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We have used a lattice gauge theory calculation of the 1S–1P splitting in the J/ψ system to obtain a determination of the strong coupling constant. Extrapolating our result to M_Z yields $\alpha_{\overline{MS}}(M_Z) = 0.105 \pm 0.004$. We review the details of this calculation and report on current efforts to clarify and reduce the corrections and uncertainties.

*Talk presented by A. El-Khadra at ICHEP 92, Dallas, Aug. 6–12, 1992.

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Abstract

We have used a lattice gauge theory calculation of the 1S–1P splitting in the J/ψ system to obtain a determination of the strong coupling constant. Extrapolating our result to M_Z yields $\alpha_{\overline{MS}}(M_Z) = 0.105 \pm 0.004$. We review the details of this calculation and report on current efforts to clarify and reduce the corrections and uncertainties.

INTRODUCTION

An important task in understanding quantum chromodynamics (QCD) is the determination of its coupling constant, α_s. The Review of Particle Properties quotes values for $\alpha_s(M_Z) \equiv g^2/4\pi$ in the range 0.10–0.14.[1] Most perturbative determinations of α_s contain nonperturbative contaminations which become small only at high energies. On the other hand, high energy determinations yield α_s at lower energies only imprecisely. Lattice gauge theory calculations provide a nonperturbative means of determining the strong coupling constant from low energy quantities.

A lattice determination of the strong coupling constant consists of 1) the identification of a system for which systematic errors are small, 2) a calculation of the lattice spacing in physical units (which sets the scale of the running coupling constant), and 3) a determination of a renormalized coupling constant.

Because heavy-quark systems are nonrelativistic it is easier to analyze and evaulate their systematic errors than those of light hadrons.[2] For these systems no extrapolation to light quark masses – an important source of uncertainty in the light hadron system – is necessary. In heavy quark systems, contrary to the light hadron system, errors arising from the omission of sea quarks (and also from the finiteness of the lattice spacing) may be systematically analyzed and quantitatively estimated. Moreover, existing lattice calculations of the light hadron spectrum depend on the light quark masses in a way that is difficult to control or to analyze quantitatively.

The lattice spacing in physical units may be provided by a comparison of a lattice calculation of any dimensionful quantity with its physical value. Many of these have been performed over the last 10 years: the mass of the proton may be the canonical example, but as discussed above it has drawbacks. Heavy-quark systems offer a better choice, the 1P–1S spin-averaged splitting, which is independent of the (heavy) quark mass for a wide range of masses spanning charm and bottom.

The determination of a renormalized coupling constant for a given lattice calculation may be obtained from a perturbative relation between the bare lattice coupling and

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the renormalized coupling constant using improved lattice perturbation theory[3] as was done in our initial paper.[4] In ongoing work, we have checked and improved this estimate by relating the renormalized coupling (e.g. in the $\overline{MS}$ scheme) to short-distance lattice quantities, which are easy to calculate precisely in Monte Carlo simulations. It remains to be shown rigorously the extent to which such couplings are identical with continuum renormalized couplings.

All existing determinations of α_s, including ours, contain corrections and uncertainties from nonperturbative effects which are estimated phenomenologically.[1] For example:

- Analyses of deep inelastic scattering must cope with higher-twist effects.
- The jet and hadronic event shape analyses from e^+e^- annihilation rely on model calculations of hadronization.
- In τ decays nonperturbative corrections to R_τ are estimated using QCD sum rules.

In the case of our calculation, the correction which is currently difficult to incorporate directly from first principles is the effect of sea quarks. This shortcoming is *temporary* and will be removed over the next few years with the improvement of algorithms and computers, yielding an entirely first principles determination.

An important aspect is to determine in what energy regime a renormalized coupling runs as predicted by perturbative asymptotic freedom. Ref. 5 proposes a program to carry out this search using lattice gauge theory. The lattice calculations are dimensionless, so once the perturbative regime has been identified, the scale must be set in GeV, as discussed above. Once the effects of sea quarks have been treated exactly, a coupling of this type will have been determined with no perturbative or non-perturbative uncertainty. There will still be uncertainties arising from finite statistics in the Monte Carlo and from extrapolations to zero lattice spacing.

The coupling determined from lattice gauge theory can be related to more familiar ones, such as the $\overline{MS}$ coupling, by perturbation theory. Thus one can eliminate one in favor of the other. Since perturbative expansions in $\overline{MS}$ always have some error associated with the truncation of perturbation theory, it makes more sense to eliminate $\alpha_{\overline{MS}}$ form those expansions and use the coupling with the smallest uncertainty.

