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An Effective Field Theory For The Neutron Electric Dipole Moment

D. CHANG,[1,2] T. W. KEPHART,[3] W.-Y. KEUNG,[4,2] AND T. C. YUAN[1]

[1] *Department of Physics and Astronomy, Northwestern University Evanston, IL 60208*

[2] *Fermilab, P. O. Box 500, Batavia, IL 60510*

[3] *Department of Physics and Astronomy, Vanderbilt University Nashville, TN 37235*

[4] *Physics Department, University of Illinois at Chicago, IL 60680*

ABSTRACT

We derive a CP odd effective field theory involving the field strengths of the gluon and the photon and their duals as a result of integrating out a heavy quark which carries both the chromo-electric dipole moment and electric dipole moment. The coefficients of the induced gluonic, photonic, and mixed gluon-photon operators with dimension ≤ 8 are determined. Implications of some of these operators on the neutron electric dipole moment are also discussed.

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1. Introduction

It is commonly believed that a nonvanishing neutron electric dipole moment (NEDM), if observed at the present level of experimental sensitivity $\sim 10^{-26}$ e–cm, would be an indication of some new physics of CP violation beyond the Standard Kobayashi–Maskawa Model. Present experimental bounds are in fact expected to improve by one or two orders of magnitude in the near future. On the theoretical side, a new mechanism of generating NEDM through a purely gluonic process envisaged by Weinberg [1] has stimulated many subsequent studies. The mechanism involves the following dimension 6 purely gluonic CP violating operator,

$$O_6 = -\frac{1}{3} g^3 f^{abc} \tilde{G}^{a\mu\nu} G^b_{\lambda\mu} G^{c\lambda}{}_{\nu} \quad , \tag{1}$$

which has been identified as the chromo–electric dipole moment (CEDM) of the gluon itself [2]. In (1), $G^a_{\mu\nu}$ is the gluon field strength, $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{a\alpha\beta}$ is its dual with $\epsilon^{0123} = +1$; g and f^{abc} are the gauge coupling and the totally antisymmetric structure tensor of $SU(3)$ respectively. Up to total derivatives, O_6 is the unique purely gluonic, gauge invariant, CP violating operator with dimension ≤ 6. (As is well known, the dimension 4 topological term

$$O_\theta = g^2 \tilde{G}^a_{\mu\nu} G^{a\mu\nu} \quad , \tag{2}$$

which is a total derivative, can provide an enormous contribution to the NEDM due to the nonperturbative QCD effect. One popular solution to this so-called strong CP problem is the Peccei-Quinn mechanism [3]. Whether this same mechanism designed to eliminate the effect of O_θ can suppress the impact of O_6 as well has been debated in the literature [4]. We will not address these issues here.)

Besides O_6, Weinberg also pointed out [1,5] that certain dimension 8 purely gluonic operators can also induce a NEDM. Such operators, together with O_6 was investigated by Morozov [6] some time ago. He observed that there are three independent dimension 8 CP odd purely gluonic operators

$$
\begin{aligned}
O_{8,1} &= \frac{1}{12} g^4 \tilde{G}^a_{\mu\nu} G^{a\mu\nu} G^b_{\alpha\beta} G^{b\alpha\beta} \quad , \\
O_{8,2} &= \frac{1}{12} g^4 \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^a_{\alpha\beta} G^{b\alpha\beta} \quad , \qquad (3) \\
O_{8,3} &= \frac{1}{12} g^4 d^{abe} d^{ecd} \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^c_{\alpha\beta} G^{d\alpha\beta} \quad ,
\end{aligned}
$$

where d^{abc} is the totally symmetric tensor of $SU(3)$ and calculated their anomalous dimensions. In renormalizable gauge theories, these operators are in general induced by two–loop diagrams when all the heavy particles in the intermediate states are integrated out. In many models of CP violation (for example, supersymmetric model, charged Higgs exchange model, and left–right symmetric model, which we shall summarily call charged models), there are two different mass scales in the intermediate states. Then the process of integrating out the heavy particles should be done in two steps. In the first step, the CP violating particles with masses of M_W or higher are integrated out. Typically the CEDM of a relatively lighter quark (say, the bottom quark) is then induced at the one–loop level. After this operator is evolved from M_W down to the quark mass threshold, the operator O_6 is induced at the one–loop level when the quark with CEDM is in turn eliminated from the effective field theory [7,8,9,10]. On several occasions [11], we have briefly discussed the potential importance of the dimension 8 operators. However, the coefficients of the dimension 8 operators that are induced in specific models have never been calculated in the literature except for the simple case of the one–particle–reducible two–loop diagrams [5,12,13] in neutral Higgs model. This work is to fill this gap for the charged models.

In all previous works, the coefficient of O_6 was obtained by standard Feynman diagram calculations using ordinary momentum space perturbation theory or background field method [7,8,9,10,14,15]. These approaches become quite unwieldy for operators of higher dimensions because of (i) the proliferation of indices of all sorts and (ii) the large number of gauge variant operators that can be generated in the intermediate steps. Background field methods based on flat connections can simplify the second problem (for example, one can look at the eight-point functions in the two-loop diagrams for the dimension 8 operators) but not the first one. These complications exist even in the one-loop calculation corresponding to the elimination of a heavy quark with CEDM. In this paper, we present a systematic and tractable way (based on a functional approach and covariant derivative expansion) to generate all such CP odd purely gluonic or photonic or mixed gluon-photon operators of higher dimensions as a result of integrating out a heavy quark that carries not only a CEDM but also an electric dipole moment (EDM).

