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Inverse Phase Transitions: Does Baryogenesis Lead to Dark Matter?

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The phase structure of a field theory can have two qualitatively different forms– the less familiar of which involves *high* temperature symmetry breaking and *low* temperature symmetry restoration and is dubbed an inverse phase transition. After a general discussion of such inverse phase transitions we present an application of this phenomenon in which the symmetry under consideration is baryon number. The model has the virtues of generating the observed quark-antiquark asymmetry (with no explicit baryon number violating interactions) while simultaneously providing the dark matter known to exist in galactic halos and clusters of galaxies. Constraints from cosmology and particle physics highly constrain the mass of this dark matter candidate: $40\text{GeV} < m_\phi < 50\text{GeV}$. In this way we demonstrate our main conclusion: the exotic phase structure of the inverted form can give rise to novel, predictive and testable cosmological phenomenon.

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1. Introduction

Our purpose in this talk is to emphasize some unusual and potentially quite useful aspects of cosmological phase transitions. Our discussion consists of two parts: the first is a general review of what shall be called 'inverse phase transitions' – phase transitions with the unusual property of *high* temperature symmetry breaking/*low* temperature symmetry restoration [1] ; the second part of our discussion gives a cosmological application of this inverse phase structure [2] [3] – a mechanism by which baryogenesis and dark matter are intimately linked, thus providing, among other things, a natural explanation of the tantalizing hitherto unexplained fact that $\Omega_{\mathrm{baryon}}/\Omega_{\mathrm{dark\ matter}}$ is unity to within an order of magnitude or two.

By way of introduction we first review the more usual phase structure of low temperature symmetry breaking/high temperature symmetry restoration. We then discuss some quantities which can *qualitatively* affect this phase structure such as spacetime curvature, a conserved charge asymmetry or a (slightly) more complicated field content. The latter is the main focus of our paper and we discuss a simple model which exhibits the inverse phase structure property. Our conclusion will be that rather natural classical potentials can give rise to this unusual phase transition phenomenon. We will then apply this mechanism to yield a novel model which ties together baryogenesis and dark matter, as mentioned above. We emphasize at the outset that our goal is not to present a single complete and phenomenologically viable model. Rather, we hope to illustrate that a rather exotic cosmological phase structure can give rise to unusual, predictive and hence falsifiable solutions to problems in astroparticle physics.

2. Inverse Phase Structure

II.1: *Zero Temperature*

The standard approach for finding the vacuum configuration of a quantum system is to minimize V^{eff} , the effective potential. We recall that V^{eff} is the classical potential V^0 together with all quantum corrections. In practice, V^{eff} cannot be determined exactly. Rather, one usually makes use of a particularly tractable approximation – the loop expansion. In this approach, an infinite number of Feynman diagrams are summed up to yield the $\mathcal{O}(\hbar)$ corrections to the classical potential. Upon minimization with respect to a spacetime independent field configuration, the vacuum state of the system is determined.

II.2: *Nonzero Temperature*

To determine the vacuum state of a system at finite temperature (assuming global thermodynamic equilibrium) one follows a similar procedure as in the zero temperature setting. A temperature dependent effective potential $V^{\mathrm{eff}}(\phi;T)$ is calculated and then minimized. $V^{\mathrm{eff}}(\phi;T)$ is the classical potential together with all quantum corrections in the presence of a thermal bath. It is again generally impossible to compute $V^{\mathrm{eff}}(\phi;T)$ exactly

[1] To our knowledge the first appearance of this possibility occurred in [1].

– rather, one usually employs a loop expansion which takes the same form as it does at $T = 0$ with well know modifications to the Feynman rules in the imaginary time formalism. (For details of this standard material see [1] and [4]) To summarize, for a host of theories, including all those to be discussed here, the one–loop finite temperature effective potential is straightforward to calculate.

