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GROUPS OF GALAXIES IN CDM UNIVERSES

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ABSTRACT

We identify dark halos from a high resolution $\Omega = 1$ cold dark matter (CDM) simulation. The overmerging of halos forms systems that are too massive to be associated with single galaxies. We convert these systems to groups and clusters assuming a universal mass-to-light ratio. The resulting groups and clusters have a significant affect on spatial clustering and pairwise velocity dispersions, and they pose a serious challenge to the model for any reasonable normalization of the initial power spectrum.

1. INTRODUCTION

We use a 144^3 particle P^3M simulation in a 100 Mpc box (Plummer softening of 65 kpc comoving and particle mass of $2.3 \times 10^{10} M_\odot$) to study the evolution of an $\Omega = 1$ ($H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) cold dark matter universe. Ben Moore (at this workshop) presented a group analysis from a simulation in a larger volume of space where galaxies were represented by single particles. Although that simulation is useful for testing group identification algorithms, it lacks sufficient spatial and mass resolution to test the CDM model. Our simulation resolves individual halos. However, the lack of gas dynamics, i.e. a dissipative baryonic component, in our simulation is partially responsible for our inability to identify massive systems as groups. Gas dynamical simulations of individual clusters with dark and baryonic matter (Katz & White 1992; Evrard, Summers, & Davis 1992) demonstrate that some galaxies can survive the merging process. Attempts to study gas dynamical effects in large volumes of space suffer from lower resolution yet can offer some insight into the sites for galaxy formation (Cen & Ostriker 1992). We convert massive halos found in our high resolution dark cosmological simulation into groups and clusters assuming a universal mass-to-light ratio in order to understand the group properties and clustering. This has been motivated by studies demonstrating the merging of halos formed at early epochs (e.g. White et al. 1987; Gelb 1992). The groups can dominate the statistics and they are essential for understanding if nature can "hide" mass and if there is a normalization of the initial CDM power spectrum matching theory with observations.

We normalize the initial, linear CDM power spectrum, $P(k)$ for comoving wavenumber k , so that σ_8 , the linear, *r.m.s.* mass fluctuation in spheres of radii $8h^{-1}$ Mpc, is unity. This is known as the $\sigma_8 = 1$ normalization and larger values of σ_8 correspond to dynamically more evolved systems:

$$\sigma_8^2 \equiv \int_0^\infty d^3k P(k) W_{\text{TH}}^2(kR); \quad W_{\text{TH}}(kR) = \frac{3}{(kR)^3} (\sin kR - kR \cos kR); \quad R = 8h^{-1} \text{ Mpc.}$$



2. TWO-POINT CORRELATIONS

We will demonstrate in §3 that it is necessary to break up massive halos in order to have groups in our simulation. Also, in Gelb (1992), we demonstrated that there are too many halos with $V_{\text{circ}} \gtrsim 350 \text{ km s}^{-1}$ and that these systems are too massive to be associated with single galaxies. For these reasons, we break up massive halos into groups assuming a universal mass-to-light ratio in a procedure outlined below.

The total luminosity in a volume V from the Schechter luminosity function is

$$\mathcal{L}_{\text{total}} = V \int_0^{\infty} \mathcal{L} \Phi(\mathcal{L}) d\mathcal{L} .$$

The total number of galaxies in a volume V with a luminosity exceeding \mathcal{L} is:

$$N(> \mathcal{L}, V) = V \int_{\mathcal{L}}^{\infty} \Phi(\mathcal{L}) d\mathcal{L} .$$

Combining these equations and defining $x \equiv \mathcal{L}/\mathcal{L}_*$, we get the total number of halos exceeding a luminosity \mathcal{L} in a cluster with total light $\mathcal{L}_{\text{total}}$:

$$N(> \mathcal{L}, \mathcal{L}_{\text{total}}) = \frac{\mathcal{L}_{\text{total}}}{\mathcal{L}_*} \frac{\int_{\mathcal{L}/\mathcal{L}_*}^{\infty} x^{\alpha} e^{-x} dx}{\Gamma(2 + \alpha)} ,$$

where $\Gamma(2 + \alpha) = \int_0^{\infty} x^{1+\alpha} e^{-x} dx = 1.0456$ for $\alpha = -1.07$.

