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HETEROTIC POMERON: A UNIFIED TREATMENT OF HIGH ENERGY HADRONIC COLLISIONS IN QCD^{*}

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ABSTRACT

A unified treatment of high energy collisions in QCD is presented. Using a probabilistic approach, we incorporate both perturbative (hard) and non-perturbative (soft) components in a consistent fashion, leading to a "Heterotic Pomeron".

1. Introduction

One of the most striking aspects of high-energy hadron-hadron scattering is the continued increase of the total cross section σ_T with the energy. There are currently two seemingly conflicting approaches to high energy hadronic collisions in QCD,^[1-3] as summarized in Table-I. We would like to focus in this paper on the following questions: How can qualitative features of rising σ_T be related to aspects of QCD? Instead of treating rising σ_T as an isolated phenomenon, can a simultaneous description of the elastic and the inelastic production be achieved by incorporating both perturbative and nonperturbative aspects of QCD?^[4]

It is well understood that the character of QCD changes depending on the nature of available probes. At short distances, the basic degrees of freedom are quarks and gluons. Collisions involving large momentum transfers, "hard" collisions, can be understood in terms of exchanges of quarks and gluons. There has been much recent discussions on the idea of "semi-hard processes" which could account for a large part of the total cross section at collider energies, with the energy dependence of various cross sections explained by perturbative QCD motivated calculations. One usually justifies this approach by appealing to the work of the "Leningrad" Group.^[1] In such a scheme, a rising total cross section is achieved by having a "hard" Pomeron singularity, the Lipatov Pomeron, above $J = 1$.

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An equally compelling argument can also be given in which the dynamical origin of the increasing total cross sections lies in “soft” hadronic physics. As one moves to larger distance scales, the QCD coupling increases and one enters the non-perturbative regime. The most promising analytic tool for a non-perturbative treatment of QCD which builds in naturally quark-gluon confinement remains the large- N expansion. In this scheme, model studies suggest that the effective degrees of freedom of QCD can most profitably be expressed in terms of “extended objects”. Indeed, low-lying hadron spectrum suggests that they can be understood as “string excitations”. In high-energy soft hadronic collisions^[6] where the interactions are mostly peripheral, it is possible to “see” the dominant excitation in terms of the exchanges of a soft Pomeron pole.^[3] A successful phenomenological model of this type, particularly for describing production processes, is the Dual Parton Model (DPM)^[2]. A rising total cross section also requires having a soft Pomeron above $J = 1$ in the forward limit, as indicated in the last column in Table-I.

Rather than treating perturbative and non-perturbative frameworks as diametrically opposite, we identify key features of each which allow a unified treatment of high energy hadronic collision in QCD. One of the main puzzles for our current understanding of high energy hadronic collisions in QCD is the relation, if any, between the perturbative (Lipatov) Pomeron and the non-perturbative (soft) Pomeron.^[6] This is precisely the subject of this talk. We begin by briefly reviewing features of Lipatov an soft Pomerons, both their differences and similarities. We next show how the key features of each can be incorporated in a unified treatment leading to a “Heterotic Pomeron”.

Table I. Two Conflicting Pictures for QCD at High Energies:

Framework:	Perturbative QCD	Non-Perturbative QCD
Emphasis:	Hard gluons, quarks	Topological expansion, <i>etc.</i>
Graphs:	Hard-gluon ladders in LLA	Soft ladders in $1/N$ exp., cylinder topology
Standard Models:	Leningrad model ^[1]	Dual Parton Model ^[2]
Vacuum Exchanges:	Hard Pomeron: $\alpha_L(0) \equiv 1 + \Delta_L > 1$	Soft Pomeron: $\alpha_0(0) \equiv 1 + \Delta_0 > 1$
Total cross sections:	Increasing	Increasing
Virtuality:	Increasing q_T^2	q_T^2 fixed and small
t-dependence:	$\alpha_L(t) \sim t$ -independent	$\alpha_0'(0) \sim 0.2 \text{ GeV}^{-2}$