DETERMINATION OF THE LATTICE SPACING

The lattice calculation yields a (dimensionless) mass or mass splitting am. The lattice spacing a is determined by comparing am with the experimentally measured value for m. It is desiarable to use a quantity in the charmonium spectrum that is independent of the quark mass m_c and insensitive to systematic errors. The spin averaged splitting between the 1P and 1S states is known to be quite independent of the quark mass, because it is so similar in the ψ and Υ systems. Since it is a spin averaged quantity, it is also expected to be insensitive to the $\mathcal{O}(a)$ finite lattice spacing errors, which are dominated by a quark-gluon $\sigma \cdot B$ interaction.

The details of the lattice calculation are described elsewhere.[4, 6] The $\mathcal{O}(a)$ corrected Wilson action for quarks was used.[7] We used 3 different lattices ($12^3 \times 24$, $16^3 \times 32$, 24^4) at different couplings ($\beta \equiv 6/g_0^2 = 5.7, 5.9, 6.1$), such that the spatial volumes are similar. The lattice spacing varies from the coarsest lattice to the finest by a factor of two. This allowed us to study residual lattice spacing errors, expected to be $\mathcal{O}(a^2)$.

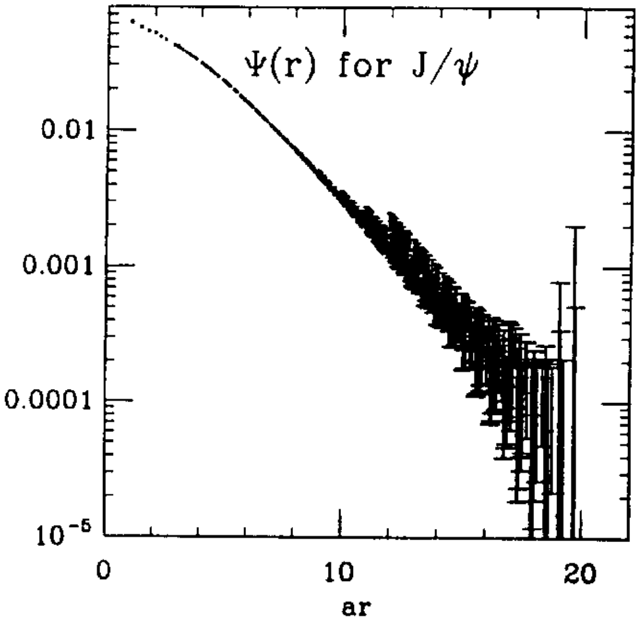

Figure 1. The wave function of the J/ψ meson.

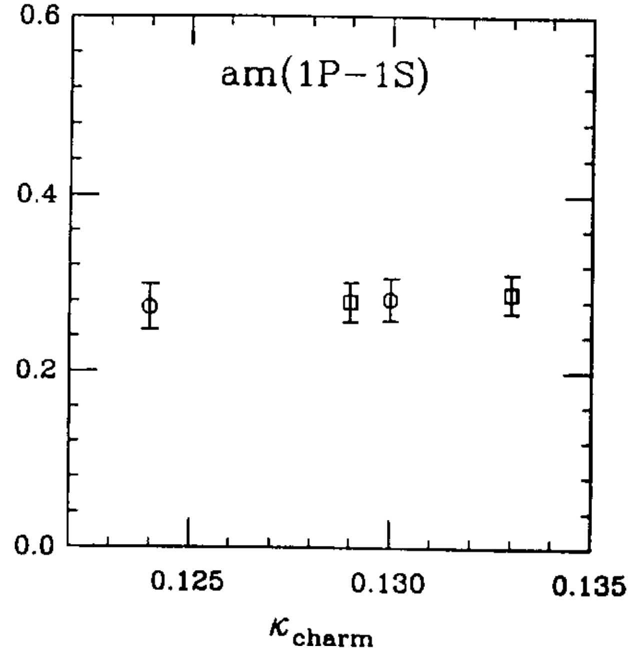

Figure 2. The 1P–1S splitting vs. κ on the $16^3 \times 32$ lattice. The circles are for $c = 1.4$, the squares are for $c = 0$.

Finite volume errors were estimated using Coulomb gauge wave functions, as described in Ref. 4, 6. Figure 1 shows as an example the wave function of the J/ψ meson calculated on the 24^4, $\beta = 6.1$ lattice.