We take the bound mass of a massive halo (those with $V_{\text{circ}} \geq 350 \text{ km s}^{-1}$; see §3) and we divide it by a specified universal value of M/\mathcal{L} . This gives us the total luminosity emitted by the cluster: $\mathcal{L}_{\text{total}}$. We then add $N(> \mathcal{L}, \mathcal{L}_{\text{total}})$ halos with luminosity exceeding \mathcal{L} .

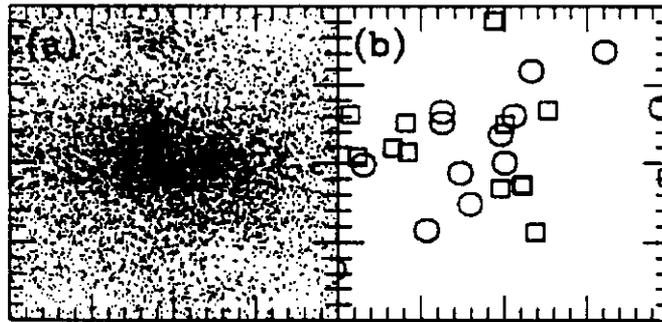


Figure 1: a $2.1 \times 10^{14} M_{\odot}$ halo (left) is split up into a cluster (right). The boxes are 2-D projections of a 2 by 2 by 2 Mpc region. Circles indicate halos with $V_{\text{circ}} \geq 250 \text{ km s}^{-1}$ and squares indicate halos with $V_{\text{circ}} \geq 200 \text{ km s}^{-1}$.

When we add in halos we need to choose positions and velocities. We do this by randomly sampling positions of particles (velocities are discussed in § 4).

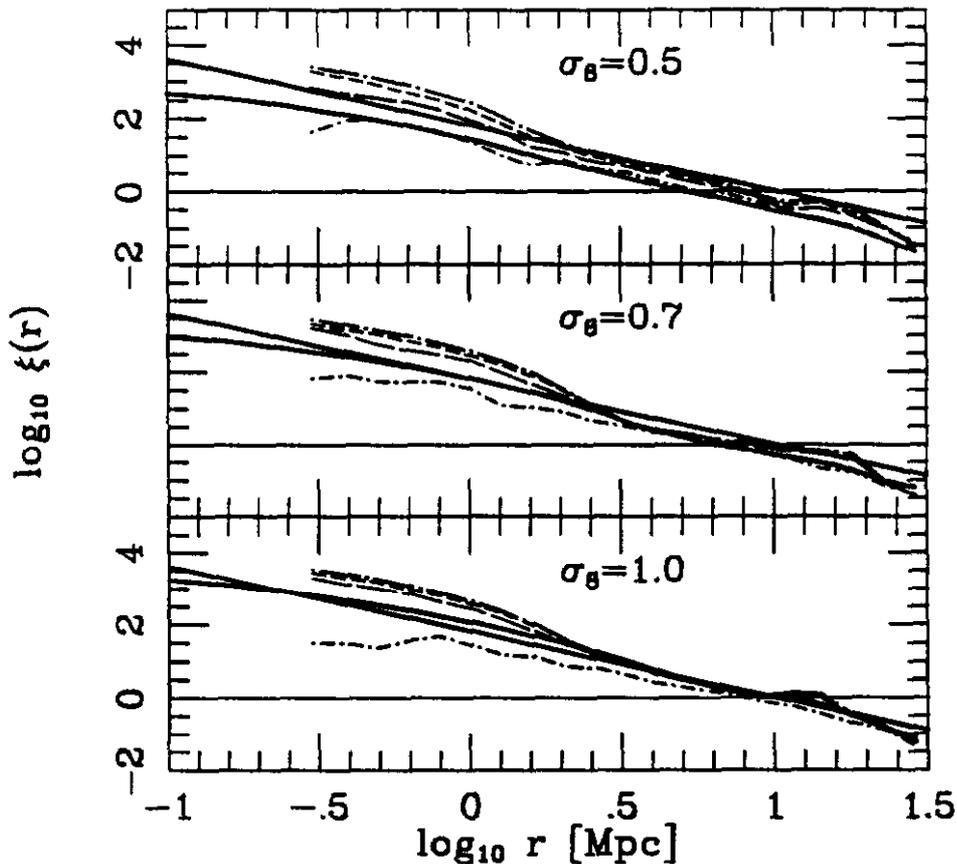


Figure 2: two-point correlations for halos with $V_{\text{circ}} \geq 250 \text{ km s}^{-1}$. No break-up (dot-short dashed lines); $M/\mathcal{L} = 250$ (long-dashed lines); 125 (short-dashed lines); 50 (dot-long-dashed lines). The straight solid lines are the observed ξ and the curved solid lines are for the mass (all particles).

