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The Electroweak Phase Transition

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Abstract

The phase transition associated with the standard electroweak model is very weakly first order. The weakness of the transition means that around the critical temperature the finite-temperature Higgs mass is much less than the critical temperature. This leads to infrared problems in the calculation of the parameters of the potential. Therefore, theories of electroweak baryogenesis, which depend on the details of the transition, must be calculated with care.

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1. Introduction

It has been 20 years since David Kirzhnits first suggested that the spontaneously broken symmetry of the electroweak theory should be restored at the high temperatures of the early Universe.[1] Since that time much has been learned about the details of the electroweak theory, and at present the only unknown parameters of the model are the masses of the top quark and the Higgs boson. The top-quark mass may soon be determined at Fermilab, leaving the Higgs mass as the only remaining parameter. However, even without a direct determination, precision electroweak measurements can limit the Higgs mass to a reasonable range, so it is worthwhile to take another look at the calculations of the electroweak phase transition.

The details of the electroweak transition are also most important for theories of electroweak baryogenesis. Kuzmin, Rubakov, and Shaposhnikov pointed out that sphaleron-mediated process can change the baryon number of the Universe.[2] This work has motivated an industry of model building to try to *produce* the baryon asymmetry at the electroweak scale. All such electroweak baryogenesis models are very sensitive to the details of the transition. Therefore, before deciding if any model fulfills the exciting promise of predicting the baryon number of the Universe on the basis of measurable low-energy physics, it is necessary to understand the dynamics of the transition in detail. Of course the first step in such a program is understanding the potential.

In this paper we will first discuss the standard calculation of the electroweak finite-temperature effective potential. We will then show how the one-loop potential is inadequate near the critical temperature where it is needed. Then we will discuss a way to estimate the magnitude of the corrections.

2. The Effective Potential

The finite-temperature, one-loop potential has been studied by many people, most recently by Anderson and Hall.[3] They showed that the potential may be written in the form

$$V_{EW}(\phi,T) = D\left(T^2 - T_2^2\right)\phi^2 - ET\phi^3 + \frac{1}{4}\lambda_T\phi^4,$$

where the constants D and E are given by

$$D = \left[6(m_W/\sigma)^2 + 3(m_Z/\sigma)^2 + 6(m_T/\sigma)^2\right]/24$$

and

$$E = \left[6(m_W/\sigma)^3 + 3(m_Z/\sigma)^3\right]/12\pi.$$

The Higgs mass is a function of temperature. We will denote as m_H the zero-temperature Higgs mass, and as $m_H(T)$ the temperature-dependent mass. The 1-loop corrected Higgs mass is $m_H^2 = (2\lambda + 12B)\sigma^2$, where λ is the tree-level quartic coupling constant, and

$$B = \frac{3}{64\pi^2\sigma^4}\left(2m_W^4 + m_Z^4 - 4m_T^4\right)$$

accounts for the one-loop, zero-temperature quantum corrections.

We will use $m_W = 80.6$ GeV, $m_Z = 91.2$ GeV, and for the zero-temperature vacuum expectation value of the Higgs field $\sigma = 246$ GeV. The temperature-corrected Higgs self coupling is

$$\lambda_T = \lambda - \frac{1}{16\pi^2}\left[\sum_B g_B\left(\frac{m_B}{\sigma}\right)^4 \ln\left(m_B^2/c_BT^2\right) - \sum_F g_F\left(\frac{m_F}{\sigma}\right)^4 \ln\left(m_F^2/c_FT^2\right)\right],$$

where the sum is performed over bosons and fermions (in our case only the top quark) with their respective degrees of freedom g_B and g_F, and $\ln c_B = 5.41$ and $\ln c_F = 2.64$. It turns out that to a good approximation the temperature-corrected Higgs self coupling λ_T is approximately equal to the tree-level value λ. In other words, the logarithmic correction from the zero-temperature 1-loop calculation pretty much cancels the temperature-dependent logarithmic correction. This can be understood by employing the renormalization scheme of Ref. 4. Expressing λ_T as $m_H^2/2\sigma^2$ will often be a useful approximation to make.

