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A NEW ACTION FOR HEAVY LATTICE FERMIONS*

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ABSTRACT

I describe a unified formalism for lattice fermions, in which the relativistic action of Wilson and the nonrelativistic and static actions appear as special cases. It is valid at all values of $m_q a$, including $m_q a \approx 1$. In the limit $m_q a \ll 1$, the formulation reduces to the light quark action of Wilson. In the limit $m_q a \gg 1$, the formulation reduces to the nonrelativistic action of Thacker and Lepage, and to the static action of Eichten.

Present and future lattice calculations involving b and c quarks include some of the most important applications of lattice gauge theory to standard model physics. These include the heavy meson decay constants, the $B\overline{B}$ mixing amplitude, and various semileptonic decay amplitudes, which are all crucial in extracting CKM angles from experimental data. They also include the extraction of α_s from the charmonium and bottomonium spectra.

There exist two main classes of methods for lattice fermions: small mass expansions and large mass expansions. The standard Wilson action contains errors which vanish as powers of $m_q a$, the quark mass in lattice units. This series of corrections does not converge when $m_q a > 1$. There exists a second class of methods for treating lattice fermions when $m_q a > 1$: Nonrelativistic QCD (NRQCD)[1] and the static approximation.[2] Because coefficients of higher terms in the Lagrangians of these methods (such as $\mathbf{D}^2/(2m)$) are explicit functions of $1/m$, the loop corrections are also explicit functions of $1/m$. These begin to diverge as ma is reduced below a value of order one, making the nonrelativistic expansion impractical. The masses of the b and c quarks are such that $m_q a$ is $O(1)$ at the lattice spacings used in current numerical work. Therefore, calculations of such crucially interesting quantities as the heavy meson decay constants f_B and f_D have often involved awkward interpolations between results in the static approximation and results using Wilson fermions through a region where neither approximation is well behaved.While such an approach is probably workable, it is clearly desirable to have a method for lattice fermions which is well behaved throughout the region of interest.

Since the large mass and small mass formalisms are both descriptions of QCD, it is not surprising that they share certain fundamental features. Like Wilson fermions

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(ψ), the fermions of NRQCD contain four components per site: a two-component quark field (ϕ) and a two-component antiquark field (χ). Further, the two actions employ the same sorts of interactions: covariant time derivatives, covariant Laplacians, etc. To find a formalism uniting the two actions, we therefore consider the following generalized Lagrangian:[3, 4]

$$\begin{aligned}\mathcal{L} &= \phi^*(\quad c_1\ \Delta_t^- + m_0 - \frac{c_2}{2}\sum_i \Delta_i^+\Delta_i^-)\phi_n + c_3\ \phi^*\sum_i \sigma_i\Delta_i\chi_n \\ &+ \chi^*(-c_1\ \Delta_t^+ + m_0 - \frac{c_2}{2}\sum_i \Delta_i^+\Delta_i^-)\chi_n + c_3\ \chi^*\sum_i \sigma_i\Delta_i\phi_n. \qquad (1)\end{aligned}$$

(Δ^+, Δ^-, and Δ are the forward, backward, and symmetric discrete difference operators, respectively. I will use m_q for the physical quark mass and m_0 for the bare quark mass on the lattice.) With the choice of parameters $c_1 = c_2 = c_3 = 1$, this is simply the standard Wilson action. When $c_1 = 1$ (times a correction factor when $ma \gg 1$), $c_2 = \frac{1}{m}$, and c_3 is negligible, it is a good, if somewhat unconventional, Lagrangian for NRQCD. The bare mass is conventionally omitted in NRQCD calculations, but we are free to leave it in the theory. The usual Dirac coupling between quarks and antiquarks is absent (having been transformed into higher derivative interactions by the Foldy-Wouthuysen transformation), but we may add back a sufficiently suppressed amount of this interaction without spoiling the nonrelativistic theory. It is thus possible to adjust the parameters for this particular NRQCD action in such a way that as m_0 is reduced, instead of blowing up, the theory turns smoothly into the Wilson theory. One must find appropriate normalization conditions to determine the parameters of the action which lead to the Wilson theory for $am_q \ll 1$, and to NRQCD and the static approximation for $am_q \ll 1$, and which work at all values of am_q.

It is illuminating in this regard to expand the equation for Wilson propagators nonrelativistically when the mass is large, to see what breaks down. After normalizing the fields by $\frac{1}{\sqrt{1-6\kappa}}$[5] (not $\frac{1}{\sqrt{2\kappa}}$ as is commonly used) one may obtain

$$0 = \left[-\mathcal{E} + \mathcal{M} + (1 - U^\dagger_{\mathbf{n},0}) - \frac{1}{2}\left(\frac{1}{m_0} + \frac{1}{(1+m_0)\,(2+m_0)}\right)\sum_i(\Delta_i)^2\right]\phi_{\mathbf{n}}, \qquad (2)$$