To obtain the lattice spacing at each value of β, we calculated the difference of the spin averaged mass of the 1S states (the J/ψ and the η_c) and the mass of the recently discovered[8] spin singlet 1P state (the h_c) in lattice units, and then compared it with the experimentally measured splitting, $M_{h_c} - (3M_{J/\psi} + M_{\eta_c})/4 = 458.6 \pm 0.4$ MeV.

In Ref. 4 we argued that only negligible errors arose from the uncertainties in the values used for the quark mass and the coefficient c of the $\mathcal{O}(a)$ correction. In the last year we have checked this by direct calculation. Our new results are displayed in Figure 2, where the 1P-1S splitting is shown as a function of the mass parameter (κ) for Wilson fermions with ($c = 1.4$) and without ($c = 0$) the $\mathcal{O}(a)$ improvement term on the $16^3 \times 32$ lattice. As expected, the quark mass dependence of the 1P–1S splitting is very small. Spin splittings, such as the $J/\psi - \eta_c$ splitting which we have also investigated are, on the other hand, very sensitive to the tuning of the quark mass and of the improvement term. This is shown in Figure 3.

Figure 2 already demonstrates that the spin averaged 1P–1S splitting does not depend on the $\mathcal{O}(a)$ errors of the Wilson quark action, which anyway have been removed by adding the improvement term. To test for the size of the remaining higher order lattice spacing errors, the calculation was performed at three lattice spacings.

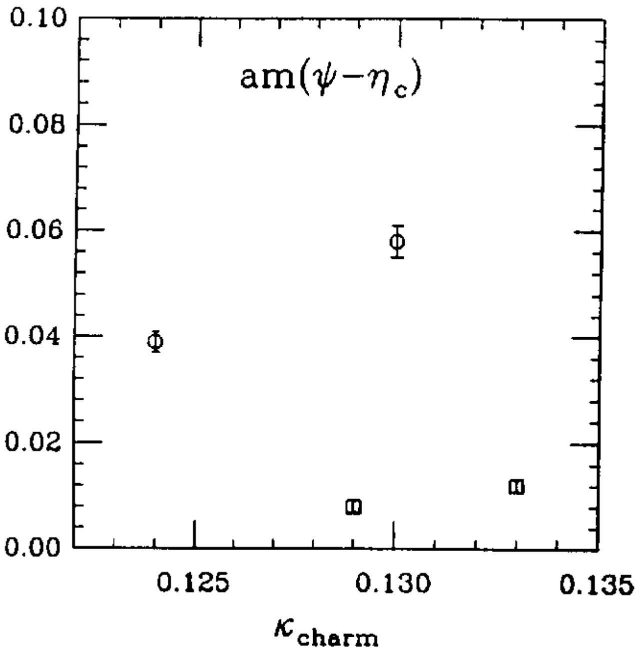

Figure 3. The J/ψ–η_c splitting vs. κ on the $16^3 \times 32$ lattice. The circles are for $c = 1.4$, the squares are for $c = 0$.

DETERMINATION OF THE RENORMALIZED COUPLING CONSTANT

In Ref. 4 we obtained an estimate of the $\overline{MS}$ coupling on each lattice with the formula

$$\frac{1}{g^2_{\overline{MS}}(\frac{\pi}{a})} = \frac{1}{g^2_{\text{eff}}} + 0.025 \quad . \tag{1}$$

where $g^2_{\text{eff}} = g_0^2/\langle TrU_P\rangle$.[9] Replacing the perturbative formula for $\langle TrU_P\rangle$ with the nonperturbative value for the plaquette yields a mean field improved relation between the $\overline{MS}$ and bare lattice coupling constants which incorporates a Monte Carlo estimate of some of the higher order effects relating renormalized coupling constants with the bare lattice coupling constant. The (non-perturbative) values for $\langle TrU_P\rangle$ are 0.549, 0.582, and 0.605 at β = 5.7, 5.9, and 6.1, respectively.

Using Eq. (1), the lattice spacings calculated as discussed in the previous section, and the parameterization for α_s of the Particle Data Group, we extract values for $\Lambda^{(0)}_{\overline{MS}}$ for each lattice spacing. In Figure 4 the results are plotted versus a^2. Within the statistical uncertainties there is only a small dependence on the lattice spacing.

