Fermi National Accelerator Laboratory

FERMILAB-Conf-92/209

An Improved Imaginary Transition γ_t Lattice for the Main Injector

K.Y. Ng and D. Trbojevic

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

August 1992

Presented at the *XVth International Conference on High Energy Accelerators,* Hamburg, Germany, July 20-24, 1992

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CHO3000 with the United States Department of Energy

Disclaimer

An Improved Imaginary γ_t Lattice for the Main Injector

K.Y. Ng and D. Trbojevic
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, U.S.A.*

Abstract

An improved imaginary-γ_t lattice for the 150 GeV Fermilab Main Injector is presented. It has the properties of small dispersion function, good tunability, small tune dependences on momentum with the presence of chromaticity sextupoles, and a large dynamical aperture. In addition, many of the quadrupoles can be recycled from the present Main Ring.

1 Introduction

An improved imaginary-γ_t lattice has been designed for the Fermilab Main Injector. Compared with the previous designs [1, 2], this lattice is much more compact, recycles a lot of Fermilab Main Ring quads, and possesses much better beam dynamic properties. Some highlights are presented here and the details can be found in Ref. 3.

2 The lattice

The lattice has been designed similar to the previous ones. It consists of matched building blocks called modules which can be put one after the other. The basic module (BASIC) consists of 2 FODO cells joined together by a low-beta insertion, where 2 doublets (instead of triplets [2]) are used (Fig. 1). The standard quads in the FODO part are 0.988 m long. For each doublet, a 2.1336 m Main Ring quad is used as the D quad, while a 2.1336 m and a 1.31953 m Main Ring quad combine together to form the F quad.

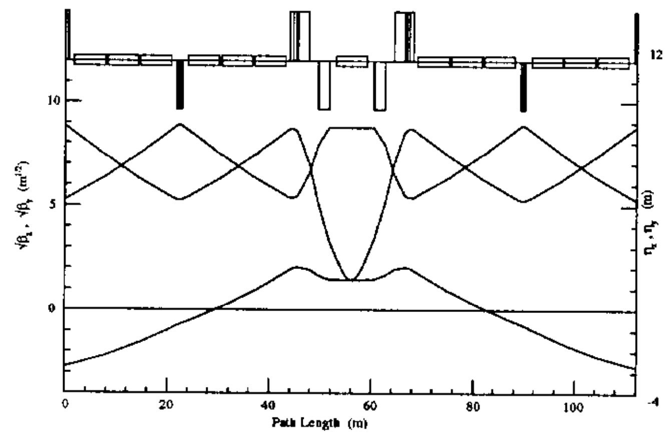

Fig. 1. The BASIC module.

*Operated by the Universities Research Association, Inc., under contract with the U.S. Department of Energy.

The injection/extraction block consists of 2 BASIC modules denoted by EXTR1 and EXTR2, one with 4 and one with 6 dipoles removed to provide for spaces to accomodate the kicker, which is to be placed in one module, and the Lambertson to be placed in the other.

The zero-dispersion straight (RFST) starts with a basic module. Instead of ending at a point with the required matching Twiss parameters, it ends with zero dispersion, because some dipoles have been removed to create the effect of dispersion suppression. We continue it with 2 zero-dispersion FODO cells of total length of ~ 80 m, and then join it with the same dispersion-suppressed basic module in the reverse direction.

The whole ring has a two-fold symmetry; each superperiod consists of the configuration [RFST, EXTR1, EXTR2, 5(BASIC), EXTR2, EXTR1, BASIC, EXTR2, EXTR1]. In total, there are 300 identical dipoles, 92 standard FODO quads, 56 identical low-beta doublets, and only two buses producing gradients of 220.6 kG/m and -216.8 kG/m for the quads. The transition gamma is $\gamma_t = i27.3$, the tunes are $\nu_x = 21.424$, $\nu_y = 11.211$, the maximum betas are $\beta_x = 89.3$ m and $\beta_y = 89.8$ m, and the dispersion ranges from -3.11 to $+3.15$ m.

