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Next-to-Leading Order Calculations In Jet Physics

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Abstract

We discuss a general method of calculating next-to-leading order cross sections and distributions for processes with multiple jets in the final state. The method utilizes universal building blocks for initial- and final-state radiation. We also present applications of such a method to $W + 0, 1$ jet processes at the Tevatron.

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1. Introduction

The searches for new physics at present and future hadron colliders, the Tevatron, the LHC, and the SSC, demand that we first develop a detailed understanding of QCD processes such as multijet production, and of QCD-associated processes such as production of Ws or Zs in association with jets. The importance of such understanding is illustrated by the search for one of the two missing ingredients of the minimal standard model, the top quark. In searching for the top quark in the single lepton + missing energy + jets mode, one must demand four distinct identified jets in addition to the isolated lepton, in order to produce an acceptable signal-to-background ratio [1]; and if one imposes such a requirement, one finds that the contribution to this final state from top-quark decay is in fact greater than the direct QCD production of a W with four jets.

It is the strength of the QCD coupling constant that forces us to such high orders in the perturbation expansion. In addition to the higher-multiplicity calculations hinted at above, developing a detailed understanding of QCD processes also requires the calculation of higher-order quantum corrections to processes with a fixed number of jets. From a theoretical point of view, such a program begins with the calculation of next-to-leading order corrections to the simplest processes, two-jet production, or vector boson + 0,1 jet production.

Beyond the general desire to refine the ability to calculate and predict jet cross-sections and distributions, there are specific failings of leading-order calculations (given by tree graphs alone), which can be alleviated through the calculation of the next order in perturbation theory. At leading order, for example, the matrix element has no dependence on the renormalization scale μ, but as one conventionally uses the one-loop running coupling constant, the latter does depend on μ. As a result, computed cross-sections and distributions have a strong dependence on the choice of this unphysical parameter. In a next-to-leading order calculation, the virtual corrections to the lowest-order hard-scattering matrix elements do have a dependence on μ, which helps compensate for the dependence in the coupling constant. For certain observables, the next-to-leading result can be quite insensitive to the choice of μ over a relatively wide range. This is illustrated in fig. 1, where we show the μ dependence of the leading-order and next-to-leading order predictions for the cross-section for $W+1$ jet (with 'standard' CDF cuts, $E_{\mathrm{Tjet}} \geq 15\,\mathrm{GeV}$, $E_{\mathrm{Tlepton}}^{\mathrm{min}} \geq 20\,\mathrm{GeV}$, $E_{\mathrm{Tmissing}} \geq 20\,\mathrm{GeV}$, $|\eta_{\mathrm{jet}}| \leq 2$, $|\eta_{\mathrm{lepton}}| \leq 1$, the jet cone size $\Delta R = 0.7$). The next-to-leading order result is considerably less sensitive to the choice of μ.

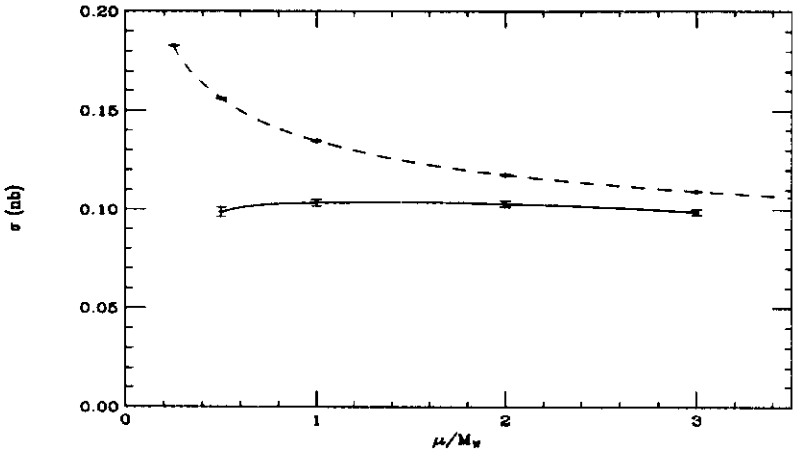

Figure 1. The renormalization scale dependence of the $W+1$ jet cross section as a function of the ratio of the renormalization scale and the W mass. (Dashed line is LO, solid line is NLO.)