II.3: *The Usual Phase Structure*

To illustrate the more familiar finite temperature field theory phase structure, let's consider a $\lambda\phi^4$ theory which exhibits spontaneous symmetry breaking at $T = 0$. The one loop finite temperature effective potential $V^{\text{eff}}_{\text{one-loop}}(\phi;T)$ is calculated as outlined above with the result

$$V^{\text{eff}}_{\text{one-loop}}(\phi;T) = V^{\text{eff}}_{\text{one-loop}}(\phi;T=0) + \frac{T^4}{2\pi^2}\int_0^\infty x^2 \, ln(1 - e^{-(x^2+y^2)^{1/2}})dx \tag{2.1}$$

where $y^2 = \frac{\lambda}{2}\phi^2 T^2$. This result becomes particularly transparent in a high temperature expansion which yields

$$V^{\text{eff}}_{\text{one-loop}}(\phi;T) \;= V^{\text{eff}}_{\text{one-loop}}(\phi;T=0) + \frac{\lambda T^2}{48}\phi^2 + \mathcal{O}(\phi^4). \tag{2.2}$$

Note that λ is positive to ensure boundedness from below. We can interpret (2.2) as providing an effective mass m_{eff}:

$$m^2_{\text{eff}} = m^2 + \frac{\lambda T^2}{24}. \tag{2.3}$$

As we raise T, the negative mass squared driving zero temperature spontaneous symmetry breaking is pushed positive thus restoring the invariant $\phi = 0$ state as the vacuum. This is the prototype of low temperature symmetry breaking/high temperature symmetry restoration.

II.4: *Further Phase Structure Dependencies:*

In addition to finite temperature effects, there are a variety of other quantities which can qualitatively affect the phase structure of a theory. Let's mention three, the first two of which, not being our main concern, will be treated schematically.

(1) Spacetime Curvature:

In a curved background (at T = 0), the equations governing fluctuations ϕ_f in a scalar field ϕ take the form

$$(\nabla^2_4 + \xi R + m^2 + \frac{1}{2}\lambda\phi^2)\phi_f = 0 \tag{2.4}$$

All terms in addition to the Laplacian may be thought of as contributing to an effective mass and hence the phase structure depends both qualitatively and quantitatively on R, the curvature.

(2) Cosmological Charge Asymmetry:

Consider a theory with a globally conserved charge Q which for some reason has a net charge asymmetry. This charge, at finite T can be stored either in thermal field excitations or in zero momentum field condensate. The question of whether there is in fact any charge stored in condensate (i.e. whether we have spontaneous symmetry breaking) amounts to determining whether the thermal modes are capable of storing all of the charge of the system [5]. If not, there *must* be a condensate storing charge. It is not hard to show, for instance, in the simple case of a single charged scalar field, that any net charge asymmetry ensures that there is some positive temperature below which charge must be stored in a condensate, i.e. charge asymmetry can drive low temperature symmetry breaking. On the other hand, in a cosmological context, such an asymmetry can drive *high* temperature symmetry breaking [6]. The reason for this is that the charge density in such circumstances grows like T^3 which is faster than the capacity of the thermal modes to store charge.

(3) Nontrivial Field Content:

Our main focus is the effect on the temperature dependent phase structure of going beyond a theory containing a single scalar field. For instance, consider the next simplest possibility: a theory with two complex scalar fields ϕ and σ. We take a general form for the classical potential

$$V(\phi,\sigma) = m_\phi^2|\phi|^2 + m_\sigma^2|\sigma|^2 + \alpha_1|\phi|^4 + \alpha_2|\sigma|^4 - 2\alpha_3|\phi|^2|\sigma|^2. \tag{2.5}$$

We take the mass parameters and the couplings to be positive and real, as well as $\alpha_1\alpha_2 > \alpha_3^2$. These choices ensure boundedness from below and also imply that the zero temperature effective potential is minimized for $\phi = 0$ and $\sigma = 0$. Now, lets examine the one-loop finite temperature effective potential which, for ease of analysis, we approximate using a high temperature expansion. A short calculation gives

$$V^{\text{eff}}_{1-\text{loop}}(\phi,\sigma;T) = V(\phi,\sigma) + [-\frac{\alpha_3 - 2\alpha_1}{6}T^2|\phi|^2 + \frac{2\alpha_2 - \alpha_3}{6}T^2|\sigma|^2]. \tag{2.6}$$

Consider the portion of parameter space for which $2\alpha_1 < \alpha_3 < 2\alpha_2$. We see that finite temperature corrections drive a *negative* mass squared for the ϕ field. In this approximation we therefore find that for $T < T_c \equiv [\frac{6}{\alpha_3 - 2\alpha_1}]^{1/2} m_\phi$ the theory is unbroken while for $T > T_c$ the minimum of the effective potential is at $|\phi| \neq 0$, $|\sigma| = 0$. This simple example thus exhibits an inverse phase structure: high temperature symmetry breaking/low temperature symmetry restoration.