We applied the mass-to-light method and computed the two-point correlation function ξ for halos with $V_{\text{circ}} \geq 250 \text{ km s}^{-1}$, see Figure 2. At $\sigma_8 = 0.5$ the enhancement in ξ is nearly sufficient for $M/\mathcal{L} = 50$ but the slope is too steep on small scales. The slope steepens at larger scales for increasing σ_8 . The results at $\sigma_8 = 0.5, 0.7$, and 1.0 have close to the observed correlation lengths but the slope at small r is too steep at all σ_8 . The no break-up case at $\sigma_8 = 1$ is almost acceptable, but the significant turnover on small scales does not match the observed slope.

White et al. (1987) found a factor of ~ 3 too many halos necessary to enhance ξ at $\sigma_8 = 0.4$ (although they used a 50 Mpc box). We find that the correlation length falls short of the observed value at $\sigma_8 = 0.5$ for $M/\mathcal{L} = 250$ and that breaking up the massive halos also produces factors $\sim 2 - 3$ too many halos compared with estimates from the Schechter luminosity function.

Before drawing more conclusions, we examine two issues further. The first has to do with mass-to-light ratios for an $\Omega = 1$ universe and the second has to do with the richness of observed groups (§ 3). For $\Omega = 1$, with $h = 1/2$, the mass density is $6.9 \times 10^{10} M_{\odot} \text{ Mpc}^{-3}$. If we divide this by the blue luminosity density in the universe $9.46 \times 10^7 \mathcal{L}_{\odot} \text{ Mpc}^{-3}$ (using $\Phi_* = 1.95 \times 10^{-8} \text{ Mpc}^{-3}$ for $h = 1/2$) the implied value

of M/\mathcal{L} for $\Omega = 1$ is ~ 750 . We would be inconsistent with the observed universe using $M/\mathcal{L} \ll 750$ in our $\Omega = 1$ models if the massive halos were characteristic of the universe as a whole. We computed the fraction of the total mass in our $(100 \text{ Mpc})^3$ volume contained in massive halos ($V_{\text{circ}} \geq 350 \text{ km s}^{-1}$). We found the percentage of mass contained in these objects to be 19.2% at $\sigma_8 = 0.5$, 29.9% at $\sigma_8 = 0.7$, and 39.9% at $\sigma_8 = 1.0$. These numbers are large when one recalls that only a few percent of galaxies are in rich groups; see Bahcall (1979) for a review.

3. GROUPS OF GALAXIES

We search for groups and we compare with Ramella, Geller, & Huchra (1989; hereafter RGH) who studied groups of galaxies from the $B(0) \leq 15.5$ CfA-2 redshift survey. For our discussion we converted all relevant quantities to Zwicky magnitudes. We replicated our $(100 \text{ Mpc})^3$ volume using periodic boundary conditions into a $(250 \text{ Mpc})^3$ volume. We then selected a wedge corresponding to the CfA-2 sky coverage: right ascension range $8^{\text{h}} \leq \alpha \leq 17^{\text{h}}$; declination range $26.5^\circ \leq \delta < 38.5^\circ$. We assumed $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and we imposed a distance cut of $R \leq 240 \text{ Mpc}$ in our analysis. We used actual positions rather than redshifts and we imposed an apparent magnitude limit of $B(0) \leq 15.5$. We assumed a Tully-Fisher relationship, see Pierce & Tully (1988), relating circular velocity to magnitudes.

We used DENMAX (our local density maxima finder, see Bertschinger & Gelb 1991) to identify all halos with $V_{\text{circ}} \geq V_{\text{circ}}^{\text{MIN}} = 50 \text{ km s}^{-1}$; then we used friends-of-friends (FOF) to identify groups in our wedge after breaking up the massive halos using the mass-to-light method. We determined a FOF linking length, l in Mpc, corresponding to a given overdensity of halos $\delta\rho/\rho$ given by $l^3 = 2/(n\delta\rho/\rho)$ (see, for example, Frenk et al. 1988) where n is the number density of halos with circular velocity exceeding $V_{\text{circ}}^{\text{MIN}}$ from our original $(100 \text{ Mpc})^3$ volume. We used FOF to identify groups of halos after breaking up the massive halos, but prior to imposing an apparent magnitude limit. Typical values of l , for $\delta\rho/\rho = 80$, ranged from 0.8 Mpc to 1 Mpc for the various assumed values of M/\mathcal{L} and σ_8 .