Because the theory contains massless particles, the perturbation expansion in QCD is not given in powers of the strong coupling constant α_s alone, but in powers of the strong coupling constant times logarithms in the jet resolution parameters, the jet cone size $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ and the minimum transverse energy $E_T^{\rm min}$ (in the form E_T^2/Q^2, where Q^2 is some characteristic hard scale in the process). If the resolution parameters are too small (that is the cuts are too 'weak'), these logarithms will become large and spoil the applicability of perturbation theory. In a leading-order calculation, there is no warning of this breakdown, whereas in a next-to-leading order calculations, these dangerous logarithms are calculated explicitly. Indeed, in the lowest-multiplicity processes, the leading-order results have *no* dependence on the resolution parameters: the $W+1$ jet and two-jet cross-sections are independent of the jet cone size, and the $W+0$ jet cross-section is independent of the minimum transverse energy. This dependence arises in a next-to-leading order calculation through the presence of real radiation inside the jet cone (that is, the possibility of a two outgoing partons forming a single jet), or through the presence of (soft) real radiation outside any jet cones. These effects are illustrated in fig. 2, showing the dependence on $E_T^{\rm min}$ in the $W+0$ jet cross-section (fig. 2a), and the dependence on ΔR in the $W+1$ jet cross-section (fig 2b).

2. Theoretical Considerations

In calculating the NLO multijet cross sections it is essential that the phase space

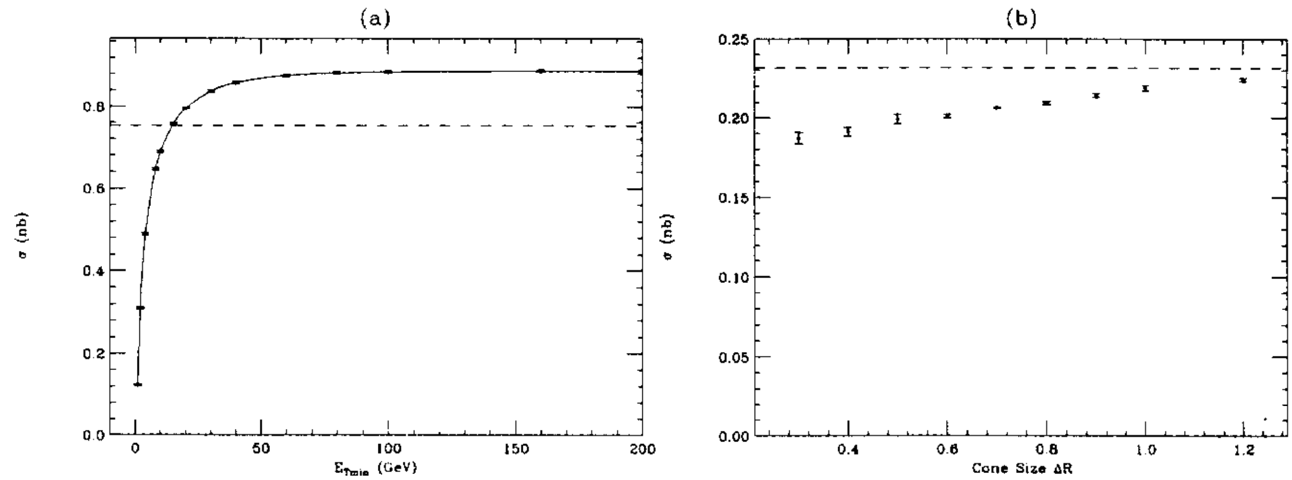

Figure 2. The dependence of the cross sections on the jet resolution parameters with standard CDF cuts. Figure 2a gives the jet $E_T^{\rm min}$ dependence of the $W + 0$ jet cross section. Figure 2b gives the cone size dependence of the $W + 1$ jet cross section. (In both figures LO is given by the dashed line, NLO is given by the points with errorbars.)

integrals be performed numerically, for example via a Monte Carlo integration technique. This allows one to take into account detector acceptance restrictions and to implement experimental jet algorithms, without any concessions to "theoretical prejudice". It also allows one to keep all correlations of the final state jets and possible leptons.