One generally expects the coefficients of the dimension 8 purely gluonic operators to be suppressed by powers of the inverse heavy masses relative to the O_6. However in the case of the charged models this is the b quark mass and the suppression is not too severe. In addition, unlike O_6 which is suppressed by the QCD renormalization effect at low energy [6,7], one combination of the dimension 8 operators (mostly $O_{8,1}$) has a positive and sizable anomalous dimension [6]. Therefore its effect can be significantly enhanced by the QCD effects at the hadronic scale. The effect of the operators with dimension higher than 8 are expected to be small unless their anomalous dimensions are not only positive but also surprisingly large in value. Nevertheless, the coefficients of operators of higher dimensions can be obtained easily by our method.

It is worthwhile to mention that the chromo-electric dipole moment effect that we calculate in this work is not the only contribution at the same level in perturbative expansion. In the charged Higgs model for example, from the viewpoint of two loop diagrams, this analysis will correspond to only the QCD improved version of Fig.1 (a). There are also diagrams like that in Fig.1(b,c). If one integrates out the t quark first in these contributions, operators of dimension 7 of the forms $\bar{Q}Q\tilde{G}G$ and $\bar{Q}\gamma_5 QGG$, or of dimension 9 of the forms $\bar{Q}Q\tilde{G}GG$ and $\bar{Q}\gamma_5 QGGG$ will be induced. And they in turn give rise to operators of dimension 8 below the b quark threshold as in Fig.2. However, at the t quark threshold, these operators of higher dimensions receive large suppression by powers of inverse t quark mass. Their eventual contributions are expected to be negligible even though some of the dimension 7 operators do receive mild QCD renormalization enhancement according to Morozov [6].

The induction of CEDM or EDM of lighter quarks (such as b quark) at the threshold of heavy CP violating particles in different charged models has been worked out before [8]. To be more model independent here, we shall simply parametrize these vertices in an effective Lagrangian and calculate their effects at the hadronic scale. In the following two sections the heavy quarks will stand for quarks, such as b and c, that is lighter than M_W and heavier than hadronic scale.

In section 2, we first show the effective action, induced by integrating out a heavy quark with both CEDM and EDM, is given by the functional determinant of a generalized Dirac operator. We then evaluate the functional determinant to first order in CEDM using the method by Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) [16] who had generalized Schwinger's operator method [17] to the case of nonabelian gauge theory. The attractive feature of this approach is that one deals with the field strength and its covariant derivative directly and thus gauge invariance is manifest throughout the calculation. The approach is essentially a covariant derivative expansion of the external gauge fields. This method has been used to determine the coefficients of the purely gluonic CP even operators up to dimension 8 [16], to derive low-energy effective Lagrangians involving anomaly-induced vertices, Goldstone-Wilczek currents, and Skyrme terms [18], etc. In the NSVZ approach, operators involved the covariant derivatives are converted into operators involved only the gauge covariant field strength through some nested multi-commutators relations which are not so easy to derive. Therefore the calculation is very tedious to say the least. Such tedious steps can be bypassed by performing a bi-unitary transformations to the functional trace, invented and improved by a number of authors [19,20,21]. Through these transformations, the covariant derivative operator is converted into a auxiliary c-number momentum at the very beginning. The functional determinant which originally was an expansion of complicated commutators of covariant derivative operators is now expressed as an expansion in the auxiliary momentum and field strengths. One can then take the average of the auxiliary momentum in order to produce the ordinary series expansion of effective action. This approach has also been widely applied to many situations [21] including the calculation of the β functions of QED and QCD, and the derivation of the effective action for the Standard Model with a very heavy Higgs, etc. We also show that the later approach reproduces easily the results previously obtained by the method of NSVZ.

In section 3, we include electromagnetism and allow the heavy quark to carry EDM as well as CEDM. A complete local Lagrangian with operators of dimension ≤ 8 is given. Besides the two operators with one–photon and three–gluon fields discussed by de Rujula *et al* [22], we also generate the CP violating four–photon operator and three operators with two–gluon and two–photon fields for completeness. We discuss briefly the phenomenology in section 4. The phenomenological consequence of this effective field theory related to the NEDM has been given in a recent letter [23]. Here we shall emphasize the analytical aspects. A bank of formulas and identities which are indispensable for deriving our final results are relegated to the Appendix.

2. Covariant Derivative Expansion

Suppose the CEDM (C) and the EDM (C') of a heavy quark Q with mass M and electric charge e_Q was induced at a large mass scale $\Lambda > M$ when the particles with mass greater than Λ were eliminated from the spectrum of the low energy effective theory. (It is not necessary here to specify the physical origin of CP violations associated with these heavy particles.) At the renormalization point $\mu = \Lambda$, the effective Lagrangian can be parametrized, up to terms of dimension five, as

$$L = -\frac{1}{2}\mathrm{Tr}\ G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{Q}(\not{P} - M - \frac{i}{2}gC\gamma_5\sigma\cdot G - \frac{i}{2}e_Q C'\gamma_5\sigma\cdot F)Q \quad . \tag{4}$$

Here $G_{\mu\nu} = G^a_{\mu\nu}T^a$ with T^a the generators in fundamental representation of a simple Lie algebra associated with color, $F_{\mu\nu}$ is the electromagnetic field strength; $\sigma\cdot G = \sigma_{\mu\nu}G^{\mu\nu}$ and $\sigma\cdot F = \sigma_{\mu\nu}F^{\mu\nu}$ with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$; and $\not{P} = \gamma_\mu P^\mu$ where $P_\mu = i\partial_\mu + \mathcal{A}_\mu$ with $\mathcal{A}_\mu = gA^a_\mu T^a + e_Q B_\mu$, the gauge connections including the color group and electromagnetism. Here we have assumed that the terms of higher dimensions are negligible. This assumption is certainly valid in most of the charged models (except maybe the supersymmetric models). As long as no other particle masses lie between Λ and M, one can evolve the above theory from Λ to M via the standard renormalization group machinery. However, when μ evolves below M, we must change the effective theory to a new one with the heavy quark Q removed from the particle spectrum [24]. The new effective theory thus obtained involves an infinite tower of non-renormalizable operators constructed out of the field strengths and their covariant derivatives but with coefficients suppressed by inverse power of the heavy quark mass M.