We only identified groups with three or more members to be consistent with RGH. RGH chose a linking distance using a galaxy number density estimated from the observed Schechter luminosity function. However, they varied their linking length with redshift to account for the sparse sampling of galaxies at large redshift. We avoided this difficulty by applying FOF with a fixed linking length prior to applying an apparent magnitude limit. We then applied the apparent magnitude limit to the resulting group catalog in a manner described below.

For field halos, i.e. those that are not in groups with 3 or more members, we simply computed $M_{B(0)}$ using the Tully-Fisher relationship, and we removed those with $B(0) > 15.5$. For the halos in groups we applied the following procedure. If the group member was not created from the break-up of a massive halo, then we eliminated it if $B(0) > 15.5$. For group members that were created from the break-up of a massive halo, we removed all of them and replaced them by the number of halos determined using the mass-to-light method with a universal M/\mathcal{L} . The lower luminosity limit was computed from $15.5 - M_{B(0)} = 5 \log_{10} d + 25.0$, where d is the distance to group centroid in Mpc. Here we do not need to relate circular velocity to luminosity; however, to be consistent with our use of $V_{\text{circ}}^{\text{MIN}}$, we never allowed \mathcal{L} to fall below \mathcal{L}_{min} determined from $V_{\text{circ}}^{\text{MIN}}$ using the Tully-Fisher relationship.

We summarize four parameters involved in the identification of groups. 1) We use $\delta\rho/\rho = 80$, the middle value considered by RGH, since we see the same levels of variation with $\delta\rho/\rho$ as reported by RGH and our conclusions do not depend critically on its choice.

2) We use $V_{\text{circ}}^{\text{MIN}} = 50 \text{ km s}^{-1}$. Our results do not depend sensitively on $V_{\text{circ}}^{\text{MIN}}$ because the low mass halos quickly fall out of sight. 3) We use $M/\mathcal{L}=125, 250, \text{ and } 500$. From a list of 36 groups, RGH find a median M/\mathcal{L} of $178h = 89$ for $h = 1/2$. We chose large values of M/\mathcal{L} because, as we will see, even $M/\mathcal{L}=125$ produces groups that are too rich. 4) There is some arbitrariness to the value of V_{circ} above which we break up the massive halos. We use $V_{\text{circ}} = 350 \text{ km s}^{-1}$. If we raise this value we get too many isolated massive halos (see Gelb 1992) which are too big to represent individual galaxies. On the other hand, if we lower this value we get even richer groups. However, the numbers of halos added become very small for smaller mass halos and higher mass-to-light ratios.

Table 1: Group Statistics in 12° Slice

Data	σ_8	$N_{\text{grp}} \geq 3 \text{ mem.}$	$N_{\text{grp}} \geq 10 \text{ mem.}$	$N_{\text{grp}} \geq 20 \text{ mem.}$	N_{gal} in field	N_{gal} in groups	$N_{1/2}$
CfA-2	N.A.	128	7	2	900	778	6
No Break-Up	0.5	30	0	0	1910	106	< 3
$M/\mathcal{L} = 125$	0.5	136	36	17	1622	1366	19
$M/\mathcal{L} = 250$	0.5	79	19	5	1579	633	11
$M/\mathcal{L} = 500$	0.5	56	6	2	1555	322	6
No Break-Up	0.7	25	0	0	1901	83	< 3
$M/\mathcal{L} = 125$	0.7	197	37	16	1589	1933	19
$M/\mathcal{L} = 250$	0.7	106	17	8	1523	843	14
$M/\mathcal{L} = 500$	0.7	58	8	3	1470	370	9
No Break-Up	1.0	58	0	0	1660	197	< 3
$M/\mathcal{L} = 125$	1.0	237	55	25	1452	2843	22
$M/\mathcal{L} = 250$	1.0	138	27	11	1341	1307	16
$M/\mathcal{L} = 500$	1.0	83	11	5	1300	610	13

The results from our simulation are shown in Table 1. We report numbers from RGH for the full 12° slice, but we impose a redshift cut of 12000 km s^{-1} . They only study groups with centroids $\leq 12000 \text{ km s}^{-1}$. We report numbers from the simulation for the full 12° slice for $R \leq 240 \text{ Mpc}$. The table shows the number of groups, N_{grp} , identified with 3 or more members, with 10 or more members, and with 20 or more members. We also show the number of halos, N_{gal} , in the field, i.e. those that are not in groups with 3 or more members. We estimated the CfA-2 field galaxies within 12000 km s^{-1} as follows.

