

Quantizing Effective Strings[★]

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ABSTRACT

This talk reviewed the theory of effective strings, with particular emphasis on the manner in which Lorentz invariance is represented. The quantum properties of an example of an effective string are derived from the underlying field theory. A comparison is made with what one would expect if one assumed that quantum effective strings were governed by fundamental string actions such as the Nambu-Goto or the Polyakov actions. It is shown that the requirements on dimensions for consistent quantizations of fundamental strings imply no contradictions for effective strings.

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There are several contexts when the physics of many particle systems, at some length scale or for some range of parameters, is simplest understood in terms of effective stringlike excitations. In some of these cases, one has a map from an underlying field theory to stringlike variables, and one can derive properties of the resulting ‘string theory’ from the field theory. This kind of ‘explicit’ effective string is the main subject of this review, which will follow for the most part the presentation of [1]. As is well known, fundamental strings have critical dimensions. One way of understanding these special dimensions is to realize that, unless additional degrees of freedom are incorporated, quantum fundamental strings, described by either the Polyakov or the Nambu-Goto actions, are only Lorentz invariant in these dimensions. The motivation for [1], which calculated the induced Lorentz transformations in a theory of effective strings from the underlying field theory, was to figure out how effective strings evade the problem of Lorentz non-invariance.

The resolution we shall find of this apparent paradox is very simple: Lorentz transformations of the effective string have a geometric universal term of dimension -1 that leads to a mixing between the dimension 2 term in the action that describes long distance physics on the string worldsheet, and an irrelevant dimension 4 term that does not affect long distance physics. The key to this structure is the fact that effective strings have a length scale—they are thick. Thus one has stringlike infrared logarithmic divergences in correlations, just as in fundamental strings, but the fact that there is a length scale alters ultraviolet properties. The short distance operator product algebra used to compute the conformal anomaly (which is at the heart of the Lorentz non-invariance of fundamental strings) is irrelevant for the Lorentz invariance of effective strings.

The problem of the quantization of fundamental strings in dimensions other than their critical dimensions, or of effective strings with conformal invariance (if such strings exist), is not addressed here. For a suggestion, see [2], and for further study of this suggestion, see [3].

The rest of the talk is as follows. After listing some theories with explicit stringlike variables, an example in 2+1 dimensions is discussed in detail. This system has been well studied in the literature, *e.g.*, [4,5,6,7]. It is shown how the quantization of the underlying field theory induces one in the effective string theory. In particular, the induced Lorentz transformations are explicitly derived. Comparisons with fundamental strings are made, including a reminder of the comparison made in the classic paper by Nielsen and Olesen[8]. Some ‘implicit’ effective strings are mentioned for contrast at the end.

An example of an explicit string is a domain ‘string’ in 2+1 dimensions, in an Ising model, separating regions on a plane where spins point up from regions where the spins point down. In each region, the order parameter, the magnetization, has a definite sign, while on the string the magnetization goes to zero. Another example is a flux tube in 3+1 dimensions, *e.g.* a Nielsen-Olesen string in the Abelian Higgs model[8]. Physical flux tubes include type II superconductors and cosmic strings. The position of the string is specified by where the order parameter goes to zero (the magnetization for the Ising case, the expectation value of the Higgs field for the gauge case). Vortex rings, for example in hydrodynamics, can also be described as a string theory, sweeping out a two dimensional world sheet. The antisymmetric tensor coupling in the string world sheet action is related to the vorticity[9]. Some of these strings have externally preferred directions (a flux tube in a superconductor has an external magnetic field) or are self avoiding.

For explicit effective strings, as will be shown in detail, it is possible to rewrite a functional integral $\int D\phi e^{iS}$ about a stringlike background in terms of modes (f) that correspond to fluctuations of the string (referred to as string configurations in the following) and other modes (a) that are separated from the string configurations by a mass gap. These modes excite the internal structure of the effective string. They may be integrated out for the purposes of studying the long distance properties of the effective string. Thus one has

$$\int D\phi e^{iS(\phi)} \Big|_{\text{about a string solution}} = \int Df Da e^{iS(f,a)} = \int Df e^{iS_{\text{eff}}(f)} .$$

So an effective string appears as a quantized string theory, a sum over different string configurations weighted by some effective string action S_{eff} .