To evaluate this, first consider the path integral representation of the vacuum to vacuum transition amplitude ${}_{\mathrm{out}}\langle 0|0\rangle_{\mathrm{in}} = \int \mathcal{D}A^a\mathcal{D}B\mathcal{D}Q\mathcal{D}\bar{Q}\ \exp\ i\int d^4x L$. Since the Lagrangian (4) is quadratic in Q and $\bar{Q}$, we can integrate out the heavy quark and obtain

$$\begin{aligned}
{}_{\mathrm{out}}\langle 0|0\rangle_{\mathrm{in}} &= \int \mathcal{D}A^a\mathcal{D}B\ \exp i S_{\mathrm{eff}}[\mathcal{A}] \quad , \\
&= \int \mathcal{D}A^a\mathcal{D}B \exp i(S_{\mathrm{qcd}}[A^a] + S_{\mathrm{qed}}[B]) \\
&\quad \times \mathrm{Det}\,(\not{P} - M - \frac{i}{2}gC\gamma_5\sigma\cdot G - \frac{i}{2}e_Q C'\gamma_5\sigma\cdot F)\,, \\
S_{\mathrm{eff}}[\mathcal{A}] &= S_{\mathrm{qcd}}[A^a] + S_{\mathrm{qed}}[B] + \Delta S[\mathcal{A}] \quad . \\
\Delta S[\mathcal{A}] &= -i\mathrm{Tr}\ln(\not{P} - M - \frac{i}{2}gC\gamma_5\sigma\cdot G - \frac{i}{2}e_Q C'\gamma_5\sigma\cdot F)\ .
\end{aligned} \tag{5}$$

The effective action, $\Delta S[\mathcal{A}]$, involves the generalized Dirac operator and is nonlocal. It corresponds to the sum of all one–loop diagrams with arbitrary numbers of external gluon attached to the heavy quark loop. Since we are only interested in the induced operators in ΔS to first order in C and C', it can be written as

$$
\begin{aligned}
\Delta S[\mathcal{A}] = & -i\mathrm{Tr}\ \Big[\ln(\not{P} - M) \\
& + \ln\left(1 - (\not{P} - M)^{-1}(\frac{i}{2}gC\gamma_5\sigma\cdot G + \frac{i}{2}e_Q C'\gamma_5\sigma\cdot F)\right)\Big] \quad , \\
= & -i\mathrm{Tr}\ \Big[\ln(\not{P} - M) \\
& - \frac{i}{2}(\not{P} - M)^{-1}(gC\gamma_5\sigma\cdot G + e_Q C'\gamma_5\sigma\cdot F) + \cdots\Big] \quad .
\end{aligned}
\tag{6}
$$

The expansion in (6) is valid as long as $|G|^2$ and $|F|^2 < M^2$. The first term of ΔS is CP even and has been studied before [16]. We will concentrate on the second term which is CP odd. To the first order in C and C',

$$
\begin{aligned}
\Delta S_{CP} = & -\frac{1}{2}\mathrm{Tr}\ \left[(\not{P} - M)^{-1}(gC\gamma_5\sigma\cdot G + e_Q C'\gamma_5\sigma\cdot F)\right] \quad , \\
= & -\frac{M}{2}\mathrm{Tr}\ \left[(P^2 - M^2 + \frac{1}{2}\sigma\cdot\mathcal{G})^{-1}(gC\gamma_5\sigma\cdot G + e_Q C'\gamma_5\sigma\cdot F)\right] \quad ,
\end{aligned}
\tag{7}
$$

where we have defined $\sigma\cdot\mathcal{G} = g\sigma\cdot G + e_Q\sigma\cdot F$.

For brevity, we present the details of following calculation without electromagnetism for the rest of this section. Incorporating this additional effect is straightforward and will be given in the next section. Setting $C' = 0$ and $e_Q = 0$ in (7) and expanding the chromo–magnetic moment in the denominator, we deduce

$$
\begin{aligned}
\Delta S_{CP} = & -\frac{gCM}{2}\mathrm{Tr}\ \left[\gamma_5(P^2 - M^2 + \frac{1}{2}g\sigma\cdot G)^{-1}\sigma\cdot G\right] \quad , \\
= & -\frac{gCM}{2}\left\{-\frac{g}{2}\mathcal{P}_2 + \frac{g^2}{4}\mathcal{P}_3 - \frac{g^3}{8}\mathcal{P}_4 + O(g^4)\right\} \quad ,
\end{aligned}
\tag{8}
$$

with $\mathcal{P}_2$, $\mathcal{P}_3$, and $\mathcal{P}_4$ defined by

$$
\mathcal{P}_2 = \mathrm{Tr}\ [\gamma_5\Delta(P)\sigma\cdot G\Delta(P)\sigma\cdot G] \quad , \tag{9}
$$

$$\mathcal{P}_3 = \mathrm{Tr}\ [\gamma_5 \Delta(P) \sigma \cdot G \Delta(P) \sigma \cdot G \Delta(P) \sigma \cdot G] \quad , \tag{10}$$

and

$$\mathcal{P}_4 = \mathrm{Tr}\ [\gamma_5 \Delta(P) \sigma \cdot G \Delta(P) \sigma \cdot G \Delta(P) \sigma \cdot G \Delta(P) \sigma \cdot G] \quad . \tag{11}$$

Here $\Delta(P) = (P^2 - M^2)^{-1}$. The $O(g^4)$ term in (8) is necessarily of dimension 10 or higher and will be ignored here.