The CfA-2 catalog has 1766 galaxies and we estimated from figure 1 in RGH that ≈ 100 galaxies are beyond 12000 km s^{-1} . RGH find 778 galaxies in groups with three or more members and only a hand full of these galaxies are beyond 12000 km s^{-1} . Therefore, the number of field galaxies within 12000 km s^{-1} in the CfA-2 catalog is approximately $1766 - 778 - 100 \sim 900$ galaxies. The last entry in the table, $N_{1/2}$, is a richness statistic defined below.

We can draw several important conclusions from the numbers in the table. If we do not break up the massive halos, then we do not have enough groups and there are no groups with 10 or more members. Therefore we need to break up our massive halos if our simulated universe is to contain groups comparable to the observed numbers! In all cases we have too many field halos. We demonstrated earlier that these are not dominated by faint halos. However, in Gelb (1992) we found that we had the correct number of halos with circular velocities between 150 km s^{-1} and 350 km s^{-1} . The reason for this discrepancy is that we have applied only the Tully-Fisher relationship to the halos here rather than a combination of the Tully-Fisher relationship and the Faber-Jackson relationship as we did in Gelb (1992). Applying the Tully-Fisher relationship to elliptical galaxies, which tend to be the most massive halos, makes the halos appear brighter than they really are. On the other hand, most of our group members result from the break-up of massive halos where we do not need to assume a relationship between circular velocity and luminosity. Therefore, we should give more emphasis to the richness of our groups than to the apparent excess of field halos.

We can constrain $M/\mathcal{L} \gtrsim 250$ based on the number of groups with 3 or more members and the total number of halos in all groups with 3 or more members. In most cases, however, we still have too many rich groups with 10 or more members. On the other hand, $M/\mathcal{L} = 500$ appears to do well. The numbers in Table 1 are consistent with the observations at $\sigma_8 = 0.5$, particularly when we consider the fact that the observed number of groups with three or four members are contaminated by projection effects which can reduce the observed numbers in groups by factors $\gtrsim 30\%$ (see RGH). This lowers the observed numbers in groups and, by definition, raises the observed numbers in the field!

To further quantify the richness of our groups, we compare the cumulative number of halos in groups with the estimates from RGH for the CfA-2 survey. The cumulative number of galaxies in groups is defined by RGH as:

$$N_{\text{gal}}(\leq N_{\text{mem}}) \equiv \sum_{N=3}^{N=N_{\text{mem}}} N N_g(N),$$

where $N_{\text{gal}}(\leq N_{\text{mem}})$ is the total number of galaxies contained in all groups with three to N_{mem} members and $N_g(N)$ is the number of groups containing N members. The results are shown in Figure 3 which was computed for a 6° slice and $\delta\rho/\rho = 80$ (we divided the numbers from our 12° slice by two) to compare with RGH (their figure 2).

