusters of galaxies and is unity for galaxies. The slightly larger correlation for the galaxies in this scale free approach is probably an indication of gravitational clustering. The near constant behavior of β for clusters indicates that the clustering process may be roughly scale invariant, in other words the structure is a fractal. Of course it's not a true fractal that maintains a power law behavior to infinite scale. Statements about a so-called "fractal universe" are excessive. At scales $\gtrsim$ 100 Mpc, the data is sufficiently poor that the power law correlation is not evident and at very large scales we know that the universe is isotropic and not fractal from the microwave observations and the relatively smooth distribution of objects on large scales *(7)*. Thus, at best the fractal is a limited fractal. It is interesting that as the sampling of the universe gets larger and deeper, more observations appear to continue to support this limited fractal hypothesis. Following Bahcall and Chokshi *(9)*, Figure 1 shows a summary of the current situation and our error bar estimates with data points for correlation of superclusters *(8)*, quasars *(9)*, X-ray clusters *(5)* and the cD's at the center of superclusters *(4)* as well as recent work by Efstathiou et.al *(10)* using the APM survey has supported this basic clustering behavior. The figure shows that for $10 \lesssim L \gtrsim 100 Mpc/h, \beta(L)$ is near constant with the current best fit value $\beta \sim 0.26$. Note that a power law correlation function with index 1.8 corresponds in 3 dimensional space to a fractal with a fractal dimension D=1.2.

Obviously, the following questions arise when we discuss the possible fractal structures in the universe: how far out does the fractal correlation extend? what can we learn from the fractal dimension D=1.2? and what physical process can give rise to a fractal structure in the distribution of observable objects? At present these questions don't have unambiguous answers. Much discussion of fractal large scale cosmological structure has either tried to assume a pure fractal structure *(11, 12)* or to emphasize how a pure fractal cannot explain the structure of the universe because of the isotropy of the microwave radiation and realtively uniform distribution of objects at large distances *(7)*. A point that can be lost in such discussion is that if the universe is fractal-like for some range of scale then some insight might be gained by looking at how such fractals can develop even though the fractal gets truncated.

If the fractal behavior is real, gravity alone has difficulty in explaining it because the clustering amplitude of clusters would then not be higher than that of galaxies. While some form of biasing *(13)* may be useful here, we will instead see if accepting the fractal interpretation offers any useful insights. In particular let us assume some sort of fractal seed or growth process to provide the fractal correlation while gravity plays the role of enhancing correlation amplitude on small scales. We will find that applying fractal analysis techniques to large scale matter distribution in the universe yields some interesting results.

There are two basic requirements to form large scale structure: (1) primordial seeds or fluctuations (density perturbations) and (2) the aggregation of matter to the seed (growth process). The correlation of seeds or density perturbations and the scaling behavior of growth process are all responsible for any fractal structure we observe today, and it is interesting to find that most structure formation theories can be fitted in the catagory of emphasizing one or the other. In the continuous clustering model *(14)*, especially the variant of Mandelbrot *(11)* where galaxies are placed on each step of a Levy flight *(11)*. The correlation between seeds is fully responsible for the fractal distribution of observed objects. The model is simple and successful in reproducing the observed correlation functions. For the Mandelbrot model the fractal dimension D enters the program through the ansatz of the probability distribution of Levy flight, for a random walk with stepsize l, $P(l) = 0$ for $l < l_0$; and $P(l) = Dl_0^D/l^{D+1}$ for $l > l_0$ where D is the fractal dimension. Thus the model is more an emperical computational device rather than a true physically motivated growth process. It also has the problem of no natural truncation of the fractal at large scales. On the other hand, in the random gaussion fluctuation model *(15)*, the seeds are randomly distributed in a gaussian manner. If the amplitude of the fluctuations is scale invariant, the model is able to to reproduce the two-point corelation on on small scale ($\lesssim 10Mpc$). When the scale gets larger, some problems appear as illustrated by the excess power observed on larger scales relative to the fall off in the model *(10)*. (If biasing is invoked to fit the cluster correlations from the galaxy correlations, then the cluster correlation function is directly proportional to the galaxy correlation function. If the galaxy function should unambigously be nagative, then so should the cluster function on that scale. As of this time, the data is not unambiguously enough to make this test.)