We will be interested in comparing S_{eff} with natural geometric actions that one would consider for structureless ‘fundamental’ string theories, *e.g.*, the Nambu-Goto action, which is just the area in spacetime of the string worldsheet, $S \sim \int \sqrt{-\det \partial_\mu X^i \partial_\nu X_i}$. An early comparison for a string in the Abelian Higgs model is [8], more recent comparisons include the 1988 TASI lectures[10] which focuses on cosmic strings. In the long wavelength limit for the effective string, and for the structureless string in light cone gauge, the action is proportional to $(\partial_\mu f^i)^2$ where the f^i are the transverse coordinates of the string in spacetime. (That is, $i = 1, \dots, D-2$, f^{D-1} lies along the string and f^D is time.) In addition an explicit effective string has a scale, a width, m^{-1} . Its field theory is nonrenormalizable. There are corrections to the purely geometric action which depend upon the short distance physics of the underlying field theory:

$$S \sim \int (\partial_\mu f^i)^2 + b[(\partial_\mu f^i)^2]^2 + \dots$$

Here $\partial \sim m^{-1}$, *i.e.* the expansion is a long wavelength expansion.

One way to quantize the Nambu-Goto string is to use the Polyakov action, whose classical equations of motion agree with those of Nambu-Goto. The Polyakov action has a larger invariance, the freedom of Weyl rescalings of the intrinsic metric. The conformal anomaly implies that this classical gauge invariance is not a symmetry of the quantum theory. In conformal gauge, this anomaly means one needs either $D = 26$ or the Weyl degree of freedom does not decouple.

Another approach to the Nambu-Goto string is to attempt quantization in light cone gauge. There one finds that unless $D = 26$, Lorentz invariance is lost (the anomaly in the Lorentz algebra vanishes for $D = 2, 3$ but there are still problems with interactions). There is no known consistent quantization of Nambu-Goto strings in any dimension between 3 and 25, which is one of the reasons for interest in how effective strings evade the Nambu-Goto string’s problems.

One procedure for studying effective strings is to take S_{eff} and write the terms relevant for long distance physics in geometric form. Quantizing the resulting geometric classical action results in the usual Nambu-Goto string. However, $\int Df e^{iS_{\text{eff}}(f)}$, induced from the underlying field theory, already defines a quantum string theory, and it is the properties of this theory which will be discussed in the following. We shall see that there is some universal stringlike behavior even though conformal invariance, a usual characteristic of

fundamental strings, does not appear. The steps involved in going from a theory with a stringlike solution to the equations of motion to an effective string action are: (1) to look at fluctuations around the string background; (2) to introduce a string coordinate, integrate out the massive excitations (possible since there is a mass gap between internal excitations and the zero mass excitations which arise due to broken translational symmetry); and then (3) to use the field theory quantization to find the string quantization (*e.g.*, the Lorentz transformations). Another possible route to writing an effective string theory is to assume a string solution and expand in the width of the string[11].

The specific example in the following is a domain string in 2 + 1 dimensions. The Lagrangian is

$$\mathcal{L} = \frac{1}{2} \left[\partial_r \phi \partial^r \phi - \lambda \left(\phi^2 - \frac{m^2}{\lambda} \right)^2 \right],$$

the metric η_{ij} has signature + - -, and the coordinates are $x^r \equiv (t, y, z) \equiv (y^\mu, z)$. This is an Ising-ferromagnetlike system. Some of the classic references for the string description of this theory (called the ‘drumhead model’ in some contexts) are [4,5,6,7] and references therein. This example is used here because it has been so thoroughly studied that many of the calculations can be done explicitly.