With the potential written in the form above, there are two other temperatures of interest that might be found directly. For high temperatures, the system will be in the symmetric phase with the potential exhibiting only one minimum at $\langle\phi\rangle = 0$. As the Universe expands and cools an inflection point will develop away from the origin at

$$\phi_{inf} = 3ET_1/2\lambda_T,$$

where T_1 is given by

$$T_1 = T_2/\sqrt{1 - 9E^2/8\lambda_T D}.$$

For $T < T_1$, the inflection point separates into a local maximum at ϕ_- and a local minimum at ϕ_+, with $\phi_\pm = \{3ET \pm [9E^2T^2 - 8\lambda_T D(T^2 - T_2^2)]^{1/2}\}/2\lambda_T$. At the critical temperature

$$T_C = T_2/\sqrt{1 - E^2/\lambda_T D},$$

the minima have same free energy, $V_{EW}(\phi_+, T_C) = V_{EW}(0, T_C)$. (Note that $V(\phi, T)$ is the homogeneous part of the free energy density whose minima denote the equilibrium states of the system. Accordingly, in this work we freely interchange between calling $V(\phi, T)$ a potential and a free energy density.)

The difference between the temperatures T_1, T_C, and T_2 is determined by the parameter

$$x \equiv E^2/\lambda_T D.$$

This parameter is shown in Fig. 1 for different values of m_H and m_T. Clearly $x \ll 1$ for the minimal electroweak model, so we can write the approximate relations

$$T_C \simeq T_2(1 + x/2); \qquad T_1 \simeq T_2(1 + 9x/16).$$

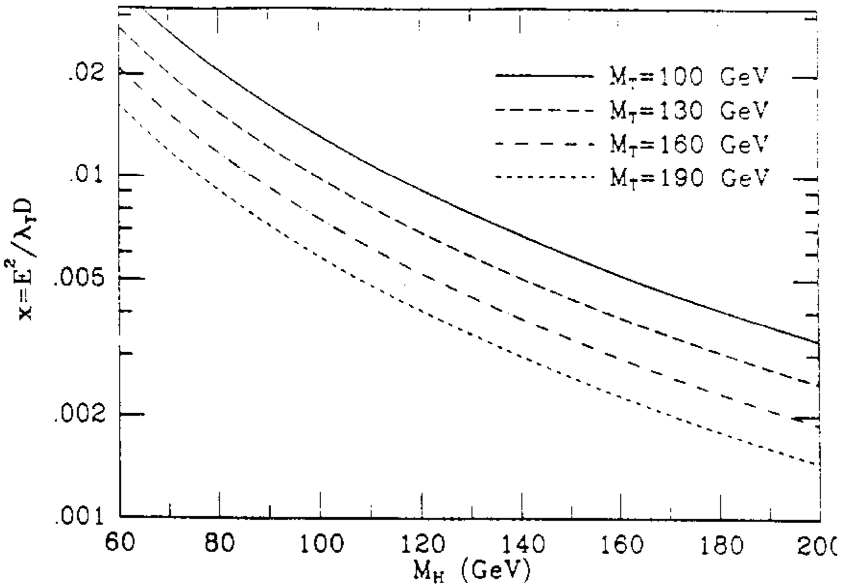

Fig. 1: The function $x = E^2/\lambda_T D$ as a function of the Higgs mass.

The potential above is the "1-loop" potential, as well known, equivalent to mean field theory. In fact, only gauge boson and top quark loops are included; scalar loops are not included. This is a good approximation at both high and low temperatures if the Higgs mass is less than about 150 to 200 GeV. However, we shall argue that around the critical temperature (exactly the temperature range of interest) it is *not* a good approximation.

It is useful to understand why the transition is first order; *i.e.*, why at T_C there is a barrier between the high-temperature phase and the low-temperature phase. It has been appreciated for a long time that a pure $\lambda\phi^4$ theory is equivalent to a Ginzburg–Landau theory, which has a second-order phase transition. The reason the electroweak theory is first, rather than second order, is that there is an additional attractive force between scalar particles mediated by the vector bosons. This additional attractive force leads to a condensate of the Higgs field at a temperature slightly above T_2. T_2 and T_C would be the same (a second-order transition) in the absence of gauge boson interactions. (Note that as $E \to 0$, i.e., as vector interactions are turned off, $T_C \to T_2$.)