Methods for performing the hard phase space integrals numerically have been developed most fully in QED, for the inclusion of photon radiation in scattering processes [2]. The essential ingredient in this method is the factorization of the cross section in the soft photon limit into an general eikonal factor containing the soft singularity multiplied by the hard cross section. Only the trivial eikonal factor has to be regulated and integrated over the soft photon region, while the remaining hard cross section can be calculated in the usual way and integrated numerically over the hard phase space.

In QCD there is no analogous soft gluon factorization, because the gluons themselves carry color charge. Furthermore, for initial state partons we must perform a mass factorization to absorb the initial-state collinear divergences in the structure functions. Within the traditional approach, the lack of factorization forces one to first square both virtual and the bremsstrahlung diagrams in their regularized form and extract the divergent pieces by hand. After that one can again use the Monte Carlo approach to deal with the hard phase space integrals [3]. This method is cumbersome and involves manipulation of a large number of terms, making its extension to more complicated jet cross sections very difficult;

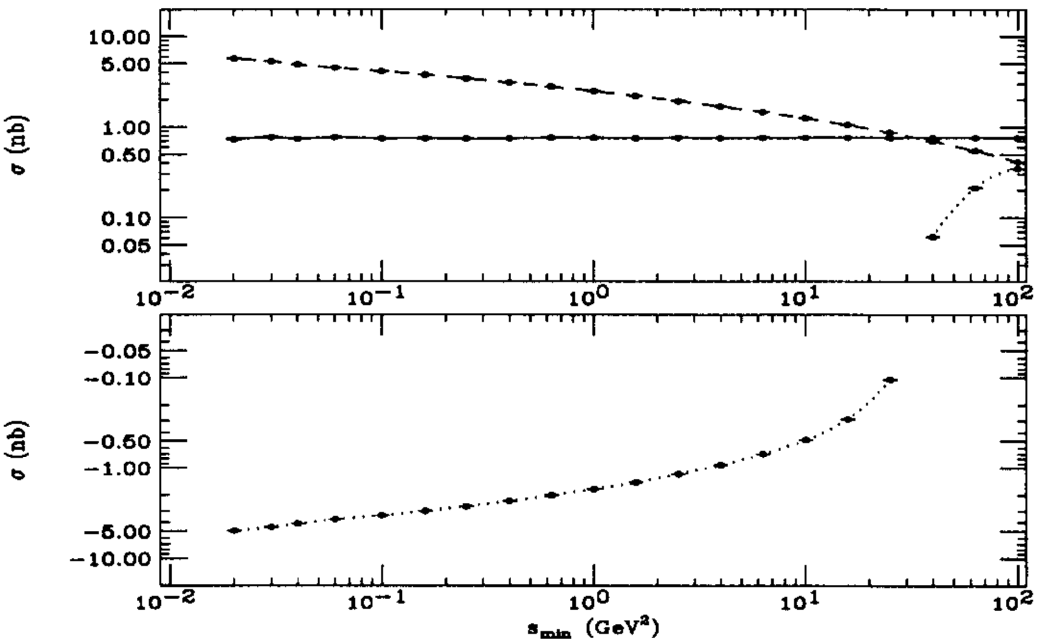

Figure 3. Cancelation of the theoretical resolution parameter for the $W + 0$ jet cross section. (Dotted curve is the soft/collinear plus virtual contribution, dashed curve is the hard resolved parton contribution and the solid line is the sum of both contributions.)

furthermore the procedure has to be repeated for each new process.

To extend the calculation to more complex processes one should understand better the singular behaviour of the matrix elements, and establish a QED-type factorization of the soft gluon singularities. A suitable factorization was found by several authors [4] through the introduction of color ordered amplitudes [5]. These gauge invariant ordered amplitudes have similar properties to QED amplitudes and exhibit the desired soft gluon behavior.