There are two minima of the potential, $\phi = \pm m/\sqrt{\lambda}$. The equation of motion is

$$\partial^2 \phi + 2\lambda \left(\phi^2 - \frac{m^2}{\lambda} \right) \phi = 0$$

which admits a solution interpolating between the two minima. Choosing this interpolation to take place along the z direction, the solution can be written as

$$\phi_{\text{cl}}(z) \equiv \frac{m}{\sqrt{\lambda}} \tanh mz.$$

This describes a domain string at $z = 0$. Other solutions with the same boundary conditions exist, corresponding to multiple string configurations, but will not be considered. It should be kept in mind that a long straight string is unstable: as will be seen, its motion is described by two dimensional massless bosonic fields, which have an infrared logarithmic divergence in their correlators. To define things carefully appropriate boundary conditions, or a small mass term, should be included in the discussion.

The field theory in this background is

$$\mathcal{L}(\phi_{\text{cl}} + \tilde{\xi}) = \mathcal{L}(\phi_{\text{cl}}) - \tilde{\xi}(\square + \Omega(z))\tilde{\xi} + O(\tilde{\xi}^3)$$

where $\square = \partial_t^2 - \partial_y^2$ and $\Omega = -(\partial_z - 2\sqrt{\lambda}\phi_{\text{cl}}(z))(\partial_z + 2\sqrt{\lambda}\phi_{\text{cl}}(z))$. The explicit form of ϕ_{cl} was used to rewrite \mathcal{L} . The spectrum and eigenfunctions of the quadratic fluctuation operators around this background are known. For \square the eigenfunctions are plane waves in (t, y) :

$e^{i(\omega t + k_y y)}$. For Ω :

eigenvalue eigenfunction

$$0 \quad \psi_0 = \frac{\sqrt{3m}}{2} \operatorname{sech}^2(mz) \equiv \sqrt{\frac{3\lambda}{4m^3}} \phi'_{\text{cl}}(z),$$

*** gap

$$3m^2 \quad \psi_1 = \sqrt{\frac{3m}{2}} \operatorname{sech}(mz) \tanh(mz),$$

$$k^2 + 4m^2 \quad \psi_k = \frac{\sqrt{m} \exp(ikz)}{\sqrt{k^4 + 5k^2m^2 + 4m^2}} \left[3 \tanh^2(mz) - \frac{3ik}{m} \tanh(mz) - \frac{k^2}{m^2} - 1 \right]$$

$$-\infty < k < \infty$$

These modes have the following physical significance:

- (a) The zero mode $\psi_0 \propto \partial_z \phi_{\text{cl}}$ is the Nambu-Goldstone boson corresponding to the translation invariance broken by the string, $\phi_{\text{cl}}(z + \beta) = \phi_{\text{cl}}(z) + 4m^3/3\psi_0(z) + \dots$
- (b) The mode with mass $\sqrt{3}m$ is also localized on the string and is referred to as the kink 'excitation' in the literature. It corresponds to a squeezing of the string: $\phi_{\text{cl}}(z(1+\beta)) = \phi_{\text{cl}}(z) + \beta z \partial_z \phi_{\text{cl}}(z)$. For this specific case, the normalized overlap of $z d\phi_{\text{cl}}/dz$ and ψ_1 is $\pi\sqrt{3}/\sqrt{8\pi^2 - 48} \approx 0.978$.
- (c) A continuum starting at mass $2m$, with k taking arbitrary real values. These extended modes are the counterparts of the spectrum obtained when expanding about a homogeneous background $\phi_{\text{cl}}(x) = \pm m/\sqrt{\lambda}$.

Naively in quantizing this system one would assume that fluctuations $\tilde{\xi}$ around the background are small. This is not true of the zero mode, which describes a fluctuation with no damping. One could have put the domain string anywhere. To quantize this system correctly one treats the zero mode exactly by introducing an (implicit) collective coordinate. As a result of introducing the collective coordinate the position of the string $f(t, y)$ will be introduced into the classical solution, $\phi_{\text{cl}}(z) \rightarrow \phi_{\text{cl}}(z - f(t, y))$. All $f(t, y)$ will be integrated over, and the zero mode in $\tilde{\xi}$ will be projected out.