3 Chromaticity and Momentum Dependence

The lattice has natural chromaticities $\xi_x = -39.18$ and $\xi_y = -19.98$. The chromaticity correction sextupoles consist of 1 SEXH, 2 SEX1's, and 2 SEXV's for each of the 24 modules but not the RFST's. For complete compensation, the SEXH's and SEX1's share the same bus with $k_{sf} = \mp 0.0565$ m^{-2}, while for the SEXV's $k_{sf} = -0.104$ m^{-2}. Because the chromaticities do not change sign during acceleration in the absence of transition, spaces of 0.4 m reserved for a SEXH and SEX1, and 0.5 m for a SEXV will be quite adequate.

The dependence of lattice properties on momentum is summarized in Table I, where we vary the momentum by $\pm 1.5\%$, although the estimated momentum spread in the future Main Injector should be less than $\pm 0.2\%$. It can be shown that because of the way dispersion is controlled, the imaginary-γ_t lattice provides momentum acceptance which is almost twice better than a conventional lattice composed of the same FODO cells that we use in our modules.

Table I: Off-momentum properties of the lattice.

$\Delta p/p_0$	1.5%	−1.5%
$\Delta\sqrt{\beta_x}/\sqrt{\beta_x}$ [%]	8.23	5.96
$\Delta\sqrt{\beta_y}/\sqrt{\beta_y}$ [%]	11.94	7.01
$\Delta D/D$ ($D>0$) [%]	−6.57	11.20
$\Delta D/D$ ($D<0$) [%]	17.20	22.40
$x_{\rm co,max}$ [mm]	45.40	42.50
$x_{\rm co,min}$ [mm]	−46.80	−43.30
γ_t	i26.44	i33.91
$\Delta\nu_x$	0.0081	0.0577
$\Delta\nu_y$	0.0476	0.0450

4 Nonlinearity and Distortion Functions

The chromaticity sextupoles represent the main source of nonlinearities, which produces tuneshift dependences with amplitude,

$$\begin{cases} \Delta\nu_x = a\,\epsilon_x/\pi + b\,\epsilon_y/\pi \ , \\ \Delta\nu_y = b\,\epsilon_x/\pi + c\,\epsilon_y/\pi \ , \end{cases} \qquad (1)$$

where ϵ_x and ϵ_y are the horizontal and vertical emittances, defined as amplitude squared divided by the betatron function. The detunings can be expressed as:

$$\begin{aligned} a &= -\frac{1}{8\pi}\sum_k (B_3 S + 3B_1 S)_k \ , \\ b &= -\frac{1}{4\pi}\sum_k (B_+\bar{S} + B_-\bar{S} - 2B_1\bar{S})_k \ , \\ c &= -\frac{1}{8\pi}\sum_k (B_+\bar{S} - B_-\bar{S} + 4\bar{B}\bar{S})_k \ , \end{aligned} \qquad (2)$$

where $S_k = (\beta_x^{3/2}B''\ell/B\rho)_k$ and $\bar{S}_k = (\beta_x^{1/2}\beta_y B''\ell/B\rho)_k$ are the reduced sextupole strengths, and B_1, B_3, $\bar{B}$, $B_\pm$ are distortion functions [4] evaluated at position k.

The nonlinearity can be minimized by minimizing the distortion functions. The sufficient conditions are:

$$\begin{aligned} &\sum_{k'} S_{k'} e^{i\psi_{xk'}} = 0 \ , \qquad \sum_{k'} S_{k'} e^{i3\psi_{xk'}} = 0 \ , \\ &\sum_{k'} \bar{S}_{k'} e^{i\psi_{xk'}} = 0 \ , \qquad \sum_{k'} \bar{S}_{k'} e^{i\psi_{\pm k'}} = 0 \ , \end{aligned} \qquad (3)$$

where the summations are over all sextupoles k', whose phases $\psi_{xk'}$, $\psi_{yk'}$, and $\psi_{\pm k'} = (2\psi_y \pm \psi_x)_{k'}$ are measured *downstream* of the point of consideration. For a FODO-cell ring, the above conditions can be easily satisfied by choosing the phase advance per cell to be either 60° or 90° in both transverse planes. Such an idea can also be extended to our non-FODO ring. Between the two special zero-dispersion modules, there are 12 modules on each side. Although there are three types of modules, their Twiss-valued structures have been designed to be very similar with phase advances $\Delta\psi_x = 0.7524$, 0.7391, 0.7450, $2\Delta\psi_y = 0.7603$, 0.7163, 0.7334 for BASIC, EXTR1, and EXTR2, respectively.