An important use of the effective potential is in the calculation of the rate of nucleation of true-vacuum bubbles. The whole picture of bubble nucleation relies on the behavior of $V_{EW}(\phi, T)$ between T_C and T_2. In the standard picture, one assumes that the system is in a near-homogeneous state around its equilibrium value (in this case $\langle\phi\rangle = 0$), so that large thermal fluctuations in the spatial correlations of ϕ are exponentially suppressed above the scale of the thermal correlation length. This assumption will not be true if there are large fluctuations caused by infrared problems near the transition.

The finite-temperature tunneling rate, $\Gamma \propto \exp(-S_3/T)$, for a theory with a potential like the electroweak potential has been shown by Dine *et al.*[4] to have an approximate analytical expression for the exponent given by

$$\frac{S_3}{T} = 4.85 \frac{m_H^3(T)}{E^2 T^3} f(\alpha); \qquad \alpha = \frac{\lambda_T m_H^2(T)}{2E^2T^2},$$

with

$$f(\alpha) = 1 + \frac{\alpha}{4}\left[1 + \frac{2.4}{1-\alpha} + \frac{0.26}{(1-\alpha)^2}\right].$$

Clearly even small changes in the parameters of the potential have an exponential effect upon the tunnelling rate. Therefore it is important to know the parameters accurately.

The correct treatment of the critical behavior of a quantum field theory in 3 + 1 dimensions at finite temperature involves reducing the theory to the effective theory of the static mode of the scalar field in 3 dimensions.[5] In lower dimensions the infrared (IR) problems are generally more severe. For instance, the effective coupling constant for the systematic loop expansion in 3 dimensions for a $\lambda\phi^4$ field theory is $\lambda T/m$. Any infrared divergences show up as a vanishing of m and mean that the loop expansion is invalid, *i.e.*, mean field theory does not adequately describe the critical behavior. This is well known to condensed matter physicists in the study of second-order phase transitions.[6] While mean field theory predicts a critical exponent of $\nu = 0.5$ for the correlation length, $\xi = m^{-1} \propto |T - T_C|^{-\nu}$, experiment gives a result $\nu \simeq 0.63$.

The departure of the correlation length from its mean-field (1-loop) value can be thought of as the effect of large thermal fluctuations in the background. These non-perturbative thermal fluctuations couple to the zero-momentum mode of the field, modifying the critical exponents.

For the electroweak transition there are IR divergences in two places. The first place is in the vector-boson masses. Recall that in the electroweak theory vector bosons acquire a mass through coupling to the Higgs, $m_V \sim g\phi$. For small ϕ, the vector boson masses vanish. This problem has been studied by three groups.[4,7,8] They assume that similar to QCD, the vector bosons acquire a "magnetic" mass of $m_{mag} \sim g^2 T$ that effectively cuts off the IR divergence. If one believes the reasonable, widely believed, but unproven conjecture of a magnetic mass, then the effect of the vanishing of the vector masses in the vicinity of $\phi = 0$ is calculable; it reduces E by a factor of 2/3.

The second place where there are infrared divergences is in the scalar sector itself. Generally for Higgs masses less than 200 GeV or so, Higgs loops may be ignored at zero temperature. This is because the Higgs self coupling $\lambda \sim m_H^2/2\sigma^2 \sim 0.08(m_H/100 \text{ GeV})^2$ is less than both the gauge coupling constants, $g = 0.66$ and $g' = 0.35$, and the top-quark Yukawa coupling constant $h_T = 0.57(m_T/100 \text{ GeV})$. However, at high temperatures the loop expansion parameter is *not* the coupling constants, but the coupling constants times T/m. If the Higgs mass is much smaller than the temperature, then the Higgs loops may be crucial.