To evaluate $\mathcal{P}_{2,3,4}$, we treat the gauge field as an external background field and apply Schwinger's operator method [17]. The main obstacle to evaluating the functional traces in (9)–(11) is that for a general position dependent background field G, P and G do not commute. The trace is not diagonal in either position space or momentum space. Methods to circumvent this difficulty were developed in Refs. [16] and [18]. The essential ingredient of the method is to put all the propagators $\Delta(P)$ in $\mathcal{P}_{2,3,4}$ to the leftmost position inside the functional trace. The algebra is performed by repeatedly using the following identities:

$$\begin{aligned} X\Delta(P) &= \Delta(P)X + \Delta^2(P)[P^2, X] + \Delta^3(P)[P^2,[P^2,X]] \\ &\quad + \Delta^4(P)[P^2,[P^2,[P^2,X]]] + \cdots \quad , \\ \Delta(P)X &= X\Delta(P) - [P^2,X]\Delta^2(P) + [P^2,[P^2,X]]\Delta^3(P) \\ &\quad - [P^2,[P^2,[P^2,X]]]\Delta^4(P) + \cdots \quad , \end{aligned} \tag{12}$$

where X is any operator. Using these rules, one can obtain the expressions for $\mathcal{P}_{2,3,4}$ that involve all the propagator $\Delta(P)$ moved to the leftmost position and a string of multi-commutators at the rightmost position inside the functional trace. $\mathcal{P}_4$ is in fact easy to handle. Since $\mathcal{P}_4$ already has four field strengths in the numerator, to obtain the dimension 8 operators one can ignore the noncommutative algebra and set the gauge connection to zero in all the propagators. The functional trace can then be easily evaluated by going to momentum space. Thus,

$$\begin{aligned} \mathcal{P}_4 &= \mathrm{Tr}\ \left[\gamma_5 \Delta^4(p)(\sigma \cdot G)^4\right] + O(G^5) \quad , \\ &= +\frac{i}{16\pi^2}\frac{1}{6M^4}\mathrm{tr}\ \gamma_5 \int d^4x (\sigma \cdot G)^4 + O(G^5) \quad , \end{aligned} \tag{13}$$

where 'tr' denotes the trace over the color gauge group and the Lorentz space. For $\mathcal{P}_{2,3}$, we get

$$
\begin{aligned}
\mathcal{P}_2 = \mathrm{Tr}\ \left[\gamma_5\Delta^2(P)(\sigma\cdot G)^2\right] - \mathrm{Tr}\ \{\gamma_5\Delta^3(P)\,(\sigma\cdot G + \Delta(P)[P^2,\sigma\cdot G] \\
+\ \Delta^2(P)[P^2,[P^2,\sigma\cdot G]] + \Delta^3(P)[P^2,[P^2,[P^2,\sigma\cdot G]]] + \cdots)\,[P^2,\sigma\cdot G]\} \quad ,
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\mathcal{P}_3 = &\mathrm{Tr}\ \left[\gamma_5\Delta^3(P)(\sigma\cdot G)^3\right] \\
&+ \mathrm{Tr}\ \left\{\gamma_5\Delta^4(P)\left([P^2,\sigma\cdot G](\sigma\cdot G)^2 - (\sigma\cdot G)^2[P^2,\sigma\cdot G]\right)\right\} \\
&- \mathrm{Tr}\ \{\gamma_5\Delta^5(P)\,([P^2,(\sigma\cdot G)^2][P^2,\sigma\cdot G] \\
&\quad +[P^2,\sigma\cdot G][P^2,(\sigma\cdot G)^2] + [P^2,\sigma\cdot G]\sigma\cdot G[P^2,\sigma\cdot G])\} + \cdots \quad .
\end{aligned} \tag{15}
$$

By using the Heisenberg equation of motion,

$$
[P_\mu, G_{\alpha\beta}] = iD_\mu G_{\alpha\beta} \quad , \tag{16}
$$

one can easily evaluate all the multi-commutators,

$$
[P^2,\sigma\cdot G] = -D^2\sigma\cdot G + 2iD_\mu\sigma\cdot GP^\mu \quad , \tag{17}
$$

$$
\begin{aligned}
[P^2,[P^2,\sigma\cdot G]] = D^4\sigma\cdot G - 2iD_\mu D^2\sigma\cdot GP^\mu + 4gD_\mu\sigma\cdot GG^{\mu\nu}P_\nu \\
- 2iD^2D_\mu\sigma\cdot GD^\mu - 4D_\mu D_\nu\sigma\cdot GP_\mu P_\nu \quad ,
\end{aligned} \tag{18}
$$

$$
[P^2,[P^2,[P^2,\sigma\cdot G]]] = -8iD_\mu D_\nu D_\alpha\sigma\cdot GD_\mu D_\nu D_\alpha + \cdots \quad , \tag{19}
$$

etc. D_μ is the covariant derivative acting on the field strength with the following rule of operations: For a matrix-valued function $\mathcal{M} = M^aT^a$, $D_\mu\mathcal{M}$ is defined to be $\partial_\mu\mathcal{M} - ig[A_\mu,\mathcal{M}]$ and $[D_\mu, D_\nu]\mathcal{M} = -ig[G_{\mu\nu},\mathcal{M}]$. In the above equations, those terms that give rise to dimension 10 operators or higher have been ignored.