2. Different Faces of Pomeron in QCD

In spite of their apparently different dynamical origins, the hard and the soft Pomerons share a structural similarity since they are both generated by summing ladder graphs. In Pomeron, one deals with the hard gluon ladders, whereas the soft Pomeron involves multiperipheral ladders. In a ladder sum, one encounters amplitudes which at high energy satisfy a recursion relation

$$\mathcal{A}_n(p, p_0; Q) = \int d^4 p' K(p, p'; Q) \mathcal{A}_{n-1}(p', p_0; Q), \quad (1)$$

where $\mathcal{A}_n(p, p_0; Q)$ corresponds to the n -rung contribution to the absorptive part of a non-forward two-to-two amplitude, $(p+Q/2)+(p_0-Q/2) \rightarrow (p-Q/2)+(p_0+Q/2)$. Let $s \equiv (p+p_0)^2$ and $t \equiv Q^2$, at high energies where $s \gg |t|$, the sum $\sum_n \mathcal{A}_n(p, p_0; Q)$ can be shown to be power-behaved in s , leading to the respective Pomerons, whose properties are summarized in Table-I.

Although both pictures can readily lead to increasing total cross sections,^{*} they exhibit

* We emphasize that, with a Pomeron intercept greater than 1, (either the hard or the soft), the single-exchange contribution alone would violate unitarity at sufficiently high energies and screening corrections must now be taken into account. It is reasonable to assume that this can be carried out within the context of a Reggeon field theory. If the triple-Pomeron coupling is small, a generalized eikonal mechanism becomes operative, leading to an expanding disk picture for the total cross section. Of course, such a representation can at best be approximate, possibly meaningful for a limited range of energies, and it most likely would break down at asymptotic energies. Our concern here is on the nature of the Pomeron itself and we do not want to address the question on the precise nature of the screening mechanism. For simplicity, we shall assume that, at for the current available collider energies, an eikonal representation is indeed operating.

other distinctive features. Two most important one are: (i) The production mechanism for a hard Pomeron leads to an increase in “virtuality”, *i.e.*, the average transverse momentum squared, $\langle q_T^2 \rangle$ increases with $\log s$, whereas a soft Pomeron exchange corresponds to processes with limited $\langle q_T^2 \rangle$. (ii) The soft Pomeron has a relatively large slope at $t = 0$, whereas the hard Pomeron has a much weaker t -dependence.[†]

These differences could in principle allow one to decide experimentally which one of these two approaches is more appropriate phenomenologically. Unfortunately, clear cut experimental tests do not exist. In fact, one finds that it is possible that an additive “two-component” picture actually work well phenomenologically. However, it should be stressed that a simple additive approach is in principle wrong, since these two components must be coupled through unitarity. It is conceivable that the interference effects between them would enter only after absorptive corrections have been taken into account, so that it is meaningful to treat them additively at the level of “eikonal”.

We emphasize that, most significantly, a “two-gluon ladder” has the topology of a “cylinder” in the color-space, it therefore survives in the leading large- N limit. Since the soft-Pomeron is also supposed to represent the effective ladder graphs emerging from the cylinder graphs, it suggests that, instead of simply adding, they should be treated as different manifestations of a more general “structure” which truly represents the asymptotic behavior of the cylinder graphs in QCD. We therefore would follow the following strategy for unification:

- (A) Identify “distinguishing” features of each approach, putting aside quantitative questions^[6] such as the precise value for the “Lipatov Pomeron intercept”,
- (B) Provide a “consistent framework” which unifies the key features of each scheme.

We carry this out next by introducing a probabilistic model.

3. Key Features and Random Walks

Let us return to the recursion relation, Eq. (1). At high energies where $s \gg |t|$, p and p_0 can be decomposed into longitudinal and transverse components. Whereas the longitudinal components determine s , their transverse components, \vec{q}_T , determine the virtuality of the process. In this limit, Q is also transverse, *i.e.*, $Q \rightarrow \vec{Q}_T$ and $t \sim -\vec{Q}_T^2$. Upon taking a two-dimensional Fourier transform with respect to \vec{Q}_T , (1) can be written as

$$\mathcal{A}_n(s, \vec{q}_T, \vec{b}) = \int \frac{ds'}{s} \int d^2 q'_T \int d^2 b' K(s'/s; \vec{q}_T, \vec{q}'_T; \vec{b} - \vec{b}') \mathcal{A}_{n-1}(s', \vec{q}'_T; \vec{b}'). \quad (2)$$

At high energies, for both cases, the kernel of this recursion relation simplifies which allows a probabilistic interpretation in terms of a “random walk” picture.