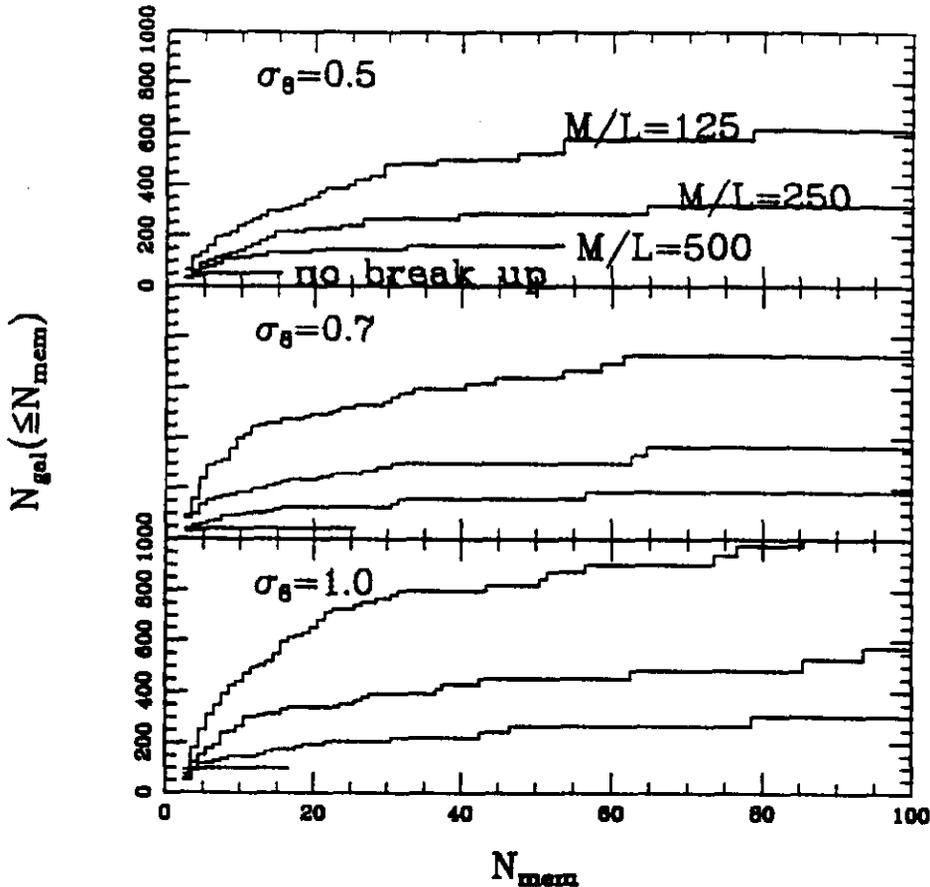


Figure 3: Cumulative halo number in groups for various M/L .

We clearly see the dramatic shortcoming of the no break-up cases at all epochs. We also find that our groups are too rich compared with RGH; the rise in the cumulative halo number is slower than the results for the CfA-2 survey indicating that our group members are concentrated in relatively larger groups. A useful statistic is $N_{1/2}$ shown in Table 1. This is the value of N_{mem} where the cumulative number of halos in groups reaches 1/2 its maximum value. The value of $N_{1/2}$ indicates that we need $M/L \gtrsim 250$. We can rule out $M/L = 125$.

We conclude from our studies comparing groups with the CfA-2 survey that we have too many halos in the field. These are dominated by bright halos that have not been broken up. However, this is partly because we have applied the Tully-Fisher relationship to all of our halos rather than a combination of the Tully-Fisher relationship and the Faber-Jackson relationship. Also, the CfA-2 estimates of groups suffer from projection effects; some of the “group” galaxies are actually field galaxies. We also find too many halos in the groups unless $M/L \gtrsim 250 = 500h$ which is much higher than observed values of M/L . In most cases our groups are too rich with far too many groups containing 10 or more members. We are forced to break up the massive halos; otherwise our catalogs produce far too few groups compared with the observed universe. The case $M/L = 500$ at $\sigma_8 = 0.5$ gives good agreement, however, with the observed properties of groups. The question is whether or not nature can hide this much mass; this will be an important consideration when we study velocities next.

4. PAIRWISE VELOCITY DISPERSIONS

We now consider constraints on σ_8 from our simulation based on pairwise velocity dispersions (σ_{\parallel}) of the resolved halos, see Figure 4. The open symbols in Figure 4 are the observed estimates from the Davis & Peebles (1983, hereafter DP) analysis of the CfA $B(0) \leq 14.5$ redshift survey. The different symbols are for different modeling parameters. The details are not important for our purposes; the scatter is small compared with the σ_8 dependence of σ_{\parallel} . The results at $r \sim 10$ Mpc are the least accurate because of distortions from peculiar motions.