To see if this is the case, consider $\lambda T/m_H$ for the standard electroweak model. For the electroweak potential near T_C, $m_H^2(T_C) = 2D(T_C^2 - T_2^2)$. Since $T_2^2/T_C^2 = 1 - E^2/\lambda_T D$, at T_C, $m_H(T_C) = T_C E\sqrt{2/\lambda_T}$. Therefore at T_C the loop expansion parameter is $\lambda_T T_C/m_H(T_C) = \lambda_T^{3/2}/E\sqrt{2}$. Now as discussed above, to a reasonable accuracy

$\lambda_T = m_H^2/2\sigma^2$ (here, of course, m_H is the zero-temperature mass). Thus

$$\lambda_T T_C/m_H(T_C) \simeq m_H^3/4E\sigma^3 \sim 1.74(m_H/100GeV)^3.$$

For m_H greater than about 80 GeV, at T_C the expansion parameter exceeds unity. Between T_C and T_2 the mass goes to zero, so the corrections are even larger.

Clearly if the Higgs mass is in excess of its present experimental lower limit of 57 GeV, the Higgs IR problem must be controlled. This problem has been studied by us.[9] We give the details in the next section.

3. Estimating the Size of the Infrared Corrections

Our proposal is to estimate the magnitude of the IR corrections to the electroweak model by studying an associated Ginzburg-Landau (GL) model. Recall that the GL model describes a scalar field theory with zero-temperature potential

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4.$$

As is well-known, this theory exhibits a second-order phase transition at $T_C^2 = 4m^2/\lambda$; above T_C the left-right symmetry is exact and the equilibrium value of ϕ is $\langle\phi\rangle$=0. Below T_C the symmetry is broken and the equilibrium value of ϕ is $\langle\phi\rangle = \pm\left[m^2(T)/\lambda\right]^{1/2}$. In the thermodynamic limit, the system will eventually settle at one value of ϕ, since any interface is energetically unfavored. Of course $\langle\phi\rangle$ only gives information about the homogeneous behavior of ϕ. Typically, there will be fluctuations around $\langle\phi\rangle$ which are correlated within the correlation length. For temperatures above and below T_C (denoted by + and − respectively)

$$\xi_+^{-2}(T) = m^2(T) = \frac{\lambda}{4}T_C^2(1+T/T_C)^2\left(\frac{T-T_C}{T+T_C}\right),$$

and

$$\xi_-^{-2}(T) = -2m^2(T) = \frac{\lambda}{2}T_C^2(1+T/T_C)^2\left(\frac{|T-T_C|}{T+T_C}\right).$$

As discussed above, this is the well known result from mean-field theory, $\xi_{MF}(T) \propto |T - T_C|^{-\nu}$; $\nu = 1/2$, where the critical exponent ν expresses the singular behavior of $\xi(T)$ as $T \rightarrow T_C$ both from above and below.

In order to handle the infrared divergences that appear near T_C, the RG is used to relate a given theory to an equivalent theory with larger masses and thus better behaved in the infrared. Within the ε expansion, one works in $4 - \varepsilon$ dimensions and finds a fixed point of order ε of the RG equations, taking the limit $\varepsilon \rightarrow 1$ in the end. To second-order in ε one obtains,[6]

$$\nu = \frac{1}{2} + \frac{1}{12}\varepsilon + \frac{7}{162}\varepsilon^2 \simeq 0.63.$$

The corrected critical exponent embodies corrections coming from the infrared divergences near T_C. The ε-corrected correlation length can be written above T_C as

$$\left[\xi_+^\varepsilon(T)\right]^{-1} = \sqrt{\frac{\lambda}{4}}\, T_C\,(1 + T/T_C)\left(\frac{T - T_C}{T + T_C}\right)^{0.63}.$$

Below T_C we obtain,

$$\left[\xi_-^\varepsilon(T)\right]^{-1} = \sqrt{\frac{\lambda}{2}}\, T_C\,(1 + T/T_C)\left(\frac{|T - T_C|}{T + T_C}\right)^{0.63},$$

so that, in both cases the ratio between the mean field and ε-corrected correlation lengths can be written as

$$\frac{\xi_{MF}(T)}{\xi_\varepsilon(T)} = \eta_C^{0.13}(T); \qquad \eta_C(T) \equiv \frac{|T - T_C|}{T + T_C}.$$