The introduction of collective coordinates is done by analogy with Fadeev-Popov ghosts. Collective coordinates for solitons are due to [12], the method applied to implicit collective coordinates is due to [13,14]. A pedagogical treatment of the general idea can be found in [15]. Use the identity

$$\int Df(t, y) \delta(g(f)) \left| \frac{\delta g}{\delta f} \right| = 1$$

inside the functional integral for ϕ : $\int D\phi e^{iS(\phi)}$. For the case here choose $g(f) = \int dz \partial_z \phi_{\text{cl}}(z - f(t, y)) \phi(z)$. In the functional integral, $\delta(g(f))$ projects out the zero mode, $\tilde{\xi} \rightarrow \xi$ and introduces an integral over the position of the string $f(t, y)$ into the measure.

The explicit form of ϕ_{cl} describes one kink, so it is being assumed that there are no overhangs, that the string is only at one z position. This means multikink solutions are neglected. These are down by e^{-mR} in the functional integral, where R is the length of the

string, when the kinks are widely separated. The Jacobian,

$$\left| \frac{\delta g}{\delta f} \right| = \int dz \partial_z \phi_{\text{cl}}(z - f(t, y)) \partial_z \phi(z) \equiv \Delta(\xi)$$

is independent of $f(t, y)$. It can be set to one using dimensional regularization[4,7].

To introduce the collective coordinate $f(t, y)$ into the rest of the action write

$$\phi(t, y, z) \equiv \phi_{\text{cl}}(z - f(t, y)) + \xi(t, y, z - f(t, y)),$$

and plug in to the functional integral to get

$$Z = \int D\xi(t, y, z) Df(t, y) e^{iS(\phi_{\text{cl}} + \xi)} \Delta(\xi) + \text{multistring configurations}.$$

All the f dependence is in the action S only, not in Δ . The measure for f is ultralocal, depending only on the value of f at a given point, not upon derivatives of f . The measure for ξ is that implied by its decomposition in terms of eigenmodes of Ω :

$$\xi(t, y, z + \alpha) = a_1(t, y) \psi_1(z + \alpha) + \int dk a_k(t, y) \psi_k(z + \alpha)$$

and the coefficient of ψ_0 has been set to zero by the delta function in the functional integral. The integral over k is schematic, it is not necessary to be precise since loop effects will not be considered.

Substituting, scaling out m, λ and making all the f dependence explicit, the action becomes

$$S = -\frac{m}{\lambda} \int d^3x \left\{ \phi'_{\text{cl}}{}^2 - \frac{1}{2} \partial_\mu f \partial^\mu f [\phi'_{\text{cl}} + \xi']^2 - \frac{1}{2} \xi \left(-\partial_\mu \partial^\mu - \hat{\Omega} \right) \xi \right. \\ \left. + 2\phi_{\text{cl}} \xi^3 + \frac{1}{2} \xi^4 + \partial_\mu \xi \partial^\mu f \xi' \right\}.$$

The operator $\hat{\Omega}$ has no zero modes and primes denote ∂_z . This action describes the two dimensional field $f(t, y)$ interacting with the massive three dimensional field $\xi(t, y, z)$, corresponding to the rest of the degrees of freedom in this background. This procedure also is the one used for studying field theories in soliton backgrounds.

Here the goal is to consider the effective field theory of the domain string with position at

$$X^2 = f(t, y), \quad X^1 = y, \quad X^0 = t.$$

As a two dimensional field theory this is a string (in a certain gauge) interacting with massive fields $a_k(t, y)$. To get the theory of the string alone, eliminate the other degrees of freedom, ξ , by using the equations of motion. (This is leading order in \hbar and corresponds to a saddle point expansion for the heavy field ξ in the functional integral):

$$\xi = -\hat{\Omega}^{-1} \phi''_{\text{cl}} (\partial f)^2 + \hat{\Omega}^{-1} \left[\partial_z^2 (\partial f)^2 - 6\phi_{\text{cl}} \hat{\Omega}^{-1} \phi''_{\text{cl}} (\partial f)^2 \right] \hat{\Omega}^{-1} \phi''_{\text{cl}} (\partial f)^2 \\ + \hat{\Omega}^{-2} \phi''_{\text{cl}} \partial_\mu \partial^\mu (\partial f)^2 + \dots$$

This is a long wavelength expansion ($\partial \sim m^{-1}$). Since $\hat{\Omega}$ has no zero mode it is invertible.