The upshot of this discussion is that a natural classical potential – without fine tuning of parameters – can yield the intuitively unexpected inverse phase structure.

The first person to realize this possible phase structure in the context of quantum field theory was Weinberg [1] who illustrated this phenomenon in an $O(n) \times O(n)$ model. He also noted that there are physical systems such as Rochelle Salts which exhibit high temperature symmetry breaking.

A natural question to ask is what are the cosmological possibilities/implications of such an inverse phase structure? A couple of groups have taken this question up. Langacker and Pi as well as Salomonson, Skagerstam and Stern [7] have applied an inverse phase transition model to try to solve the monopole problem and more recently Kephart, Weiler and Yuan (see also [8]) have classified a variety of interesting phase transitions/restorations which can occur in extensions of the Weinberg–Salam model. We will come back to the latter work at the end of our discussion; for now we move on to present our own cosmological application of such inverse phase transitions.

3. Baryogenesis, Dark Matter and the Width of the Z

There are three motivations for the work to be presently described [2] [3]. First, we are aware that any theory of baryogenesis must have baryon number violating interactions. However, we are also aware that baryon number violating interactions have not been observed. Is it possible to have baryogenesis in a theory for which L_{Universe} is baryon–number conserving? As we shall see, the answer is yes if one makes use of an inverted phase structure. The second reason is that anomalous baryon number violating standard model interactions indicate that any viable baryogenesis scheme must occur at about the weak scale to avoid being washed out. We will see that the present scenario can naturally occur at such a scale. Finally, and we feel most compelling, is that the present work provides a natural explanation for the mysterious fact $\Omega_{\text{baryon}}/\Omega_{\text{dark matter}}$ is unity to within a factor of ten to one hundred, a ratio which is *a priori* completely arbitrary.

III.1: *Essentials of the Model*

Sakharov identified the necessary ingredients for a dynamical theory of baryogenesis as consisting of baryon number violating interactions, CP violation and a period during which the universe was out of thermal equilibrium. Our main point of departure with respect to most models is to replace explicit baryon number violating interactions by spontaneous violation [2]. We will have nothing special to say about CP violation (which we will generate using complex Yukawa couplings) nor about an out–of–equilibrium scenario (which we will parameterize by the super–thermal abundance of a certain species of particle).

We will realize spontaneous baryon number violation by introducing a complex scalar field ϕ which is colourless but nonetheless has a nonzero baryon number. The potential for ϕ (with another scalar field σ) will be chosen, as presented earlier, to yield high temperature symmetry breaking/low temperature restoration of $U(1)_B$. That is, above some critical temperature T_c we will have $<\phi>\neq 0$ while below T_c we recover $<\phi>=0$. Additionally we will introduce two more complex scalars ϕ_1 and ϕ_2 which will play the role of 'lepto-quarks'. The field ϕ_1 will be able to decay to a u, d pair while ϕ_2 can decay to a u, e pair. A coupling $\kappa_1\phi^2\phi_1\phi_2^*$ will induce a mixing term for $<\phi>\neq 0$ allowing ϕ_1 to have a second decay channel to u, e and similarly ϕ_2 will have a decay channel to u, d. Thus when $<\phi>\neq 0$ there will be apparent baryon number violating decays. Now, we say 'apparent' because our theory is chosen to be $U(1)_B$ invariant – any apparent baryon number violation amongst the quanta is precisely compensated by a back reaction on $<\phi>$. That is, the $U(1)_B$ current is conserved and consists of contributions from the usual quarks and from $<\phi>$. Any change in one is precisely compensated for by the other. It thus behooves us to distinguish between baryogenesis and 'nucleogenesis' – the former does not occur in this model, but the latter does. In other words our 'nucleogenesis' amounts to rearranging an initial condition of $n_B - n_{\overline{B}} = 0$ into a later state of $0 = (n_B - n_{\overline{B}})|_{quarks} + (n_{\overline{B}} - n_B)|_{<\phi>}$. This however is not the end of the story. At temperatures below T_c, $U(1)_B$ is restored – that is $< \phi >$ goes back to zero. The antibaryonic charge that was stored in $< \phi >$ reappears as a cold bath of scalar antibaryonic ϕ particles. Our proposal is that these particles constitute the dark matter [3].