It is interesting to note that the dimension 4 topological term O_θ is also induced by $\mathcal{P}_2$

$$\mathcal{P}_2 = \mathrm{Tr}\ \left[\gamma_5 \Delta^2(p)(\sigma \cdot G)^2\right] + O(G^3) \quad . \tag{20}$$

This term is in fact logarithmic ultraviolet divergent. Introducing a momentum cut off Λ_{UV}, we get

$$\mathcal{P}_2 = \frac{i}{16\pi^2}\left(\ln \Lambda_{UV}^2 + \text{finite}\right) \mathrm{tr}\ \gamma_5 \int d^4x (\sigma \cdot G)^2 + O(G^3) \quad . \tag{21}$$

This divergent term comes with no surprises since the induced operator has a lower dimension than the quark CEDM operator in (4). Thus, this term does not belong to the threshold effects. Instead, it should be interpreted as the operator mixing between the quark CEDM and the topological term due to the QCD renormalization group evolution. The divergent coefficient in (21) implies that the anomalous dimension is $\gamma_{Q\theta} = -2M$, which agrees with Morozov's result [6].

In the following, we will simply sketch the calculation of the other terms in $\mathcal{P}_2$ and $\mathcal{P}_3$ which are somewhat lengthy. After applying the commutator relations (16)-(19) to $\mathcal{P}_{2,3}$, one can choose the position space representation to evaluate the functional trace since the operators involving the field strength and its covariant derivative are diagonal in this representation. One obtains at this stage the following generic expression

$$\int d^4x < x|\Delta^n(P) P_\mu P_\nu \cdots P_\gamma|x > \mathcal{F}^{\mu\nu\cdots\gamma}[G](x) \quad , \tag{22}$$

where $\mathcal{F}^{\mu\nu\cdots\gamma}[G]$ is only a function of the field strength and its covariant derivative. The final thing one has to know is the diagonal matrix element $< x|\Delta^n(P) P_\mu P_\nu \cdots P_\gamma|x >$. For the operators with dimension ≤ 8 considered in this work, one can set the gauge connection to zero in these matrix elements, which can then be easily evaluated by going to momentum space. The algebra is tedious but straightforward. The answer is given by

$$\begin{aligned}\Delta S_{CP} = \frac{i}{32\pi^2} g C \mathrm{tr}\, \gamma_5 \int d^4x \Bigg\{ & \frac{gM}{2} \left(\ln \Lambda_{UV}^2 + \text{finite}\right) (\sigma \cdot G)^2 \\ & + \frac{1}{4M} \left[-\frac{g}{3} \sigma \cdot G D^2 \sigma \cdot G \; + \; \frac{g^2}{2} (\sigma \cdot G)^3 \right] \\ & + \frac{1}{24 M^3} \Bigg[g \left(\frac{1}{5} D_\mu D_\nu \sigma \cdot G D^\nu D^\mu \sigma \cdot G - g^2 G_{\mu\nu} G_{\mu\nu} (\sigma \cdot G)^2 \right) \\ & \qquad - \frac{3}{4} g^2 D^2 \sigma \cdot G (\sigma \cdot G)^2 + \frac{1}{2} g^3 (\sigma \cdot G)^4 \Bigg] \Bigg\} \quad . \end{aligned} \tag{23}$$

So far, the gauge group has not been specified. The two operators of dimension 6 in Eq.(23) come from P_2 and P_3 in Eqs.(9–10). They are actually related to each other through Eq.(A.21) in the Appendix, as a consequence of the Bianchi identity. We do not include those dimension 8 operators which vanish when the equation of motion for the sourceless external gauge field $D_\alpha G^{\alpha\beta} = 0$ is imposed. We have dropped all the total derivatives except the topological term in order to obtain the desired form of (23).

It is clear that the NSVZ approach is a covariant derivative expansion and one might wonder if there exists a more efficient way to perform the calculation. Indeed, a more elegant formalism of covariant derivative expansion has been advocated in the literature [19,21,20]. We now illustrate how such this technique can simplify our calculation. The first step is to make two consecutive unitary transformations $\hat{g} = e^B e^A$ to the functional trace, where $A \equiv -ip_\mu x^\mu$ and $B \equiv iD_\mu \partial/\partial p_\mu$ with p_μ an auxiliary Fourier momentum parameter. The transformation e^A is a simple Fourier transform to momentum space. The second one e^B is to shift the momentum by the covariant derivative. The next step is to take the average over the auxiliary momentum parameter p_μ. Take the trace in (8) for illustration, we have

$$\mathrm{Tr} \left[\gamma_5 (P^2 - M^2 + \frac{1}{2} g \sigma \cdot G)^{-1} \sigma \cdot G \right]$$

$$= \frac{1}{V}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left\{\hat{g}\left[\gamma_5(P^2 - M^2 + \frac{1}{2}g\sigma\cdot G)^{-1}\sigma\cdot G\right]\hat{g}^{-1}\right\} \quad , \qquad (24)$$

where V denotes the volume of momentum space. Since the trace is invariant under the unitary transformation, all the p–integral does is to average all values of p. Furthermore, p_μ commutes with everything except $\partial/\partial p_\mu$, we thus have

$$[A, P_\mu] = -p_\mu \ , \ \ [B, P_\mu] = -[D_\nu, D_\mu]\frac{\partial}{\partial p_\nu} \ , \ \ [B, p_\mu] = iD_\mu \quad . \qquad (25)$$

