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EXTRACTING PROPERTIES OF HEAVY-LIGHT MESONS

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A multistate smearing method suitable for the extraction of both ground and radially excited heavy-light states is applied to the pseudoscalar channel.

1. Introduction

The dynamics of QCD simplifies in two limits: (1) the usual chiral limit ($m_q = 0$) and (2) the heavy quark limit ($m_Q \to \infty$) [1]. Since heavy-light mesons have only one dynamical light (valence) quark, these systems are also well suited to the study of constituent quark ideas [2] and the chiral quark model [3]. These properties make heavy-light systems ideal for lattice studies.

In actual lattice calculations, many practical issues must be addressed. One particular problem in the study of the pseudoscalar state is that the heavy-light correlator becomes noisy rather rapidly in time. Unfortunately, this is an unavoidable feature of heavy-light systems. To understand the origin of this behaviour, recall that the noise to signal ratio (N/S) is determined from the expectation of the square of the correlator divided by the square of the expectation of the correlator and that new states can couple in the square of the correlator[4]. In the heavy-light case,

$$\mathrm{N/S} \approx \mathcal{N} \exp(\Delta M \; T)$$

where ΔM is the mass difference between the pseudoscalar ground states of the heavy-light system, $m(q\bar{Q})$, and the average of the light-light, m_π, and heavy-heavy, $m(Q\bar{Q})$ systems. The normalization $\mathcal{N}$ depends on the overlap of the specific operators used in the correlator with the physical ground states. The mass difference

$$\Delta M = m(q\bar{Q}) - [m_\pi + m(Q\bar{Q})]/2$$

is rigorously nonnegative.

As $m_Q \to \infty$, $m(q\bar{Q}) - m_Q \equiv \mathcal{E}_{q\bar{Q}}$ approaches a constant independent of m_Q, but the binding energy $\mathcal{E}_{Q\bar{Q}} \equiv 2m_Q - m(Q\bar{Q})$ is positive and grows linearly with m_Q. Hence $\Delta M = \mathcal{E}_{q\bar{Q}} - m_\pi/2 + \mathcal{E}_{Q\bar{Q}}/2$ grows linearly with m_Q. Thus at fixed physical time the ratio of noise to signal grows exponentially with m_Q. For the static effective action, there is no dependence on m_Q and $\mathcal{E}_{Q\bar{Q}} = 0$; but, now, the static energy of the heavy-light pseudoscalar meson depends on the lattice cutoff $1/a$; i.e. $\mathcal{E}_{q\bar{Q}} = \mathcal{E}_0 + \mathcal{E}_1/a$. Hence at fixed physical time the ratio N/S grows exponentially with $1/a$.

It is important, therefore, to develop new techniques which allow the extraction of the properties of heavy-light states from relatively short times. Here, such a new smearing method is applied to the pseudoscalar channel.

2. Multistate Smearing Method

The basics of the multistate smearing method are reported by Duncan [5]. Let $\Psi^{(a)}_{\rm smear}(\mathbf{r})$ ($a = 1, 2, ..., N$) be a set of linearly independent, orthogonal wavefunctions with S-wave symmetry. These wavefunctions are generated by a Hamiltonian, H and ordered by eigenvalue, $E^{(a)}$. The basic multismeared correlator, $S^{(a)}$ ($a = 1, 2, ..., N$), is given by:

$$S^{(a)} \quad (\mathbf{r}, T) = \sum_{\mathbf{r}'} \Psi^{(a)}_{\rm smear}(\mathbf{r}', T)$$
$$< q(\mathbf{r}+\mathbf{r}', T) Q^\dagger(\mathbf{r}', T)\ Q(\mathbf{r}', 0) q^\dagger(\mathbf{o}, 0) >$$

The heavy quark operator, Q, simply generates a product of gauge links along the time direction, while the color and spin indices of the multismeared correlator are the same as for the light quark propagator $< q(\mathbf{r}, T)\ q^\dagger(\mathbf{o}, 0) >$.

For given quantum numbers and a well-chosen set of smearing wavefunctions it is possible to accurately extract the low-lying heavy-light states with only a small number of smearing functions.