The soft-gluon factorization of these building blocks allows one to use methods developed in QED to integrate out analytically the soft and collinear region of the bremsstrahlung phase space. This scheme was applied to study the singular behavior of final state partons and isolate the singular parts of phase space using the Lorentz invariant minimal invariant mass $s_{\min}$ as the theoretical resolution parameter [6]. If the invariant mass of a parton pair is smaller than the minimal invariant mass, its singular contribution is integrated out within the $s_{\min}$ cone. This is then added to the virtual corrections, thus canceling the final state soft and collinear divergences. The soft and collinear factors are universal, easy to evaluate in their regularized form and independent of the hard process. This makes the approach very general and applicable to any process involving final state partons. Finally one integrates over the remaining hard phase space using a Monte Carlo technique. In

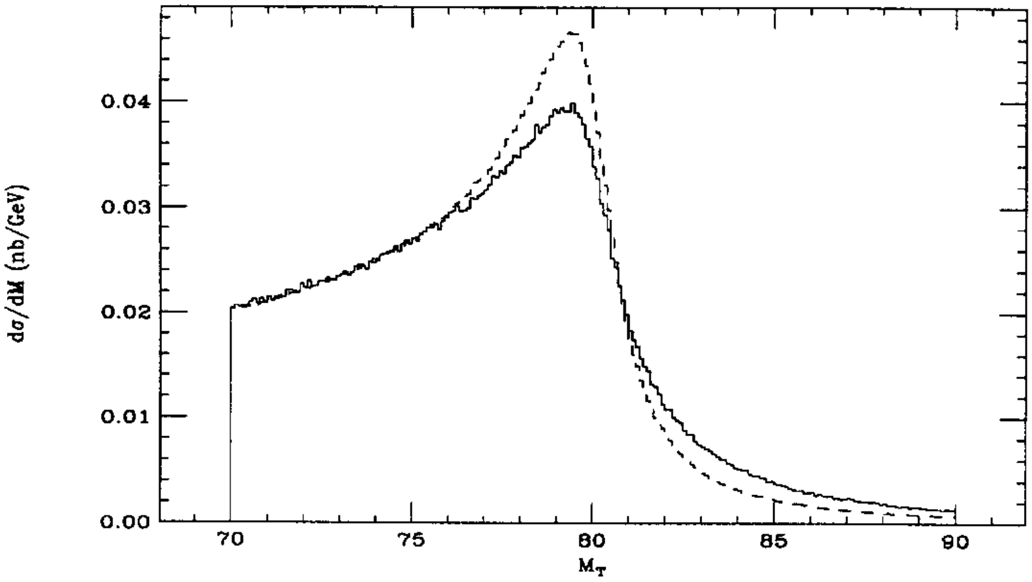

Figure 4. The transverse mass distribution for $W + 0$ jets. (Dashed line is LO, solid line is NLO.)

the sum, the dependence on the theoretical parameter $s_{\min}$ disappears. This is illustrated in fig. 3 for $p\bar{p} \rightarrow W + 0$ jets. The figure shows the two independent contributions, virtual+soft+collinear (unresolved partons) and real emission (resolved partons within the jet cone, or below minimum energy cut-offs), each growing logarithmically; the former are negative, the latter positive, and the sum is independent of $s_{\min}$. The $s_{\min}$-independence of the result as $s_{\min} \rightarrow 0$ also provides a strong check on the calculation and program; any error which disturbs the cancellation of $\ln s_{\min}$'s will yield a logarithmically diverging answer.

To include initial state radiation we can cross the unphysical but finite all-outgoing parton cross section and perform an analytic continuation of the appropriate transcendental functions. We then introduce a universal crossing function, which is the appropriate initial state collinear factor integrated over the correct phase space within the s_{min} cone minus the improperly added final state collinear factor. This yields the correct treatment of the initial state collinear behaviour. Only the crossing function is affected by mass factorization which then renders it finite. The crossing function is unique for each type of incoming parton; it is a simple convolution of a splitting functions with the appropriate structure functions and is independent of the hard process. Once calculated, it can be used for all processes involving incoming partons regardless of the final state, and without further consideration of the question of mass factorization. The crossing functions are analogous to structure functions in both appearance and use [7].