Using (25), one can show [21]

$$\mathrm{Tr}\left[\gamma_5(P^2 - M^2 + \frac{1}{2}g\sigma\cdot G)^{-1}\sigma\cdot G\right]$$

$$= \frac{1}{V}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\gamma_5\left((p_\mu + \mathbf{G}_{\nu\mu}\frac{\partial}{\partial p_\nu})^2 - M^2 + \frac{1}{2}g\sigma\cdot(G+\delta G)\right)^{-1}\right.$$
$$\left.\sigma\cdot(G+\delta G)\right] \quad ,$$

$$= \frac{1}{V}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\gamma_5\left(p^2 - M^2 + \{p^\mu, \mathbf{G}_{\nu\mu}\}\frac{\partial}{\partial p_\nu} + \mathbf{G}_{\nu\mu}\mathbf{G}_\lambda{}^\mu\frac{\partial^2}{\partial p_\lambda \partial p_\nu}\right.\right.$$
$$\left.\left. + \frac{1}{2}g\sigma\cdot(G+\delta G)\right)^{-1}\sigma\cdot G\right] \ , \qquad (26)$$

where

$$\mathbf{G}_{\nu\mu} = \sum_{n=0}^{\infty}\frac{(n+1)}{(n+2)!}[D_{\mu_1}, \cdots [D_{\mu_n}, [D_\nu, D_\mu]] \cdots]\frac{i^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \quad , \qquad (27)$$

and

$$\delta G_{\alpha\beta} = \sum_{n=1}^{\infty}\frac{1}{n!}[D_{\mu_1}, \cdots [D_{\mu_n}, G_{\alpha\beta}] \cdots]\frac{i^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}} \quad . \qquad (28)$$

From (28), we note that δG contains at least one ordinary derivative. We thus dropped the term δG in the numerator in the last line of (26) since it gives zero when acting on unity at its right. Eq.(26) is a compact formula for the functional trace which has 'integrated' all the tedious non-commutative algebra encountered step by step in the NSVZ approach. To obtain local operators, one simply expands the field strength and its covariant derivatives in the denominator in Eq.(26). Note that the ordinary momentum derivative still acts nontrivially on the free propagator. After all the derivatives are evaluated, one is left with the trivial momentum integral. Using this compact expression, one can reproduce the effective action (23) directly. Alternatively, one expands the chromo-magnetic moment first, and then applies the unitary transformation technique to the functional traces of $\mathcal{P}_{2,3,4}$ given in (9), (10), and (11). We have checked that all three approaches give the same result.

3. Incorporation Of Electromagnetism And The Effective Action

It is straightforward to extend previous calculation to include the electromagnetic effect when $C' \neq 0$. First of all, it is impossible to write down any gauge invariant dimension 6 purely photonic or mixed gluon-photon operators. Secondly, dimension 8 operators involving the photon and gluon field strengths and two or four derivatives can be eliminated by the equations of motion $\partial^\mu F_{\mu\nu} = 0$ and $\partial^2 F_{\mu\nu} = 0$. Therefore the additional dimension 4 and 8 operators must be all constructed out of field strengths $G_{\mu\nu}$ and $F_{\mu\nu}$. The complete effective action can be obtained from (23) by the following substitutions

$$g^n C(\sigma \cdot G)^n \rightarrow gC(\sigma \cdot \mathcal{G})^{n-1}\sigma \cdot G + e_Q C'(\sigma \cdot \mathcal{G})^{n-1}\sigma \cdot F \quad ,(n = 2,3,4) \tag{29}$$

and

$$g^4 C G_{\mu\nu} G^{\mu\nu}(\sigma \cdot G)^2 \rightarrow gC\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\sigma \cdot \mathcal{G}\sigma \cdot G + e_Q C'\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\sigma \cdot \mathcal{G}\sigma \cdot F \quad . \tag{30}$$

Thus the complete CP odd effective action is given by

$$\Delta S_{CP} = \Delta S_C + \Delta S_{C'} \quad , \tag{31}$$

where

$$
\begin{aligned}
\Delta S_C = \frac{i}{32\pi^2} g C \mathrm{tr}\gamma_5 \int d^4x \Big\{ & \frac{gM}{2}\left(\ln \Lambda_{UV}^2 + \text{finite}\right)(\sigma\cdot G)^2 \\
& + \frac{1}{4M}\left[-\frac{g}{3}\sigma\cdot G D^2 \sigma\cdot G \;+\; \frac{g^2}{2}(\sigma\cdot G)^3\right] \\
& + \frac{1}{24M^3}\Big[g\left(\frac{1}{5}D_\mu D_\nu \sigma\cdot G D^\nu D^\mu \sigma\cdot G - g^2 G_{\mu\nu}G_{\mu\nu}(\sigma\cdot G)^2\right) \\
& \quad - \frac{3}{4}g^2 D^2\sigma\cdot G(\sigma\cdot G)^2 + \frac{1}{2}g^3(\sigma\cdot G)^4 \\
& \quad + \frac{1}{2}g^2 e_Q\left(3(\sigma\cdot G)^3\sigma\cdot F - 2G_{\mu\nu}G^{\mu\nu}\sigma\cdot G\sigma\cdot F \right.\\
& \qquad \left. -4F_{\mu\nu}G^{\mu\nu}(\sigma\cdot G)^2\right) \\
& \quad + \frac{1}{2}g e_Q^2\left(\sigma\cdot F\sigma\cdot G\sigma\cdot F\sigma\cdot G + 2(\sigma\cdot F)^2(\sigma\cdot G)^2\right. \\
& \qquad \left. -2F_{\mu\nu}F^{\mu\nu}(\sigma\cdot G)^2 - 4F_{\mu\nu}G^{\mu\nu}\sigma\cdot F\sigma\cdot G\right)\Big]\Big\} \quad ,
\end{aligned}
\tag{32}
$$