[†] Depending on the approximation used, the hard Lipatov Pomeron can either be a fixed cut or a series of poles accumulating above $j = 1$. For our qualitative discussion, we assume that it can be treated as an effective t -independent J -plane singularity.

We begin by first working out the example of a one-dimensional random walk, which is specified by a normalized elementary one-step probability distribution, $\int_{-\infty}^{\infty} \omega(r) dr = 1$. The (relative) probability distribution after n steps in r is then related to that for $n - 1$ steps by a linear relation, $\Psi_n(r) = \int_{-\infty}^{\infty} dr' \omega(r - r') \Psi_{n-1}(r')$. We assume $\omega(r) = \omega(-r)$ so that $\langle r \rangle_1 = \int_{-\infty}^{\infty} r \omega(r) dr = 0$. It follows that $\langle r \rangle_n = 0$ and $\langle r^2 \rangle_n$ increases with n , the number of steps taken. Simple examples for $\omega(x)$ are step-function, $\frac{1}{\lambda} \theta(\lambda - |r|)$, gaussian, $\frac{1}{\sqrt{\pi\lambda}} e^{-r^2/\lambda^2}$, and exponential, $\frac{1}{2\lambda} e^{-|r|/\lambda}$.

For n large, we can treat the relative probability as continuous in n , *i.e.*, $\Psi_n(r) \rightarrow \Psi(n, r)$. Since the dominant contribution to the recursive integral comes from the region where $r \sim r'$, we can expand the integrand about (n, r) , and obtain a diffusion equation $\frac{\partial \Psi(n, r)}{\partial n} = D_r \frac{\partial^2 \Psi(n, r)}{\partial r^2}$, where the "diffusion coefficient" is related to the elementary one-step fluctuation by $D_r = \frac{1}{2} \langle r^2 \rangle_1$.

A directed random walk corresponds to a situation where $\omega(y) = 0$ for $y < 0$ so that $\lambda \equiv \langle y \rangle_1 = \int_0^{\infty} y \omega(y) dy \neq 0$ and the relative probability after n steps satisfies the recursion relation, $\Psi_n(y) = \int_0^y dy' \omega(y - y') \Psi_{n-1}(y')$. A simple consequence of a directed random walk is the fact that the distribution in the number of steps taken in reaching a large and fixed distance, Y , is Poisson-like. For instance, in the case of an exponential step distribution, $(1/\lambda) e^{-y/\lambda} \theta(y)$, one has $\Psi_{n+1}(y) = (y^n / \lambda^{n+1} n!) e^{-y/\lambda}$, leading to the result that the average number of steps taken, the average multiplicity, equals to the distance travelled divided by the average one-step length, $\langle n \rangle = y / \langle y \rangle_1 = y / \lambda$.

Let us next consider a two-dimensional walk in which $\omega(r, y) \sim \omega_r(r) \omega_y(y)$ where only the walk in the y -direction is directed, *i.e.*, $\omega_r(r) = \omega_r(-r)$ and $\omega_y(y) \propto \theta(y)$. The joint probability now satisfies the recursion relation

$$\Psi_n(x, y) = \int_{-\infty}^{\infty} dr' \int_0^y dy' \omega(r - r', y - y') \Psi_{n-1}(r', y'). \quad (3)$$

Treating $\Psi_n(r, y)$ as continuous in n for n large and expanding the integrand in (3) about (n, r, y) , one obtains a generalized diffusion equation

$$\frac{\partial \Psi(n, r, y)}{\partial n} = \lambda \frac{\partial \Psi(n, r, y)}{\partial y} + D_r \frac{\partial^2 \Psi(n, r, y)}{\partial r^2}. \quad (4)$$

We next demonstrate that the structure of both the hard and the soft Pomerons can be interpreted as simultaneous random walks in appropriate spaces. Concentrating first on the hard Pomeron, where Eq. (2) corresponds to the celebrated Lipatov equation. The most remarkable feature of this equation is the fact that it is not infrared singular, *i.e.*, the kernel is regular at $q_T = q'_T$, in spite of the fact that various individual terms contributing to the kernel are singular. Much recent discussions have focused on the detailed structure of this equation, *e.g.*, the precise intercept of the Lipatov Pomeron under a variety of physically motivated

modifications to this equation.^[6] Although these are extremely interesting questions, for our present purpose, we only need to identify certain qualitative features of the Lipatov equation.