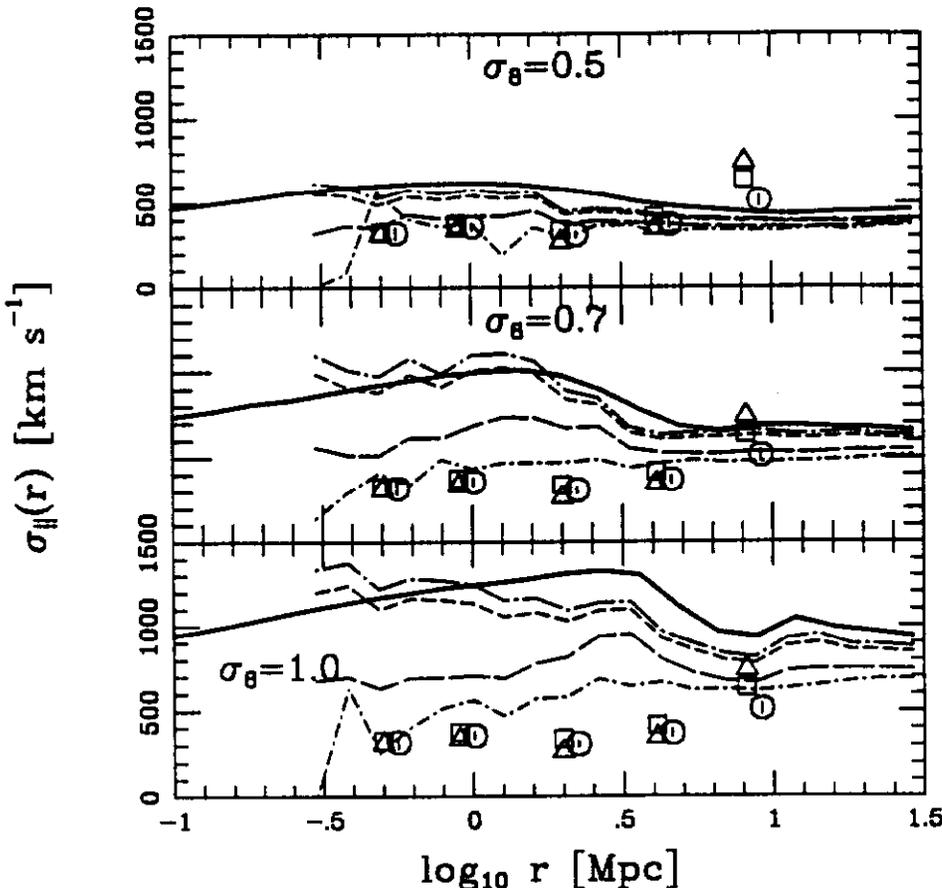


Figure 4: Pairwise velocity dispersions for halos with $V_{\text{circ}} \geq 150 \text{ km s}^{-1}$. No break-up (dot-short-dashed lines); mass (solid lines); and $M/L = 500$ with $\beta = 0.5$ (long-dashed lines), $\beta = 0.8$ (short-dashed lines), and $\beta = 1$ (dot-long-dashed lines).

Based on the no break-up cases in Figure 4, observational data constrains $\sigma_8 \lesssim 0.7$. The case $\sigma_8 = 0.5$ is an excellent match to the observed data. The case $\sigma_8 = 1.0$ is ruled out; the pairwise velocity dispersions are too high by factors ~ 2 for $r \gtrsim 1$ Mpc. Note that this is true even though there is a velocity bias of about a factor of two!

Couchman & Carlberg (1992; hereafter CC), in a slightly lower resolution simulation, find a pairwise velocity dispersion for their halos and their mass in reasonable agreement with our results for $\sigma_8 = 1$; however, they only report results at $r \sim 1$ Mpc. CC do not report pairwise velocity dispersions on larger scales where we find the disparity with the observations to be large. CC also find that their halos are significantly antibiased with respect to the mass (i.e. fall below the mass) on small scales.

We also study pairwise velocity dispersions after breaking up the massive halos. The results are shown in Figure 4 assuming $M/\mathcal{L} = 500$. We randomly sampled the positions of the massive halos to assign positions to the added halos. Next we describe how velocities are assigned to the added halos.

We use the 1-dimensional velocity dispersion of each massive halo as the *r.m.s.* for random numbers (see Gelb 1992); this quantity is labeled $\sigma_1(\text{MH})$ where MH is used to denote the original massive halo. We label the i -th (for $i = x, y, z$) component of the center-of-momentum velocity of the massive halo as $v_i(\text{MH})$. We then compute three gaussian random numbers, r_i , with mean zero and a 1-dimensional standard deviation $\sigma_1(\text{MH})$ for each group member. We define the velocity of the added group member as

$$v_i[\text{group member}] = v_i(\text{MH}) + \beta^{1/2} r_i ,$$

for some constant $\beta \leq 1$ discussed next.