and

$$
\begin{aligned}
\Delta S_{C'} = \frac{i}{32\pi^2} e_Q C' \mathrm{tr}\gamma_5 \int d^4x \Big\{ & \frac{e_Q M}{2}\left(\ln \Lambda_{UV}^2 + \text{finite}\right)(\sigma\cdot F)^2 \\
& + \frac{1}{24M^3}\Big[\frac{1}{2}g^3\left((\sigma\cdot G)^3\sigma\cdot F - 2G_{\mu\nu}G^{\mu\nu}\sigma\cdot F\sigma\cdot G\right) \\
& \quad + \frac{1}{2}g^2 e_Q\left(\sigma\cdot F\sigma\cdot G\sigma\cdot F\sigma\cdot G + 2(\sigma\cdot F)^2(\sigma\cdot G)^2\right. \\
& \qquad \left. -2G_{\mu\nu}G^{\mu\nu}(\sigma\cdot F)^2 - 4F_{\mu\nu}G^{\mu\nu}\sigma\cdot F\sigma\cdot G\right) \\
& \quad + \frac{1}{2}e_Q^3\left((\sigma\cdot F)^4 - 2F_{\mu\nu}F^{\mu\nu}(\sigma\cdot F)^2\right)\Big]\Big\} \quad .
\end{aligned}
\tag{33}
$$

Equations (31), (32), and (33) are the main results of this work. The results are valid for any color gauge group where the heavy quark Q transforms in any representation. Besides the purely gluonic operators, there are also operators involving the covariant derivatives in (32). For instance, besides the Weinberg operator tr $\gamma_5(\sigma\cdot G)^3$, we also induce tr $\gamma_5\sigma\cdot GD^2\sigma\cdot G$. However we show in the Appendix that, up to total derivatives, all the latter operators can be expressed solely in terms of the field strength. To apply this to the NEDM we have to fix the color gauge group to be $SU(3)$. In the Appendix, we also show that for $SU(3)$ all the operators in (32) and (33) can be expressed in terms of the chosen basis of $\tilde{F}_{\mu\nu}F^{\mu\nu}$, O_θ, O_6, $O_{8,i}(i=1,2,3)$, and the following set of dimension 8 operators

$$
\begin{aligned}
O_{8,4} &= \frac{1}{3} e_Q g^3 d^{abc} \tilde{F}_{\mu\nu} G^{a\mu\nu} G^b_{\alpha\beta} G^{c\alpha\beta} \quad , \\
O_{8,5} &= \frac{1}{3} e_Q g^3 d^{abc} \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^c_{\alpha\beta} F^{\alpha\beta} \quad , \\
O_{8,6} &= \frac{1}{2} e_Q^2 g^2 \tilde{F}_{\mu\nu} F^{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta} \quad , \\
O_{8,7} &= \frac{1}{2} e_Q^2 g^2 \tilde{G}^a_{\mu\nu} G^{a\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \quad , \\
O_{8,8} &= \frac{1}{2} e_Q^2 g^2 \tilde{F}_{\mu\nu} G^{a\mu\nu} F_{\alpha\beta} G^{a\alpha\beta} \quad , \\
O_{8,9} &= \frac{1}{12} e_Q^4 \tilde{F}_{\mu\nu} F^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \quad .
\end{aligned} \tag{34}
$$

Therefore, after integrating out the excitation of the heavy quark Q with mass M, we have the following effective action for the NEDM,

$$
S_{\text{eff}}^{NEDM} = S_{\text{QCD}} + S_{\text{QED}} + S_{\text{light quarks}} + \Delta S_C + \Delta S_{C'} \quad , \tag{35}
$$

with

$$
\begin{aligned}
\Delta S_C = -\frac{C}{16\pi^2} \int d^4x \Big[-\frac{1}{2M} O_6 + \frac{1}{M^3} \Big(&\frac{1}{6} O_{8,1} + \frac{1}{4} O_{8,3} + \frac{1}{4} O_{8,4} \\
&+ \frac{1}{8} O_{8,5} + \frac{1}{6} O_{8,6} + \frac{1}{3} O_{8,8} \Big) \Big] \quad ,
\end{aligned} \tag{36}
$$

and

$$
\Delta S_{C'} = -\frac{C'}{16\pi^2} \frac{1}{M^3} \int d^4x \left[\frac{1}{8} O_{8,5} + \frac{1}{6} O_{8,7} + \frac{1}{3} O_{8,8} + 2 O_{8,9} \right] \quad . \tag{37}
$$

Note that we have not included the terms $\tilde{G}G$ and $\tilde{F}F$ in (36) and (37). There is no topological effect from the $\tilde{F}F$ contribution of the $U(1)$. The effect of $\tilde{G}G$ term has been discussed in Ref.[25]. Besides the kinetic terms, $S_{\text{light quarks}}$ in Eq.(35) contains the EDMs and CEDMs of the light quarks as well.

4. Conclusions

Using the functional approach together with the covariant derivative expansion, we have deduced an effective action for the neutron electric dipole moment that are related to the electric dipole moment and chromo–electric dipole moment of a heavy quark. Local operators of dimension ≤ 8 and their Wilson coefficients are worked out explicitly. The coefficients for the Weinberg's three–gluon operator and the one–photon three–gluon operators agree with previous works [7,8,22]. The coefficients for the other dimension 8 operators are new. This effective action is the starting point for the hadronic matrix element calculation. Implication of the dimension 8 purely gluonic operators to the neutron moment has been discussed in a recent letter [23], in which we showed that at the hadronic scale, due to the QCD renormalization effects, these operators of higher dimensions may dominant over the dimension 6 three–gluon operator. Thus, the dimension 8 purely gluonic operators listed in Eq.(3) may provide either a dominant mechanism generating an observable neutron moment or a stringent constraint on the underlying dynamics of CP violation beyond the Standard Model.