Introducing rapidities $y = \log s$, $y' = \log s'$, and $r = \log(q_T^2/q_0^2)$, $r' = \log(q_T'^2/q_0^2)$,^{*} the angular part of \vec{q}_T can be integrated out, and, (2) becomes

$$\mathcal{A}_n^{(h)}(y, r, \vec{b}) \simeq \int_0^y dy' \int_{-\infty}^{\infty} dr' K_L(y - y'; r - r') \mathcal{A}_{n-1}^{(h)}(y', r'; \vec{b}). \quad (5)$$

The Lipatov kernel is factorizable, $K_L(y - y'; r - r') \simeq R_h(y - y') V_h(r - r')$, where $R_h(y) \sim e^y$ and the Fourier transform of $V_h(r)$, denoted by $\chi(\nu)$, is analytic for $|\text{Im}\nu| < 1/2$. The absence of the \vec{b} -integration, which is an approximation, reflects the fact that the Lipatov kernel for (1) asymptotically has a weak t -dependence.

Observe that, Eq. (5) is structurally similar to (3). (By dividing an appropriate power of s , $s^{1+\Delta_L(0)}$, it is possible to normalize the kernel K_L so that $\int dy \int dr K_L(y, r) = 1$. Indeed, one finds that $\alpha_L \equiv 1 + \Delta_L(0)$, $\Delta(0)_L \propto \chi(0)$, is precisely the Lipatov Pomeron intercept.) Therefore, the properties of the hard Pomeron can be understood in terms of a simultaneous random walk in “rapidity” and “log of virtuality”, as summarized in Table-II. In particular, we emphasize that diffusion in the r -space can be described by an equation like (4):

$$\frac{\partial \Psi(n, \vec{b}, r, y)}{\partial n} = \lambda_h \frac{\partial \Psi(n, \vec{b}, r, y)}{\partial y} + D_r \frac{\partial^2 \Psi(n, \vec{b}, r, y)}{\partial r^2}, \quad (6)$$

which leads to a spread in virtuality with increasing rapidity: $\langle q_T^2 \rangle \sim e^{\text{const.} \sqrt{Y}}$. However, a hard Pomeron does not lead to diffusion in the impact parameter space.

Table II. Hard and Soft Pomerons as Random Walks:

	Hard Pomeron	Soft Pomeron
Rapidity: $y = \log s$	Directed-random-walk	Directed-random-walk
Log of Virtuality: $r = \log(q_T^2/q_0^2)$	Random-walk, Diffusion.	No, q_T^2 fixed and small.
Impact parameter: \vec{b}	No, \vec{b} (approx.) fixed.	Random-walk, Diffusion.

* We have introduced here a scale, q_0 , below which the LLA used for deriving the Lipatov equation is questionable. We can use this as a cutoff below which a non-perturbative description must be used. However, this will not affect the key “diffusion” feature which we would like to identify next.

We will be more sketchy for the case of a soft Pomeron. Note that, by definition, a soft ladder structure also involves a strong cutoff in q_T^2 , and, for simplicity, we assume that all q_T^2 's are of the order q_0^2 . The kernel must also be cutoff in t , thus leading at high energies to a recursion relation

$$\mathcal{A}_n^{(s)}(y, r_0, \vec{b}) \simeq \int_0^y dy' \int d^2 b' K_s(y - y'; \vec{b} - \vec{b}') \mathcal{A}_{n-1}^{(s)}(y', r_0; \vec{b}'), \quad (7)$$

where $r_0 \simeq 0$. That is, a soft Pomeron corresponds to a simultaneous random walk in the rapidity and the impact parameter space, as summarized in Table-II. Under a factorizable approximation, one has $K_s(y - y'; \vec{b} - \vec{b}') \simeq R_s(y - y') I_s(\vec{b} - \vec{b}')$, where $R_s(y) \sim e^{c y}$ and $I_s(\vec{B})$ decreases rapidly for \vec{B}^2 large. Instead of (6), for n large,