The action for one domain string, with the massive fields integrated out is then

$$S(f) = - \int dt dy \left[\frac{1}{2\pi\alpha'} \left(1 - \frac{1}{2}(\partial f)^2 - \frac{1}{8}((\partial f)^2)^2 - \frac{1}{16}((\partial f)^2)^3 + \dots \right) \right. \\ \left. + b(\partial f)^2 \square(\partial f)^2 + \dots \right]. \\ \frac{1}{2\pi\alpha'} = \frac{m}{\lambda} \int dz (\phi'_{\text{cl}})^2, \quad b = \frac{m}{8\lambda} \int dz (z\phi'_{\text{cl}})^2.$$

As f is a Nambu-Goldstone boson, coming from the breaking of translation invariance, only derivatives of it appear. The first three terms in the derivative expansion (including the constant) come from the kinetic terms in the original action. They are independent of the details of the potential except for the overall factor of α' . The top line in S (as was shown by [5]) is the Nambu-Goto action for the string $S_{N-G} = \sqrt{1 - (\partial f)^2} = \sqrt{-\det h_{\mu\nu}}$. The induced metric on the string world sheet $h_{\mu\nu}$ is $h_{\mu\nu} = \frac{\partial X^i}{\partial y^\mu} \frac{\partial X^j}{\partial y^\nu} = \eta_{\mu\nu} - \partial_\mu f \partial_\nu f$. The second line is partially the intrinsic curvature, but also has a contribution that is not in any obvious way geometrical. Since the coefficient of this term is not universal, this is not surprising. This expansion is up to $O(\partial^8, \hbar, \text{boundary terms})$. To include higher order \hbar effects, one needs to include loop effects in the underlying field theory and then find the new solution to the equations of motion and expand around it. If regularization schemes other than dimensional regularization are used for the Jacobian Δ , it may also contribute at order \hbar .

The quantization of the underlying field theory induces a quantization of the string theory. For instance, to find the Lorentz transformations in the theory, start with the transformations in the ϕ field theory. This example is worked out in detail in [1]. The canonical Lorentz generators are $M_{rs} \equiv \int dy dz [j_{0r}x_s - j_{0s}x_r]$, where $j_{rs} \equiv -\eta_{rs}\mathcal{L} + \partial_r\phi\partial_s\phi$ are the translation currents. Upon quantization, M_{rs} becomes an operator: in the usual way $P_\phi = \partial_0\phi = -i\delta/\delta\phi$ where

$$[P_\phi(y, z), \phi(y', z')]_{\text{e.t.}} = -i\delta(y - y')\delta(z - z').$$

This induces a quantization of the string coordinate, $[P_f(y), f(y')]_{\text{e.t.}} = -i\delta(y - y')$. One can rewrite P_ϕ using the chain rule (using the components of ξ , the a_k , and $\langle g|h \rangle = \int dz g(z)h(z)$):

$$P_\phi = -i \frac{-\phi'_{\text{cl}}(z - f(t, y))}{\Delta(\hat{a})} \frac{\delta}{\delta f} + \left[-\frac{\phi'_{\text{cl}}(z - f(t, y))\hat{a}_k \langle i|\partial_z|k \rangle}{\Delta(\hat{a})} + \psi_i(z - f(t, y)) \right] \frac{\delta}{\delta \hat{a}_i}.$$

Ordering ambiguities will change the transformation in subleading order in \hbar . These will not introduce anomalies because it is known that it is possible to regulate the underlying ϕ^4 theory and keep Lorentz invariance. So in terms of the field theory for f and the components

of ξ , the Lorentz transformations become

$$\begin{aligned}
[M_{0y}, f] &= i(t\partial_y f + y\partial_0 f) \\
[M_{0y}, \xi] &= i(t\partial_y \xi + y\partial_0 \xi) \\
[M_{\mu z}, f] &= i \left[-y_\mu + f\partial_\mu f + \frac{\partial_\mu f}{\Delta(\hat{a})} a_j \langle 0|z\partial_z|j\rangle - \frac{\partial_\mu a_j}{\Delta(\hat{a})} \langle 0|z|j\rangle \right] \\
[M_{\mu z}, a_j] &= i \left[\partial_\mu a_k \langle j|z|k\rangle - \partial_\mu a_i a_k \frac{\langle 0|z|i\rangle \langle j|\partial_z|k\rangle}{\Delta(\hat{a})} \right. \\
&\quad \left. - \partial_\mu f \left(\langle j|z|0\rangle + \langle j|z\partial_z|k\rangle a_k - a_i a_k \frac{\langle 0|z\partial_z|k\rangle \langle j|\partial_z|i\rangle}{\Delta(\hat{a})} \right) + f\partial_\mu a_j \right].
\end{aligned}$$