Up till now, little attention has been paid to the scaling of the aggregation of matter to seeds beyond linear gravitational perturbation theory. In general matter undergoes a stochastic motion in space until it is gravitationally bound by seeds to form clumps and the growth rate of the clump is controlled by the diffusing flux of matter onto the seed. We note here that the underlying physics of this kind of growth process can be modelled by DLA (diffusion limitted aggragation) and studies on the model show that the resulting aggragate has a well-defined scaling behavior *(16)*. In the growth of a tranditional DLA fractal, the interaction of the diffusing particles with the aggregate is short-ranged and the aggregate doesn't grow until the diffusing particles get attached, so the aggregate is connected. In the cosmological case, because the gravity is long-ranged, the star-cluster is more loosely bounded. In this paper we don't intend to go into the details of a particular growth model but rather to show that based on the growth process, the fractal dimension D can serve as an interesting constraint on the growth space, which is defined as the possible trajectories of the stochastic motion of matter clumps. The overall space is 3-dimensional, but the stochastic motion is not necessarily 3-dimensional.

The aggregate grows by absorbing particles which are randomly moving in $d-$ dimensional growth space, the outer raduis R of the aggregate grows with time, but dR/dt is limited by some value v. which is proportion to the density u of moving particles, because of the "shadow" effect which parts of a cluster begin to block the interior sites. In our case, "shadow" effect also occures for different reason - when the material is used up, the sites adjacent to "void" cannot grow. So, $dR/dt < v \sim u$. dR/dt is related to the change of mass $M(\sim R^D)$ of the aggregate by $\frac{dR}{dt} = (\frac{dM}{dt})/\frac{dM}{dR}$. dM/dt is also the rate which the diffuing particles are first bounded by the aggregate:

$$\frac{dM}{dt} \sim u \cdot R^{d-2}. \tag{1}$$

So

$$\frac{dR}{dt} \sim uR^{d-1-D} < v \sim u, \tag{2}$$

thus

$$d - 1 - D \leq 0, d \leq 1 + D, \tag{3}$$

or

$$D \geq d - 1 \tag{4}$$

This is the causality bound on the fractal grown from a diffusion limited process *(17)*. The observed fractal dimension $D = 1.2$ implies that the dimension d of the growth space is less than 2.2. In other words the growth space should involve 2-dimensional sheet-like object. This fact can constrain the properties of topological defects which might serve as seeds for large scale structure. This result favors the light domain wall *(18)*, wakes of string *(19)*, superconducting string (the explosive model) *(20)*, the pancake model *(21)* or collapsing textures *(22)*. (Of course it say nothing about other problems these model may have such as the microwave background $y-$parameter constraint on the explosive models etc.)

One consequence of embedding a fractal structure generation mechanisam into the well established Big Bang framework *(7)* is the prediction that the fractal correlation should break down at some scale. As pointed earlier by Peeble *(1)*, a pure fractal contradicts the observed large scale angular correlation function. It also has problems with microwave background isotropy. Since the growth process is limited by the diffusion of particles onto the aggregate, it can drop below the expansion rate of the universe. Futhermore, when the random motion of the matter is not constrained the growth will be 3-dimensional. From equation (4) we know that it is impossible to grow a $D = 1.2$ pure fractal in 3-dimensions via any kinematic growth process. The breakdown scale of the fractal correlation can be estimated from the constraint of Microwave background anistropy $\delta T/T \lesssim 10^{-5}$ in the extreme case of a sheet-like seed model, say light domain walls from a late-time phase transition. In the late-time phase transition scenario, the cosmological seed and density pertubation is generated after the decoupling of microwave background which minimizes the CBR anistropy *(18, 23)*. To grow a fractal extend to scale L, the aggregration of matter onto the seeds will perturb the cosmic microwave background *(24)*,

$$\delta T/T \sim (9/32)(3\pi)^{1/2}(H_0 L)^3 \Omega_{wall}, \tag{5}$$

and the density perturbation $\delta\rho/\rho$ induced by a wall is estimated to be

$$\delta\rho/\rho \sim (3\pi^2/20)\Omega_{wall}. \tag{6}$$

. The fractal growth process can only proceed when $\delta\rho/\rho > 1$, or $\delta T/T \gtrsim (H_0 L)^3$. So $\delta T/T < 10^{-5}$ implies $L \lesssim 100/hMpc$. This is a natural result because the horizon size at the time of structure formation serves as a cutoff to the fractal correlation. The horizon size R at the time of a late-time phase transition ($z \sim 1000$), $R \sim 3000Mpc/h/\sqrt{1+z} \sim 100/hMpc \sim 200Mpc$ with Hubble constant $h = H_0/(100km/sec/Mpc) = 0.5$. This agrees reasonably well with the previous argument.

In summary we have discussed the large scale correlations as a result of generic growth process and found two preliminary results. Namely the dimension of the allowed stochastic motion which can yield a limited fractal correlation and the break down scale of the fractal correlation scale, are obtained. It is possible that the detailed growth process may be traced back to the recombination epoch where neutral hydrogen is grown from electron-proton plasma, which we will persue in a seperate paper.