If we are interested in studying the behavior of the theory above T_C we can use the fact that $\xi(T) = m^{-1}(T)$ to obtain an ε-corrected mass,

$$m_\varepsilon(T) = \eta_C^{0.13}(T)m(T),$$

To apply what is known about the IR behavior of the GL model to the electroweak model, we present here the simplest possible approach, by studying the GL model defined by the free energy density,

$$V_{GL}(\phi,T) = \frac{m^2(T)}{2}\phi^2 + \frac{\lambda_T}{4}\phi^4; \qquad m^2(T) \equiv 2D\left(T^2 - T_2^2\right),$$

where D, T_2, and λ_T are defined above. This is simply $V_{EW}(\phi,T)$ with $E \to 0$. This model exhibits a second-order phase transition at $T = T_2$. Recall that this is the temperature at which the barrier disappears in the 1-loop electroweak potential. Thus, we are interested in the behavior of this model for temperatures above T_2. The claim is that for $T \lesssim T_C$ and in the neighborhood of $\langle\phi\rangle = 0$ this model can be used to give us an *estimate* of the infrared corrections to the electroweak potential. Note that our choice of the mass is such that the correlation length for fluctuations around equilibrium is the same in both models. Thus, from the results above, the ε-corrected mass is

$$m_\varepsilon^2(T) = 2D\eta_2^{0.26}(T)\left(T^2 - T_2^2\right); \qquad \eta_2(T) = \frac{|T - T_2|}{T + T_2}.$$

The value of $\eta_2(T)$ at $T = T_C$ can be found using T_C and T_2:

$$\eta_2(T_C) = \frac{1 - \sqrt{1 - E^2/\lambda_T D}}{1 + \sqrt{1 - E^2/\lambda_T D}}.$$

In Fig. 2 we show $m_\varepsilon^2(T_C)/m^2(T_C) = \eta_2^{0.26}(T_C)$ as a function of the Higgs mass for several values of the top mass. It is clear that the infrared corrections are quite large for all values of parameters probed. Comparing the electroweak potential and the GL model for $T = T_C$, we notice that the electroweak model is even flatter near the origin and the infrared problem should be even more severe. For larger values of ϕ the cubic term becomes important increasing the flatness of the electroweak model compared to the GL model (leading again to more severe infrared problems).

This is discussed further in Ref. 9, along with a different procedure for matching the electroweak model to a GL model. Again, it must be emphasized that this is merely an

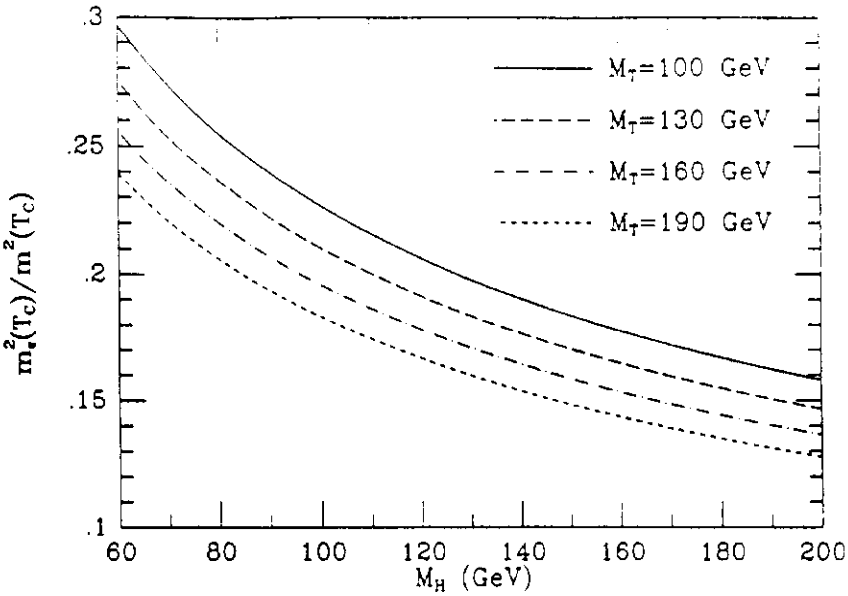

Fig. 2: $m_\epsilon^2(T_C)/m^2(T_C)$ as a function of the Higgs mass for various values of m_T.

estimate of the size of the infrared corrections. Clearly if m near T_C changes a lot, then the calculation of the tunnelling rate will be greatly modified.