The quantity β is the ratio of “galaxy temperature” to gas temperature (see Sarazin 1988). The “galaxy temperature” is a measure of the kinetic energy of the galaxies and the gas temperature, which is assumed to be in hydrostatic equilibrium with the group dark matter, is directly related to the velocity dispersion of the group. Observational estimates yield $\beta \sim 0.8$ (Evrard 1990).

Figure 4 shows results with $\beta = 1, 0.8,$ and 0.5 . These results indicate that the pairwise velocity dispersions are too high at $\sigma_8 = 0.5, 0.7,$ and 1.0 if we break up the massive halos. CC did not report the high pairwise velocity dispersions associated with groups. They find that merging decreases the numbers of halos in high dispersion regions, and they reference Bertschinger & Gelb (1991) where we first discussed why this effect can significantly reduce pairwise velocity dispersions. CC used FOF to identify halos with a prescription for preserving merged systems as distinct halos, but they comment that only $\sim 20\%$ of their “galaxy precursors” survive as distinct “galaxies”. This might explain why CC still find antibiasing and lower velocity dispersions. If we demand that the massive halos represent groups and if we use reasonable mass-to-light ratios, then we are forced to add back many halos. On the other hand, if we ignore the large masses associated with these systems, then we must find a mechanism whereby nature can hide a lot of mass.

We conclude that even small β cannot save $\sigma_8 \gtrsim 0.7$. The case $\sigma_8 = 0.5$ still has pairwise velocity dispersions that are high compared with the observations for $M/\mathcal{L} = 250$ and the model requires $\beta \lesssim 0.5$ which is on the low side of observational estimates (see Sarazin 1988, table 2). The same conclusion holds for $M/\mathcal{L} = 500$ at $\sigma_8 = 0.5$ except that the $\beta = 1/2$ case is a reasonable match to the observed pairwise velocity dispersions.

On the other hand, we have found that $M/\mathcal{L} = 500$ at $\sigma_8 = 0.5$ might solve some of the problems with the models. The numbers of halos and group properties are in good agreement with the observations. However, the correlation length ($r_0 \sim 6$ Mpc) falls short of the observed value $r_0 = 10$ Mpc. We have found in this section that the velocities for $M/\mathcal{L} \gtrsim 250$ at $\sigma_8 = 0.5$ are marginally consistent with the observed pairwise velocity dispersions and in good agreement with the observed pairwise velocity dispersions for $\beta \lesssim 0.5$ and $M/\mathcal{L} = 500$. If CDM is to survive on small scales, nature must conspire to hide a lot of dark matter. Whether or not it can do so is a controversial subject (Peebles 1986).

There is a difficulty in determining the bound mass of halos used in our mass-to-light method. Observationally, mass is inferred dynamically from galaxies. Galaxy tracers do not probe the mass in the outskirts of groups. We are investigating the bound mass of massive halos as a function of local density; imposing a density cut may lower our mass-to-light estimates. In any case, it appears to $\sigma_8 \gtrsim 0.7$ is ruled out in agreement with Davis et al. (1985).

5. CONCLUSIONS

Considerable attention has been paid to massive halos ("godzillas") which might represent groups. Breaking up these massive halos into groups removes the turnover of the two-point correlation function on small scales and it increases the correlation length on larger scales. Unfortunately, the groups do more harm than good unless we assume very high mass-to-light ratios. They significantly increase the number of halos, they give the wrong shape of the two-point correlation function, they significantly increase the pairwise velocity dispersions, and they make groups that are too rich for reasonable mass-to-light ratios.

In agreement with White et al. (1987) we find that we need to restore halos in massive systems to get the required two-point correlation length for $\sigma_8 \sim 0.5$. However, the fact that the model then has a factor ~ 3 too many halos and produces the wrong shape of the two-point correlation function is a serious shortcoming of the model. In agreement with Couchman & Carlberg (1992) we find a velocity bias of a factor ~ 2 for $\sigma_8 = 1$. However, restoring the merged halos in massive systems which have high velocity dispersions significantly increases the pairwise velocity dispersions. We can rule out $\sigma_8 \gtrsim 0.7$; even $\sigma_8 = 0.5$ requires $\beta \lesssim 0.5$ which is small compared with observed estimates.

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