$$\frac{\partial \Psi(n, \vec{b}, r_0, y)}{\partial n} = \lambda_s \frac{\partial \Psi(n, \vec{b}, r_0, y)}{\partial y} + D_b \nabla^2 \Psi(n, \vec{b}, r_0, y), \quad (8)$$

where the diffusion coefficient D_b is related to the one-step fluctuation in b^2 . This diffusion leads to $\langle b^2 \rangle \propto \log s$, which, when translated back into t , corresponds to the well-known shrinkage of the forward peak due to the exchange of a soft Pomeron.

4. Unification and Heterotic Pomeron

We are now in the position to construct a model which incorporates both diffusion in virtuality, (6), and diffusion in impact parameter, (8). Note that, whereas diffusion in r can take place at any fixed value of \vec{b} , diffusion in impact parameter space in a soft process can take place only at small virtuality, $r_0 \simeq 0$. This can be realized in a two-channel simultaneous random walk in the $y - r - \vec{b}$ space. Let us label the allowed channels by "s" and "h", (for soft and hard respectively.) We introduce four elementary one-step distributions, $K_{i,j}(y, r, \vec{b}; y', r', \vec{b}')$, the relative probability of starting from the j th channel at y', r', \vec{b}' and ending in the i th channel at y, r, \vec{b} after one step.

For an ordinary random walk, $K_{i,j}$ should depend only on the differences $y - y', r - r', \vec{b} - \vec{b}'$. However, our situation is more restricted, *e.g.*, a soft process can participate only if the virtuality is small. This can be simulated by assuming that $K_{sh} \propto \delta(r - r_0) \delta(r' - r_0)$, and $K_{ss} \propto \delta(r - r_0)$. Similarly, the fact that very little diffusion in impact parameter takes place in a hard process can be simulated by assuming that $K_{hh} \propto \delta(\vec{b} - \vec{b}') \propto K_{hs}$. Lastly, for directed walk in rapidity, we must have $K_{ij} \propto \theta(y - y')$. That is, the desired 2×2 one-step probability distributions can be chosen as $K_{ss} = g_{ss} R_s(y - y') I_s(\vec{b} - \vec{b}') \delta(r - r_0)$, $K_{sh} = g_{sh} R_s(y - y') I_s(\vec{b} - \vec{b}') \delta(r - r_0) \delta(r' - r_0)$, $K_{hs} = g_{hs} R_h(y - y') \delta(\vec{b} - \vec{b}') V_h(r - r')$, and $K_{hh} = g_{hh} R_h(y - y') \delta(\vec{b} - \vec{b}') V_h(r - r')$, where R_h and R_s can be taken from that used in (5) and (7) respectively.

Let the relative probabilities of arriving in the i th channel after n steps be $\Psi_{(n;i)}(\mathbf{y}, \mathbf{r}, \vec{b})$, and let $\Psi_n(\mathbf{y}, \mathbf{r}, \vec{b})$ be the two-vector with $\Psi_{(n;s)}(\mathbf{y}, \mathbf{r}, \vec{b})$ and $\Psi_{(n;h)}(\mathbf{y}, \mathbf{r}, \vec{b})$ as its upper and lower components respectively. It follows that

$$\Psi_{(n)}(\mathbf{y}, \mathbf{r}, \vec{b}) = \int_0^{\mathbf{y}} d\mathbf{y}' \int d\mathbf{r}' \int d^2b K(\mathbf{y}, \mathbf{r}, \vec{b}; \mathbf{y}', \mathbf{r}', \vec{b}') \Psi_{(n-1)}(\mathbf{y}', \mathbf{r}', \vec{b}'), \quad (9)$$

with $\Psi_{(0;i)} \sim G_i \delta(\mathbf{y}) \delta(\mathbf{r} - \mathbf{r}_0) \delta(\vec{b})$. We note that, because of the structure of $\{K_{ij}\}$, one always has $\Psi_{(n;s)}(\mathbf{y}, \mathbf{r}, \vec{b}) \propto \delta(\mathbf{r} - \mathbf{r}_0)$. That is, the soft interactions take place at small virtuality only.