The distortion functions B_1, B_3, and $\bar{B}$ are canceled every 4 adjacent modules and their maxima are 6.2, 17.9, and 2.3 m$^{-1/2}$, respectively, contributed roughly by 4 modules only. Among these, B_3 appears to be much bigger than the others. This is due mostly to the fact that the contributions of the five sextupoles within a module add more constructively for B_3. For B_+, the average phase advance per module is $\Delta\psi_+ = 2\Delta\psi_y + \Delta\psi_x = 1.490$. Thus, B_+ will be canceled every 2 adjacent modules instead, with maximum of only 2.2 m$^{-1/2}$ contributed by only 2 modules.

The distortion function B_- rotates with a tune of $\nu_- = 2\nu_y - \nu_x = 0.9612$ which is extremely slow. Therefore, cancellation between adjacent modules will not be possible. However, due to the near integral value of ν_- and the two-fold symmetry of the ring, each sextupole in the first 12 modules is *nearly* canceled by a complementary sextupole in the other 12 modules. Exact cancellation will occur if ν_- is exactly an integer. But this is not desirable since the difference resonance should be avoided. Each of the 12 adjacent modules has nearly the same phase advance $2\psi_y - \psi_x$, and we find $|B_-|$ reaches a maximum of 18 m$^{-1/2}$. Note that this value is insensitive to how close the difference resonance is.

With this arrangement of sextupoles, the nonlinearity of the chromaticity sextupoles cancels mostly. We obtain the tuneshift dependences on amplitude as

$$\begin{aligned} \nu_x &= 21.42005 - 97.1\epsilon_x/\pi - 42.5\epsilon_y/\pi \ , \\ \nu_y &= 11.18993 - 42.5\epsilon_x/\pi + 2.20\epsilon_y/\pi \ , \end{aligned} \qquad (4)$$

where the emittances ϵ_x and ϵ_y are in m. The detunings here are actually smaller than those of the existing FODO-lattice of the Fermilab Main Injector by 50%. As a result, harmonic sextupoles are not required. In our previous design of the same imaginary-γ_t lattice [2], these special precautions had not been taken, and even after we added a family of harmonic sextupoles to lower the nonlinearity, the detunings were $a = 443$ m^{-1}, $b = -241$ m^{-1}, and $c = 1330$ m^{-1}, still very much larger than the values here.

5 Tunability and Misalignment Errors

The nominal tunes are $\nu_x = 21.42$ and $\nu_y = 11.19$. When we keep ν_y fixed and vary ν_x between 21.1 and 21.8, or keep ν_x fixed and vary ν_y between 11.05 and 11.55 by varying the field gradients of the FODO quads, the horizontal beam size ($\sqrt{\beta_x}$) and vertical beam size ($\sqrt{\beta_y}$) do not change by more than 2%.

If all the misalignment errors of the quads are random, uncorrelated, and have a variance $\langle z^2 \rangle$, where $z = x$ or y, the closed-orbit offset $z_{co}(s)$ at location s has a variance given by

$$\langle z_{co}^2(s) \rangle = \frac{\langle z^2 \rangle \beta_z(s)}{2 \sin \pi\nu_z} \sum_i \left[\beta_z \frac{B'\ell}{B\rho} \right]_i \cos[\pi\nu_z - |\psi_{zi} - \psi_z(s)|] \,.$$

In the above, B' is the field gradient of the ith quad and ℓ its length, the summation runs over all the quads in the lattice, and the thin-lens approximation has been assumed. From a SYNCH output file, we obtain misalignment sensitivity factors:

$$S_z = \left[\frac{\langle z_{co}^2(s) \rangle}{\langle z^2 \rangle} \right]_{\max}^{\frac{1}{2}} = \begin{cases} 24.6 & \text{horizontal} \\ 48.6 & \text{vertical} \end{cases}$$

which are about a factor of two less than those for the previous imaginary-γ_t design [2]. This is because low-beta doublets of shorter lengths have been used.