Again it is possible to get rid of ξ by substituting its derivative expansion (good for low energies) in terms of f . Since ξ scales as ∂^2 , the first terms depending on the massive modes ξ in the Lorentz transformations come in at order ∂^3 . In the action dependence upon ξ enters first at order ∂^4 . The universal (independent of the potential) parts of the Lagrangian for a theory with canonical kinetic term are

$$\frac{1}{2\pi\alpha'} \left[1 - \frac{1}{2}(\partial f)^2 - \frac{1}{8}(\partial f)^4 \right]$$

and the corresponding universal parts of the Lorentz transformation which are symmetries of this up to $O(\partial^6)$ are

$$[M_{\mu z}, f] = i[-y_\mu + f\partial_\mu f] \quad [M_{0y}, f] = i(t\partial_y f + y\partial_0 f).$$

One can see that to this order the Lorentz generators form a representation of the Lorentz algebra. The irrelevant term in the action proportional to $(\partial f)^4$ gives an indication that S is proportional to area (the preceding lower order term is the action for a free scalar field). Since the Lorentz transformations begin with a universal dimension -1 term, y_μ (for a standard kinetic term for the underlying field theory), there are cancellations between renormalizable and nonrenormalizable terms in S_{eff} under the symmetry group. The nonrenormalizable terms reflect dependence upon the short distance behavior of the underlying field theory. These symmetries are up to order ∂^6 , \hbar^2 and boundary terms. One can also check explicitly the invariance of the measure to this order.

The nonlinear transformations for the field f are appropriate for a Nambu-Goldstone boson. One can turn the logic around and say that since the string breaks translation invariance, the action for its Nambu-Goldstone bosons is determined by the nonlinear transformations under the broken symmetry. Using the Volkov-Akulov formalism, [16] did this for the super Nielsen-Olesen vortex.

It is possible to compare this to the fundamental string in the light cone gauge, where one imposes the conformal invariance constraints $T_{++} = T_{--} = (\partial_t X \pm \partial_y X)^2 = 0$ on the Polyakov string in conformal gauge. One chooses $X^+ = x^+ + p^+ \tau$ and then uses the constraint on $T_{\pm\pm}$ to solve for X^- in terms of X^2 . With this choice the complete action is $S = \int \beta(\partial_\mu X^2)^2$, which is renormalizable. However, the conformal anomaly for dimension

$D \neq 26$ means that the constraints cannot be imposed, *i.e.*, solving for some of the degrees of freedom using the constraints is inconsistent. One way this shows up is that the Lorentz transformations do not close in dimensions larger than 3. In 3 dimensions, there is only one nonlinear Lorentz generator and there is no problem with the closure of the algebra—one has to go to the interacting string to see problems.

To compare directly to the induced string theory in light cone gauge, one would need to quantize the ϕ^4 theory in light cone gauge. Quantization in light cone gauge is rather difficult to do for interacting (non-integrable) scalar field theories. The gauge implied by the standard quantization of the underlying ϕ^4 field theory is not light cone, as one can check explicitly. Another comparison of light cone gauge fundamental and effective strings was made by Olesen[17]. He put unusual boundary conditions on Nambu-Goto strings of length R and showed that the Lorentz anomaly was proportional to α'/R^2 , which disappears as R gets large.