It is appropriate at this point to comment that in specifying $\{K_{ij}\}$ we have introduced a set of symmetric ‘‘coupling matrix’’, $\{g_{ij}\}$. Whereas $g_{ss} = 0(1)$, the other three must be of the order of the QCD running coupling constant at a large virtuality, i.e., $g_{hs} \propto g_{sh} \propto g_{hh} \propto 0(\alpha_s)$. Note also that $\{K_{ij}\}$ are relative probabilities, no longer normalized to unity as was done earlier. In particular, the choice $R_h(\mathbf{y}) \sim e^{\mathbf{y}}$ and $R_s(\mathbf{y}) \sim e^{c\mathbf{y}}$, $c \sim 1/2$, correspond precisely to the large energy behaviors for two-gluon and two-meson exchanges, appropriate for the hard and the soft processes respectively. Introduce a complex angular momentum J via a Laplace transform, one has $\tilde{R}_h(J) = 1/(J - 1)$ and $\tilde{R}_s(J) = 1/(J - c)$.

Let $\Psi(\mathbf{y}, \mathbf{r}, \vec{b}) = \sum_n \Psi_n(\mathbf{y}, \mathbf{r}, \vec{b})$ and denote $\tilde{\Psi}(J, \nu, t)$ as its multiple-transform, (Laplace in \mathbf{y} , Fourier in \mathbf{r} and \vec{b}). It follows from the recursion relation that a formal solution can be expressed as $\tilde{\Psi}(J) = (I - \tilde{K})^{-1} \Psi_0$. We point out that the high energy behavior of $\Psi(\mathbf{y})$ will be controlled by the ‘‘right-most’’ singularity of $\tilde{\Psi}(J)$ in the complex J plane, which is given by the condition $\det(I - \tilde{K}) = 0$.

If soft interactions were turned off, the determinantal condition would lead to the Lipatov Pomeron, $\alpha_L = 1 + \Delta_L$, which, as mentioned earlier, is approximately t -independent. In fact, the Lipatov Pomeron, without further refinements, corresponds to a fixed cut. Conversely, if the hard processes were turned off, one would obtain a soft Pomeron, $\alpha_0(t) = 1 + \Delta_0(t)$, which has a ‘‘normal’’ t -slope in the forward region. For simplicity, we assume that $0 \leq \Delta_s \leq \Delta_L = 0(\alpha_s)$. In our unified treatment, a new singularity, to the right of both α_L and α_0 , emerges. This new singularity is a *simple pole*, which we referred to as the ‘‘Heterotic Pomeron’’. In an approximate treatment, the location of the Heteroti Pomeron, $\alpha_H(t) \equiv 1 + \omega^*(t)$, can be found as the solution to the equation $\sqrt{\omega^*(\omega^* - \Delta_L)(\omega^* - \Delta_0(t))} = g_{sh}^2 G(t)$, where $G(t)$ is positive and peaked at $t = 0$. Details of this analysis will be presented in a regular publication.^[7]

5. Discussion

The fact that Heterotic Pomeron is a pole, with a slope of the order of that for the soft Pomeron, might come as a surprise to some. We will have much more to say about this point elsewhere. Here, we close by pointing out some potentially important consequences of our unified treatment of QCD at high energies.

Since Heterotic Pomeron is a pole, a well-defined perturbative Reggeon calculus can be carried out. Because of the factorization property, it naturally leads to diffractive dissociation

events, or more generally, it allows the study of “rapidity gap” physics at collider energies. We also mention that, in this unified treatment,^[7] diffusion in virtuality becomes “limited”, the Heterotic Pomeron coupling to hadron is “soft-dominated”, and truly hard processes become dominant only in the region where $|t| \geq 0(\log s)$. On a more phenomenological side, we mention that our treatment allows a systematic expansion of cross sections in terms of “hard” and “soft” events, which goes beyond the simple “additive” approach. Furthermore, since Heterotic Pomeron intercept is greater than one, absorptive corrections must again be taken into account. It thus provides a new starting point for handling screening corrections, which could have a profound effect on our understanding of both the near forward hadronic collisions and the small- x physics in deep-inelastic scattering.

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