The investigation presented here used an existing set of 50 configurations (separated by 2000 sweeps) generated by ACPMAPS on a $16^3 \times 32$ lattice at $\beta = 5.9$. The configurations were fixed to Coulomb gauge and light quark propagators with $\kappa = .158$ were used. Only the four lowest energy smearing functions are included ($N = 4$).

The appropriate smearing functions can be obtained by an iterative process:

- Start with a reasonable smearing function for the ground state. An exponential $\exp(-R/R_0)$ with $R_0 = .5a$ was chosen.
- Measure the ground state wavefunction at moderate time T. The singlet piece of the wavefunction (obtained by projecting on the spin and color singlet piece of the smeared correlator $S^{(1)}(\mathbf{r}, T)$) was measured at times $(3-6)a$.
- Tune the parameters in the Hamiltonian, H to give the best fit for the lowest eigenfunction to the measured ground state wavefunction.
- Generate the required number $N(4)$ of smearing wavefunctions from this tuned H.
- Use this set of smearing functions to measure the multismeared correlators.
- To take maximum advantage of these measurements, define a $N \times N$ (4×4) coupling matrix,

$$C^{ab}(T) = \sum_{\mathbf{r}} \Psi^{\dagger(b)}_{\rm smear}(\mathbf{r}) S^{(a)}(\mathbf{r}, T)$$

Diagonalizing the coupling matrix at each time T gives eigenvalues, $\lambda^i(T)$, which are related to the mass of the i^{th} heavy-light state by

$$\mathcal{E}^i(T) = -log(\lambda^i(T))/log(\lambda^i(T+1))$$

and eigenvectors which give the coupling of the set of smearing functions to that i^{th} state.

- Finally, the improved wavefunctions for the heavy-light states can be used to retune the parameters in H and then the whole process repeated with better smearing functions.

T	E_1	E_2	E_3	E_4
1	0.714	1.042	1.247	1.457
2	0.651	0.978	1.102	1.302
3	0.651	0.921	1.087	1.229
4	0.650	1.102	1.430	3.211

Table 1
Convergence of masses with multistate smearing

The critical element in this procedure is the choice of the generating function H. We propose a lattice Hamiltonian, H = T + V where the potential energy V(R) is just the static energy measured on the same configurations used to study the heavy-light spectrum. For the kinetic piece, the lattice form of the relativistic kinetic energy

$$\mathrm{T} = \sqrt{\sum_i \hat{p}_i^2 + \mu^2} - \mu$$

is used with the reduced mass μ as a tunable parameter. The simpler nonrelativistic form is not adequate for light quarks ($\kappa = .158$).

3. Masses and Wavefunctions

Using the four state smeared correlator described in the previous section an initial study for the pseudoscalar channel was carried out. In the first pass (using an exponential smearing function for the ground state) the value $\mu = .45$ was obtained. Using this reduced mass parameter in H, the multismeared correlator was measured and the coupling matrix $\mathcal{C}^{ab}(T)$ was determined. The results for the static energies $\mathcal{E}^i(T)$; are presented in Table 1. No error estimates are included. The ground state has reached a plateau within two time slices. There is also some evidence for the first excited state.

The eigenvector corresponding to the heavy-light ground state is quite stable in time. For time $T = 4$ its couplings to the smearing states are: $(.96, -.28, .07, .02)$.

R	$\psi(T=2)$	$\psi(T=4)$	$\psi(T=6)$	fit
1	.623	.612	.608	.638
2	.384	.389	.385	.382
3	.234	.250	.250	.223
4	.144	.157	.155	.125
5	.089	.100	.096	.068
6	.058	.068	.061	.038
7	.043	.051	.046	.023
8	.038	.045	.040	.018

Table 2
Ground state wavefunction $\psi(R)$ at various times T. The ground state smearing wavefunction (fit) is also shown

Finally one can look at the wavefunction for the ground state. The singlet piece is shown in Table 2 at time $T = 2, 4, 6$. All wavefunctions are normalized to one at the origin. For comparison, the wavefunction for the first (lowest energy) smearing state is also shown. As can be seen from Table 2 some further improvement in the smearing functions would be possible using a slightly smaller value for the reduced mass parameter in H. Additional studies are in progress.

References

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