The effective strings approximate structureless strings for length scales $\gg m^{-1}$. Besides the long wavelength limit, one can make another comparison with structureless strings, as Nielsen and Olesen did in their original paper on flux tubes in the Abelian Higgs model[8]. The solution $\phi_{cl}(m, \sqrt{\lambda}, z)$ depends on the parameters m, λ of the potential. They asked the question: is it possible to tune λ so that fluctuations on the scale m^{-1} don't excite the string width? (They were looking for structureless strings in $D \neq 26$.) The energy of fluctuations associated with the field f to leading order is $S \sim \frac{1}{\alpha'} \int (\partial f)^2$ where the tension $\alpha' \sim [L^2]$, scales as length squared. So for large α' fluctuations have low energy, and for small α' fluctuations have large energy. Asking the energy of fluctuations to be too small to excite the internal structure of the string means $\sqrt{\alpha'} \gg m^{-1}$. In terms of the model discussed in this talk, $1/\alpha' = \int dz (\phi'_{cl})^2 \sim m^3/\lambda$. So the constraint becomes $\lambda \gg m$. Since the effective field theory was a derivative expansion assuming m large, taking λ larger is very strong coupling—analysis done in perturbation theory cannot be trusted anymore, and all bets are off. (For the Nielsen-Olesen vortex their constraint was also strong coupling, $e \gg 1$.) This is like the $\hbar \rightarrow \infty$ limit, as they put it, so expanding about a classical solution in a functional integral is meaningless. For example the ϕ^4 theory, when Euclidean, is in the universality class of the high temperature Ising model in this limit.

Before closing, it is worthwhile to mention some implicit strings for contrast. For these, stringlike behavior is seen or expected but the strings are not fully characterized. One example is compact QED in $2 + 1$ dimensions. Polyakov[18] demonstrated that there is a linear potential between charges, which could be attributed to a string. Another classic example is QCD, where string theory was first used. At long distance, quarks and gluons are confined and one wants to use variables providing a natural description of the physics. Strings are a popular paradigm, but the specific definition of a QCD string is unclear. For perspective, we mention three clues that suggest strings as a good description.

First of all, in the data Regge behavior was observed, *i.e.*, for mesons and baryons, one found the relation $M^2 \sim J$ which one can model by a relativistic string (all the energy comes from stretching it, $\rho = T$, density = tension), and the ends move at the speed of light. Then $J \propto MvR = ML$ and $M = TL$, and so one gets the above mass/angular momentum relation. This rule has been seen to be a good heuristic up to $J = 19/2$, with the relation getting better and better at higher spin[19].

Secondly, strings were expected from lattice descriptions of QCD. At strong coupling[20],

one can rewrite the sum of gauge configurations as the sum over surfaces. Initially, it was thought that this rewriting was in terms of noninteracting surfaces, but Weingarten showed otherwise[21]. One could try to fix up the noninteracting surface theory to account for the interactions in various ways, for example by adding degrees of freedom, but the resulting models became quite involved and hard to work with.

A third connection between strings and QCD was found in the field theory description of QCD by 't Hooft[22]. In the large N limit of $SU(N)$ gauge theory (fixing g^2N), planar diagrams dominated and could be viewed as tracing out the world sheet of a string (in group space). Large N accounts for some aspects of phenomenology, so it might be expected that the stringlike picture coming from large N has some validity. In addition to the vast amount of work done on QCD and strings several years ago, there have been a few attempts to look at strings and QCD again. I would like to mention in particular the paper by Polchinski[23] and the review [24]. In [25] there is a summary of a lot of the older data with the aim of isolating what particularly stringy properties it implies.

In conclusion, this was a review of the quantization of effective strings, in the case where there were explicit stringlike solutions to the equation of motion. The procedure was to rewrite the field theory in the one string sector, introduce collective coordinates for the position of the string and use equations of motion to substitute for the massive fields. Only physical degrees of freedom were present in the action. The result was a nonrenormalizable effective field theory on the string worldsheet, which had terms to arbitrarily high order in the derivative expansion, and leading order in \hbar . The fundamental string's constraints had no relevance for the effective string, which was not conformally invariant and at short distances was no longer a string. For example, Lorentz transformations mixed relevant and irrelevant terms in the action for the effective string. It was possible to take the thin string limit by going to long wavelengths. Naively, strong coupling would also give a thin string limit, but on closer inspection, in this limit, all of the analysis has dubious validity.

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