where $\mathcal{E}$ is the energy eigenvalue obtained from the transfer matrix and $\mathcal{M} = \mathcal{E}_{p^2=0} = \ln(1+m_0)$. This is a lattice Schrödinger equation not unlike the one obtained from NRQCD, but it has some unusual features. Most important, the two "masses" in the equation, the rest mass $\mathcal{M} = \ln(1+m_0)$, and the mass governing the energy-momentum relation $\frac{1}{M} = \frac{1}{m_0} + \frac{1}{(1+m_0)\,(2+m_0)}$, are completely different. $\mathcal{M}$ plays little dynamical role in heavy quark systems and is usually omitted from NRQCD and static approximation calculations. The dynamically more important condition $\partial\mathcal{E}/\partial p^2 = 1/(2m_q)$ is used to fix the mass in NRQCD. This condition is also the same as the usual mass condition for Wilson fermions when $am \ll 1$, since when $m_0 a$ is small,

$$\frac{1}{m_0} + \frac{1}{(1+m_0)\,(2+m_0)} \approx \frac{1}{\mathcal{M}} \approx \frac{1}{m_0}. \qquad (3)$$

However, if the rest mass $\mathcal{M}$ is used to fix the fermion mass for Wilson fermions when $am > 1$, the energy-momentum mass condition $\partial\mathcal{E}/\partial p^2 = 1/(2m_q)$, which is usually more important, will be completely incorrect. The two masses can be put back into agreement with the use of the Lagrangian[3, 4]

$$\begin{aligned}\mathcal{L} = -\bar{\psi}_n\psi_n \quad &+ \quad \kappa_t\bar{\psi}_n(1-\gamma_0)U_{n,0}\psi_{n+\hat{0}} + h.c. \\ &+ \quad \kappa_s\sum_i \bar{\psi}_n(1-\gamma_i)U_{n,i}\psi_{n+\hat{i}} + h.c.\end{aligned} \tag{4}$$

Thus, it seems that an action closely related to the Wilson action is a member of the class of actions suitable for NRQCD. Further, in NRQCD and in the static approximation, $\mathcal{M}$ plays little dynamical role. It can usually be ignored, and is usually omitted. This suggests that for that majority of calculations in which the mass gap between states containing different numbers of quarks is unimportant, the standard Wilson action itself can be used when $am > 1$ as long as $\mathcal{M}$ is ignored and $\partial\mathcal{E}/\partial p^2 = \frac{1}{2m}$ is used to fix the quark mass, as is done in NRQCD.

This proposal is obviously correct in free field theory, where we can calculate the behavior of quark propagators exactly to see that the proposed interpretation makes sense. It is certainly correct in mean field theory, too. Mean field improvement of these fermions, as of Wilson fermions, is simply the absorption of a "mean link" u_0 into an effective $\tilde{\kappa} \equiv u_0\kappa$ and then proceeding as with free field theory.[6] (A plausible estimate of the mean link in this context is probably $u_0 \approx 1/8\kappa_c$.) It remains to be shown whether the theory is somehow spoiled by renormalization.

Perturbatively, Green functions must be expanded in p^2 and α_s. The coefficient of each term in the expansion is an explicit function of the quark mass, since the theory must be solved exactly in ma. (The is also the case for the loop corrections of NRQCD.[7]) If these functions become singular or badly behaved in some way, the theory could conceivably break down. The one loop perturbative corrections contain all of the ugliest features of Wilson and NRQCD perturbation theory simultaneously, and have only been begun. There is, however, one numerical calculation by El-Khadra[8] indicating that nothing too surprising occurs. The one-loop correction to the local current normalization for Wilson fermions with the naive normalization is[9]

$$\langle\psi|V_4^{loc}|\psi\rangle = \frac{1}{2\kappa(1-0.17g^2)}. \tag{5}$$

The correct normalization with mean field improvement is

$$\langle\psi|V_4^{loc}|\psi\rangle = \frac{1}{(1-\frac{6\kappa}{8\kappa_c})(1-0.06g^2)}. \tag{6}$$

The remaining perturbative correction, $0.06g^2$, becomes an explicit (so far uncalculated) function of m (or κ) in the new formalism. The small mass limit of this function is $0.06g^2$. This function must not become singular if the theory is to make sense. This

normalization is straightforward to calculate numerically. As described in Ref. [8], the nonperturbative calculation agrees with Eq. 6 to within a few per cent. It disagrees with Eq. 5 by around a factor of two. It can be seen that for this quantity, not only is the unknown function of m not singular, it is approximately equal to 1.

Putting the new action on a secure footing will ultimately require: 1) determination of the bare parameters of the action with mean field theory and full perturbation theory, 2) nonperturbative tests of the perturbative results, and 3) phenomenological tests of the resulting action in calculations of well understood physical quantities. Not much of this program has yet been accomplished. However, from what is known now, there appears to be no insuperable obstacle to developing a formulation of lattice fermions practical for all values of the quark mass, not just $m_q a \gg 1$ or $m_q a \ll 1$.

Care will clearly be required in formulating normalization conditions which capture the most important physics in both the relativistic and nonrelativistic regions. (Identifying $\partial\mathcal{E}/\partial p^2$ rather than $\mathcal{M}$ as the fundamental mass condition is example number one of these.)

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