4. Conclusions

In this work we have argued that it is possible to study the critical behavior of a weak first order transition which has a spinodal instability at some temperature T_2 by mapping its behavior around equilibrium, $\langle\phi\rangle$, to an effective Ginzburg-Landau model above its critical temperature T_2. In this way, both models have the same spinodal instability at $\langle\phi\rangle$ so that infrared corrections can be estimated from well-known ε-expansion methods. This approach is completely general and can in principle be applied to any sufficiently weak first order transition. It suits the standard electroweak model particularly well due to the closeness of its critical temperature T_C to the spinodal instability temperature T_2. In fact, the difference between the two temperatures should provide a qualitative measure of the weakness of the transition.

Incorporating the ε-expansion results leads to a larger correlation in the spatial fluctuations of the order parameter, which can be translated into a smaller (infrared corrected) mass for excitations around $\langle\phi\rangle$. Thus, the strength of the transition is considerably weaker than one would estimate from the naïve 1-loop potential. We do not claim here to have obtained the ε-corrected effective potential, but an estimate of the infrared corrections which are not included in the 1-loop result. Our results provide a simple way to examine the importance of these corrections around T_C, offering an estimate of the strength of the transition. If the η parameter is close to unity at the critical temperature T_C the transition is well described by the 1-loop result. Otherwise, the transition is weakly first order, and one should be very careful when adopting the usual vacuum decay formalism to study the transition.

In Ref. 9 we also discuss two other simple ways of estimating the strength of the transition based on the thermal dispersion around equilibrium[10] and on the sub-critical bubbles method.[11] When applied to the 1-loop electroweak potential both approaches suggest that there will be large fluctuations around equilibrium, indicating that the 1-loop result does not fully describe the dynamics of the transition. In fact, the results here show that the actual dynamics of the transition may be much more complex than the usual scenario based on vacuum decay calculations.

Since the electroweak potential plays such a crucial role in electroweak baryogenesis scenarios, a lot of work needs to be done. In this work we require help from field theorists, condensed-matter theorists, and perhaps even lattice gauge theorists. It should be an exciting next few years.

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References

1. D. A. Kirzhnits *JETP Lett.* **15**, 529 (1972); D. A. Kirzhnits and A. D. Linde, *Phys. Lett.* **72B**, 471 (1972).

2. V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).

3. G. W. Anderson and L. J. Hall, *Phys. Rev. D* **45**, 2685 (1992).

4. M. Dine, R. Leigh, P. Huet, A. Linde, and D. Linde, Stanford University preprints SU–ITP–92–6 and SU–ITP–92–7.

5. P. Ginsparg, *Nucl. Phys.* **B170,** 388 (1980).

6. See, for example, K.G. Wilson and J. Kogut, *Phys. Rep.* **12C**, 75 (1974); E. Brézin, in *Methods in Field Theory*, Les Houches 1975, Session XXVIII, ed. R. Balian and J. Zinn-Justin, (North-Holland, Amsterdam 1976).

7. D. Brahm and S. D. H. Hsu, Caltech preprints, CALT-68-1705 and CALT-68-1762 (1991).

8. M. E. Carrington, *Phys. Rev.* **D45,** 2933 (1992).

9. M. Gleiser and E. W. Kolb, University of California Institute for Theoretical Physics preprint NSF–ITP–92–102.

10. M. Gleiser, *Phys. Rev. D* **42**, 3350 (1990); E. W. Kolb and Y. Wang, *Phys. Rev. D* **45**, 4421 (1992); M. Gleiser and E. W. Kolb, Ref. 9.

11. M. Gleiser, E. W. Kolb, and R. Watkins, *Nucl. Phys.* **B364**, 411 (1991); M. Gleiser and E. W. Kolb, Fermilab preprint 91/305-A, *Phys. Rev. Lett.*, in press (1991); N. Tetradis, DESY preprint 91-151 (1991).