The extent to which Eq. (1) correctly estimates the higher order corrections has since been tested by comparing Monte Carlo results for several short distance quantities with perturbative predictions using the improved coupling constant of Eq. (1), following Ref. 3. The conclusion is that Eq. (1) correctly gives the bulk of the corrections, but that the Monte Carlo results are typically a few per cent higher than the perturbative predictions. For example, one can use the plaquette expectation value to define the coupling constant $g_P^2 = -3\ln\langle TrU_P\rangle$ and relate g_P^2 to the $\overline{MS}$ coupling perturbatively by:

$$\frac{1}{g^2_{\overline{MS}}(\frac{\pi}{a})} = \frac{1}{g_P^2} - 0.040 \quad . \tag{2}$$

This procedure raises the value of $\alpha_s(5\ GeV)$ a few per cent over that obtained from Eq. (1).

RESULTS

The dominant source of uncertainty in our final result arises from the conversion from the zero light quark running coupling constant of the lattice calculation to the four quark running coupling of the real world. This effect has not yet been included from first principles. For the masses of light hadrons, there is no way of estimating this correction even phenomenologically. For a nonrelativistic system, the dominant effects of the omision of sea quarks are expected to lie in their effect on the static potential. We used this effect as an estimate of the corrections and uncertainty arising from this source. The procedure used to obtain this correction and systematic error estimate is described in detail in Ref. 4. The resulting correction is $\Delta g^{-2} = -0.110 \pm 0.030$.

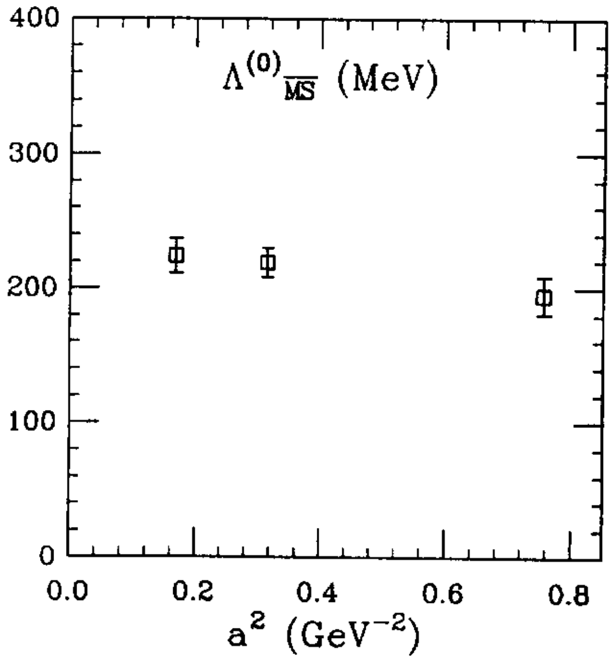

Figure 4. $\Lambda^{(0)}_{\overline{MS}}(5\mathrm{GeV})$ as a function of a^2.

The final result quoted in Ref. 4 is

$$\alpha_{\overline{MS}}(5\ GeV) = 0.174 \pm 0.012. \qquad (3)$$

We do not expect the further investigations discussed in this talk to change this result significantly. This result corresponds to $\Lambda^{(4)}_{\overline{MS}} = 160^{+47}_{-37}$ MeV, using the parameterization of the Particle Data Group. Extrapolating to the mass of the Z, we obtain $\alpha_{\overline{MS}}(M_Z) = 0.105 \pm 0.004$.

This is about 2σ below the combined results from LEP analyses of event shapes: $\alpha(M_Z) = 0.120 \pm 0.006$, using $O(\alpha_s^2)$ perturbation theory.[11]

A similar calculation to ours has been performed for both the ψ and Υ systems using the nonrelativistic formulation of lattice fermions.[10] For the Υ system, the systematic errors and corrections are quite different from the ones reported here. The results are compatible.

FUTURE PROSPECTS

Over the next few years, Monte Carlo simulations directly including the effects of sea quarks will eliminate the uncertainty that currently dominates the total error. They will leave residual errors of only a few % in $\alpha_s(5\ GeV)$, a level of uncertainty far below what is currently obtainable with conventional determinations.

As lattice and perturbative determinations of the strong coupling constant improve, it will eventually become necessary to replace the $\overline{MS}$ coupling constant with a standard of comparison defined from some physical process. This will insure that uncertainties such as those associated with the convergence of perturbation theory, which are intrinsic to only one regulator, not be propagated to all determinations. The process used for the standard of comparison should be one which is easy to calculate in all regulators. The heavy quark potential at short distances is one possible candidate.

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