6 Dynamical Aperture

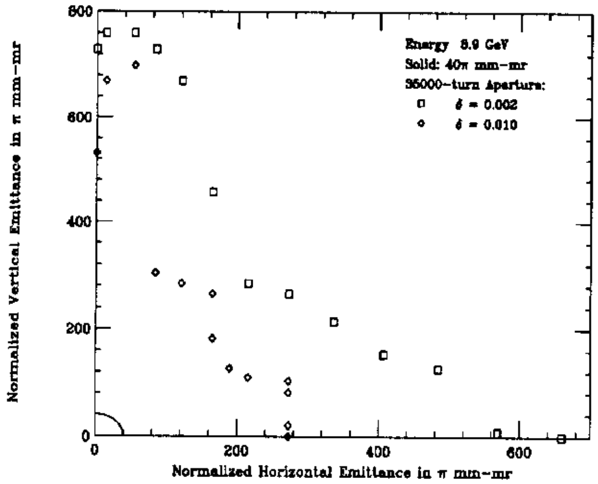

Fig. 2. 35000-turn survival apertures at injection energy 8.9 GeV with momentum offsets of $\delta = 0.002$ and 0.010.

The dynamical aperture was studied by TEAPOT tracking [5]. The dipole multipoles in the ± 25.4 mm range were provided from the magnetic measurements of the Main Injector prototype dipoles. The vertical magnetic field in the plane of the ideal orbit has the expansion $B(x) = B_0 \sum_{n=0}^{\infty} b_n x^n$, where B_0 is the vertical bending field at the ideal orbit, and the b_n's ($b_0 = 1$) are the normal multipole coefficients. At the injection energy of 8.9 GeV, the sextupole in the dipoles is positive with $b_2 = 0.0654$ m^{-2} while the decapole contribution is $b_4 = 4.27$ m^{-4}. A particle was launched at the center of the dispersionless rf straight with a certain horizontal displacement x. The vertical displacement y was varied and the maximum recorded for a survival of 35,000 turns or approximately 0.38 sec of the injection. The initial angular displacements x' and y' were set at zero. Sometimes the initial y displacement was fixed while the x displacement was varied instead. The whole x-y space was scanned resulting in the aperture plot shown in Fig. 2. The tracking had been done with particle momentum offsets of 0.2% and 1%. Due to the small detunings exhibited in Eq. (4), the aperture turns out to be fairly large. Even at 1% momentum offset, the allowable normalized emittance is larger than 200π mm-mr in both planes, while the required aperture is only 40π mm-mr. At 19 GeV, the eddy currents have a maximum. However, the 35,000-turn survival aperture for 1% momentum offset admits normalized emittances 200π mm-mr and 500π mm-mr in the horizontal and vertical planes, respectively, much larger than those at injection.

7 Conclusion

We have presented an improved imaginary-γ_t lattice, which is very compact and requires construction of only one type of dipole and one type of quad. All other quads can be recycled from the Main Ring. This lattice possesses very appealing beam dynamic properties like small dispersion, nice tunability, small tune dependences on momentum, and large momentum acceptance. The special precautions taken in the design and the deployment of chromaticity sextupoles lead to large cancellation in nonlinearity. For this reason no additional families of sextupoles are necessary to cancel the amplitude induced second-order tune shifts. Because of this, the 35000-turn survival dynamical aperture has been very much larger than the 40π mm-mr requirement.

References

[1] D. Trbojevic, D. Finley, R. Gerig, and S. Holmes, *Design Method for High Energy Accelerator without Transition Energy,* Proc. of 2nd European Part. Accel. Conf., Nice, France, June 1990, pp 1536-1538.

[2] K.Y. Ng, D. Trbojevic, and S.Y. Lee, *A Transitionless Lattice for the Fermilab Main Injector,* Proc. of 1991 IEEE Part. Accel. Conf., May 6-9, 1991, San Francisco, California, pp 159-161.

[3] S.Y. Lee, K.Y. Ng, and D. Trbojevic, *Design and Study of Accelerator Lattices without Transition,* to be published in Phys. Rev.

[4] T. Collins, *Distortion Functions,* Fermilab-84/114, 1984. K.Y. Ng, *Distortion Functions,* KEK Report 87-11, 1987.

[5] L. Schachinger and R. Talman, *TEAPOT — A Thin Element Accelerator Program for Optics and Tracking,* SSC Central Design Group Internal Report SSC-52,1985.