The assumption that ϕ is the dark matter limits the choice of what ϕ can be. We have considered two possibilities: ϕ as a gauge singlet and ϕ as an $SU(2)_W$ triplet with $Y = 0$. The latter possibility (see also ([9]) is the more interesting of the two and will be the subject of what follows. One immediate check is to determine whether or not the neutral member of this triplet is the lightest as the dark matter candidate must be electrically neutral. A one loop calculation shows that this is indeed the case after weak gauge symmetry breaking [3]. Next, let's give the Lagrangian of our model:

$$L = L_{\text{standard model}} + L' + L'' + L_{\text{kin}} \tag{3.1}$$

with

$$L' = \kappa_1 \phi^2 \phi_1 \phi_2^* + \kappa_2 \phi^2 \phi_1 \phi_3^* + f_1 \phi_1 U^T C D + f_2 \phi_2^* U^T C E + f_3 \phi_3^* U^T C E + h.c.; \tag{3.2}$$

and

$$L'' = -m_\phi^2 |\phi|^2 - \alpha_1 |\phi^4| - \alpha_2 |\sigma|^4 + 2\alpha_3 |\phi|^2 |\sigma|^2; \tag{3.3}$$

and L_{kin} containing the kinetic terms for the new scalar fields. A few remarks are in order. First, the ϕ_3 field has been introduced (with quantum numbers the same as ϕ_2) to ensure that there are enough Yukawa couplings to have CP violation (the inclusion of more than one generation could be used in place of this choice). Second, all quarks and leptons are taken to be right handed and hence $SU(2)_W$ singlets. Third, the ϕ field has $|b_\phi| = 1/2$ and hence the lightest component (the neutral member) is stable. Fourth, introducing all of these scalar fields is quite unattractive. We reemphasize, though, that our goal is to illustrate some unusual physics which follows from making use of inverse phase transitions. We therefore suspend the usual reluctance for such additions which we would have if our goal had been to build a fully realistic model.

Using standard methods we can compute the quark–antiquark excess produced in this model in terms of n, the out of equilibrium abundance of ϕ_i particles at temperature T [2]. (As mentioned earlier we are parameterizing the out of equilibrium nature of our scenario by n which we assume to be $\mathcal{O}(T^3)$ even though we take $M > T$.) The result is

$$B = \frac{n_B}{s} \simeq \frac{n}{M^2} Im[f_2 f_3^* \kappa_1^* \kappa_2| < \phi > |^2][\frac{30}{\pi^2 g_* T^3}][1 + \frac{2nM}{g_* T^4}]^{-3/4}. \tag{3.4}$$

In this expression, g_* is the number of light degrees of freedom at temperature T and M is a typical ϕ or ϕ_i particle mass. Minimization of the effective potential gives $< \phi > \sim T$ and hence we must have $T \sim M$ in order to generate a sufficiently large B. With these values it is well within the capabilities of this model to yield $B \sim 10^{-10}$, the observed value.

III.2: *Dark Matter*

We have seen that we can generate a quark–antiquark asymmetry of the correct magnitude in our approach, with the expense of generating an equal and opposite charge in the vacuum. As the temperature drops further we know that $U(1)_B$ is ultimately restored. What happens to the antibaryonic charge in $< \phi >$ when this VEV is driven back to zero? This antibaryonic charge appears as a bath of scalar ϕ particles with total antibaryonic charge equal in magnitude to n_B. Such a cold bath of massive weakly interacting particles is a natural dark matter candidate. In fact, it is a rather novel dark matter candidate for two reasons. First, it is antibaryonic. Second, its relic abundance is fixed by conservation laws rather than by the usual dynamical calculation involving annihilation rates in the early universe. To see this we note that baryon charge conservation yields