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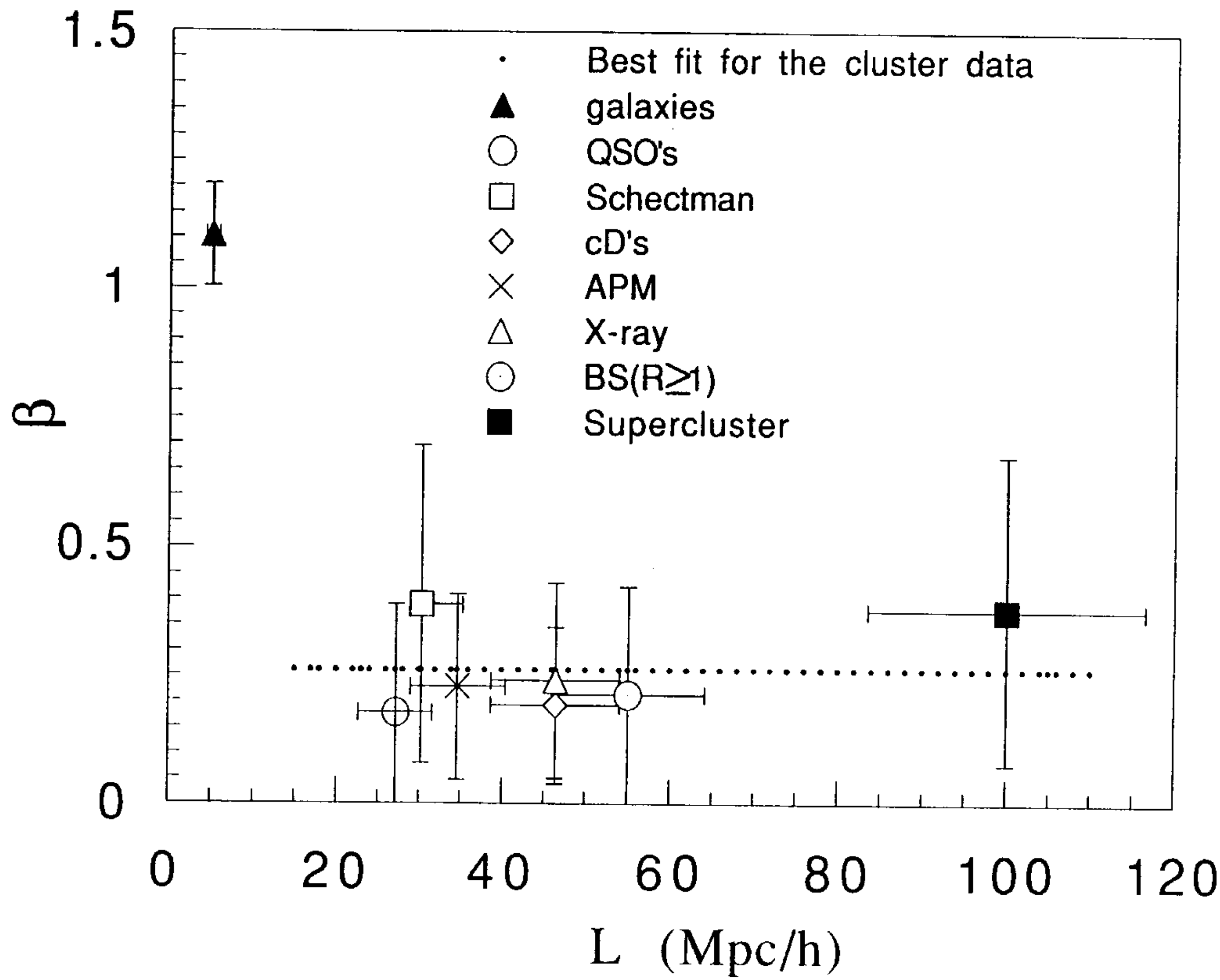
Figure Captions

Fig. 1: The two-point correlation function is written in the scale invariant form: $\xi(r) = \beta(r/L)^{-1.8}$, where $L = n^{-1/3}$ is the mean distance between objects a catalogue, n is the mean density, β is the dimensionless correlation amplitude. The best fit to the updated observational data gives $\beta \approx 0.26$. The error-bars represent $\pm 50\%$ uncertainty in the density, $\pm 50\%$ uncertainty in the correlation amplitude and 20% uncertainty in determining the power-law index.



1.5
1
0.5
0
β
Best fit for the cluster data
galaxies
QSO's
Schectman
cD's
APM
X-ray
BS(R≥1)
Supercluster
0
20
40
60
80
100
120
L (Mpc/h)