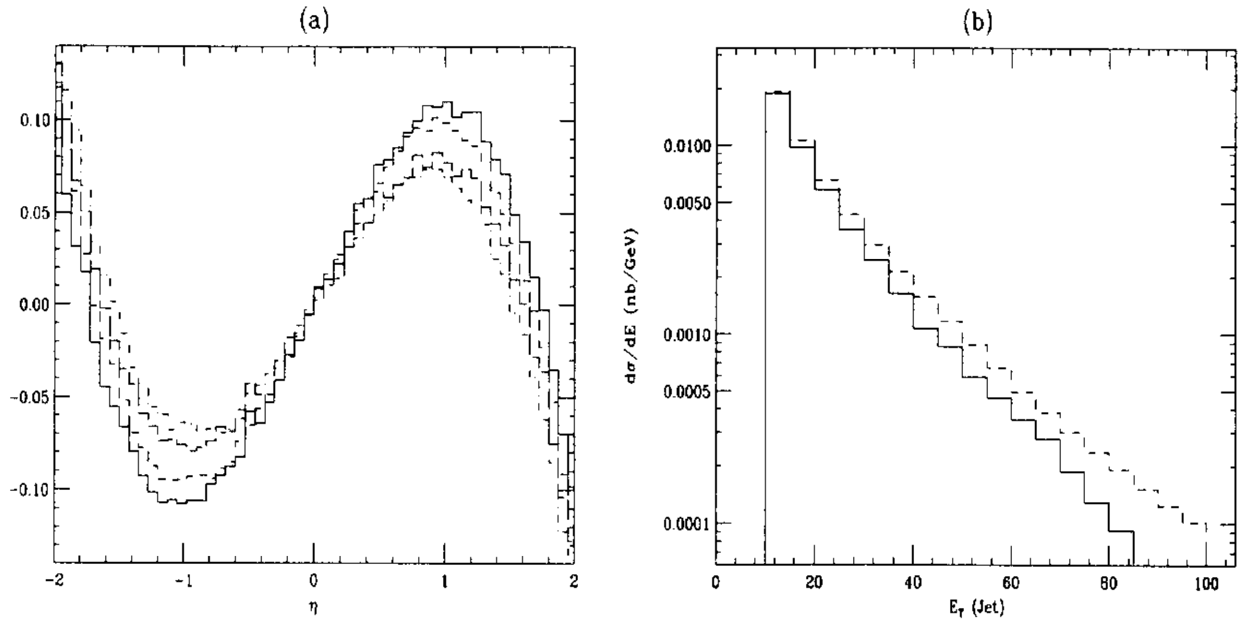

Figure 5a. The lepton asymmetry as a function of the charged lepton rapidity in the $W+0$ jet cross section for two sets of structure functions, HMRSB (dashed line is LO, solid line is NLO) and KMRSB0 (dotted-dashed line is LO, long-dashed line is NLO). **Figure 5b.** The jet transverse energy differential $W+1$ jet cross section. (Dashed line is LO, solid line is NLO.)

3. Phenomenology

With a numerical program implementing the scheme described in the previous section, one can investigate the importance of next-to-leading order corrections to a variety of experimentally-measurable quantities. We display three examples here.

The transverse mass of the W boson, in terms of the electron momentum and the missing E_T vector, can be used to measure the W mass. This distribution is relatively stable against radiative corrections, as shown in fig. 4. For the particular cuts used, the total cross section is nearly the same at leading as at next-to-leading order. There is a radiative tail above the W mass which is compensated by a lower cross section in the region around the peak.

The lepton asymmetry in $W+0$ jets is very stable against radiative corrections, as can be seen in fig. 5a; on the other hand, in the region near rapidities of 1, it has some sensitivity to the structure function used. The asymmetry could thus be used to constrain the form of the structure functions [8].

The jet E_T distribution in $W+1$ jet, on the other hand, shows a rather dramatic softening at next-to-leading order compared to the leading-order prediction, as can be seen in fig. 5b.

The computations displayed in the figures herein were performed on the Fermilab

ACPMAPS parallel machine, and we thank the Fermilab lattice group for providing us with computer time on it.

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