$$(n_B - n_{\overline{B}}) = b_\phi (n_\phi - n_{\overline{\phi}}) \tag{3.5}$$

where b_ϕ is the baryon number of the ϕ field. It is straightforward to see that only the excess asymmetry particles will survive until today and hence

$$n_\phi = n_B / b_\phi \tag{3.6}$$

from which we have

$$\Omega_\phi = \frac{m_\phi}{b_\phi m_p} \Omega_B. \tag{3.7}$$

To avoid overclosure of the universe we thus determine an upper bound on the mass of the ϕ particle:

$$m_\phi \leq b_\phi m_p \frac{1 - \Omega_B}{\Omega_B}. \tag{3.8}$$

In our model $|b_\phi| = 1/2$ and taking $\Omega_B = .01$ we find $m_\phi \leq 50$ GeV.

We can use particle physics to go further. Recall that ϕ is a triplet and that the electrically neutral member is our dark matter candidate. The charged partners contribute to the width of the Z unless the multiplet is sufficiently heavy. In fact, we get a lower bound from this constraint of $m_\phi \geq 40$ GeV. Hence, particle physics and cosmology combine to yield a rather narrow window of acceptable mass values:

$$40\text{GeV} \leq m_\phi \leq 50\text{GeV}. \tag{3.9}$$

We therefore see that this exotic cosmological phase structure yields unusually constrained physics. Equally important, we have a natural explanation of why the relic baryon abundance is close to that of the dark matter abundance: they both derive from precisely the same source.

III.3: *Direct Searches*

Even without viewing this model as anything but illustrative, it is worthwhile to pursue its signatures as far as possible. Along this line of inquiry it is natural to ask about whether direct dark matter searches and accelerator data shed any light on this scenario. At present the answer is no. For direct dark matter searches this is because our candidate does not have neutral current interactions and hence only scatters off of nuclei via double

W exchange. For the types of detectors envisioned by Goodman and Witten [10] we expect about one event per year per kilogram of detector. The dominant accelerator signature is the pion spectra produced from $Z \rightarrow \phi^0 + \pi^+ + \overline{\phi}^0 + \pi^-$ mediated by the charged members of the ϕ multiplet. The details of this are given in [3]; suffice it to say here that it appears that presently available accelerator data does not impinge on the model under study.

4. Gauge Symmetry Phase Transitions

The final issue we shall discuss is the inclusion of gauge symmetries amongst those which can undergo inverse phase transitions. This is a subject that has been studied in [1] [11] [12] [13]. Much the same way as we have shown that global symmetries can be broken at high temperatures and restored at low temperatures, gauge symmetries can also exhibit such behaviour. For example, the gauge symmetries of the standard model can be broken for a period of time in the early universe and restored at a sufficiently high scale to avoid conflict with experimental data. For an example of this phenomenon we can return to the model we have been discussing and replace the σ field (which occurs in the inverse phase transition potential) by the Higgs field of the standard model. As shown in [3](following the work of [13]), a simple study of the structure of the finite temperature effective potential in this context reveals a number of interesting possibilities. Amongst these are periods, for example, in the early universe in which at a very high scale all symmetries are restored, followed by an intermediate scale in which all are broken, finally followed by a low temperature theory in which $SU(3)_c \times U(1)_{em} \times U(1)_B$ are restored. The cosmological implications and restrictions on such relatively complicated stages of symmetry change have yet to be fully understood and are sure to embody interesting physics.

5. Conclusions

In this talk we have emphasized one general point that has been known for some time but has received comparatively little attention: the phase structure of a field theory can quite naturally follow an inverted form in which a symmetry is broken at high temperatures and restored at low temperatures. We have given one interesting application of such a scenario in which baryon number is the symmetry of consideration. In this model the issues of baryogenesis and dark matter are seen to naturally merge; combined with particle physics considerations we find the scenario to be hearteningly predictive. Furthermore, it gives a natural explanation of the puzzling 'coincidence' that the relic abundance in baryons differs from the critical closure density by only one to two orders of magnitude. Beyond the particulars of this model, our main conclusion is that there are a rich set of possible cosmological phase transitions leading to the presently observed set of symmetries. Some of these possibilities leave a very definite and hence testable signature and are worthy of further detailed study.

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