In the following, we summarize the phenomenological results [23]. The operators $O_{8,4}$ and $O_{8,5}$ may not give an important contribution to NEDM because of the heavy mass suppression and the absence of QCD enhancement [22]. The effects in NEDM from other operators in Eq.(34) are of higher orders in α_{em} and thus negligible. However, they could have observable effects in systems with strong electromagnetic field, such as the large–Z nucleus. Now we concentrate our attention to the effective Lagrangian of the form:

$$L_{\rm eff}(\mu) = \cdots + C_6(\mu)O_6(\mu) + \sum_{i=1}^{3} C_{8,i}(\mu)O_{8,i}(\mu) \quad , \tag{38}$$

where $C_{8,i}(m_b)$ and $C_6(m_b)$ are related to C_b, the CEDM of the b quark, via Eq.(36). The coefficient $C_b \equiv C_b(m_b)$ depends on details of the models of CP violation. The scale μ dependence follows the standard renormalization group equation. The size of the NEDM can be estimated using the naive dimensional analysis [27] accompanied with the unknown nonperturbative correction factors ξ_6, $\xi_{8,i}$ which are naively of order about one,

$$D_N(O_6) \simeq (eM_\chi/4\pi)g^3(\mu)C_6(\mu)\xi_6 \quad ,$$

$$D_N(O_8) \simeq (eM_\chi^3/16\pi^2)g^4(\mu)C_{8,i}(\mu)\xi_{8,i} \quad . \tag{39}$$

Here $M_\chi = 4\pi F_\pi \simeq 1.19$ GeV is the chiral symmetry breaking scale. The strong coupling is set at $g(\mu) = 4\pi/\sqrt{6}$ as in Ref.[1]. With the QCD enhancement mostly for the component $i = 1$, we obtain

$$D_N(O_8)/D_N(O_6) \simeq 3.6\xi_{8,1}/\xi_6 \quad . \tag{40}$$

The naive dimensional analysis is certainly not reliable because it is ambiguous about normalization of the operators. One has to rely upon educated guess to determine the normalization. Recently, Chemtob[13] used the QCD sum rule method to provide a more systematic estimate of the hadronic matrix elements of the operators O_θ, O_6, and O_8. In this scheme if one assumes the nucleon pole dominance, the results are $\xi_6 = 0.07$, $\xi_{8,1} = 0.08$, which correspond to smaller D_N compared to the dimensional estimates given above. However, their ratio is still about 1. Therefore it is reasonable to conclude that the ratio of matrix elements can be more reliably estimated than the individual elements themselves. So is the conclusion that the O_8 operators give the dominant contribution to the NEDM. Using the current experimental bound[11] 10^{-25} e cm and the matrix elements of Chemtob, one can put a constraint on the CEDM of the b quark, *i.e.*

$$C_b < 0.6 \; G_F m_b/16\pi^2 \quad . \tag{41}$$

If the chromo-electric dipole moment is given to the charm quark initially, the ratio $D_N(O_8)/D_N(O_6)$ will be even an order of magnitude larger because the quark mass suppression factor is less severe. In conclusion, the induced O_8 operators can place strong constraint on parameters of the CP violation.

APPENDIX

In this Appendix, we collect all the formulas that are essential for deriving Eqs.(36) and (37) from Eqs.(32) and (33). To this end, we first prove that, for $SU(N)$ with $N \geq 4$, there are only four independent dimension 8 CP odd purely gluonic operators. For $SU(3)$ and $SU(2)$ one can further reduce the number of independent operators down to three and two respectively. These results were stated in [6] without proof. We provide here an independent proof of these nontrivial facts because the identities used in the proof may be useful for other purposes as well.

We kick off with some well-known Dirac trace identities.

$$
\begin{aligned}
\mathrm{tr}\,\gamma_5\sigma_{\mu\nu}\sigma_{\alpha\beta} &= 4i\epsilon_{\mu\nu\alpha\beta} \quad , \\
\mathrm{tr}\,\gamma_5\sigma_{\mu\nu}\sigma_{\alpha\beta}\sigma_{\lambda\rho} &= 4\left[g_{\mu\alpha}\epsilon_{\nu\beta\lambda\rho} - g_{\mu\beta}\epsilon_{\nu\alpha\lambda\rho} - g_{\nu\alpha}\epsilon_{\mu\beta\lambda\rho} + g_{\nu\beta}\epsilon_{\mu\alpha\lambda\rho}\right] \quad , \\
\mathrm{tr}\,\gamma_5\sigma_{\mu\nu}\sigma_{\alpha\beta}\sigma_{\lambda\rho}\sigma_{\xi\eta} &= 4i\left\{(g_{\lambda\xi}g_{\rho\eta} - g_{\lambda\eta}g_{\rho\xi})\epsilon_{\mu\nu\alpha\beta} + (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})\epsilon_{\lambda\rho\xi\eta}\right. \\
&\quad \left. - \left[g_{\mu\alpha}g_{\lambda\xi}\epsilon_{\nu\beta\rho\eta} - (15 \text{ permutations of } \mu \leftrightarrow \nu, \alpha \leftrightarrow \beta, \lambda \leftrightarrow \rho, \xi \leftrightarrow \eta)\right]\right\} \quad .
\end{aligned} \tag{A.1}
$$

Since there is no rank 5 totally antisymmetric tensor in 4 dimension, the ϵ tensor in 4 dimension satisfies the identity

$$
\epsilon^{[\mu\nu\alpha\beta}g^{\lambda]\xi} \equiv \epsilon^{\mu\nu\alpha\beta}g^{\lambda\xi} + \epsilon^{\nu\alpha\beta\lambda}g^{\mu\xi} + \epsilon^{\alpha\beta\lambda\mu}g^{\nu\xi} + \epsilon^{\beta\lambda\mu\nu}g^{\alpha\xi} + \epsilon^{\lambda\mu\nu\alpha}g^{\beta\xi} = 0 \quad . \tag{A.2}
$$

Let T^a be the generators of $SU(N)$ in the fundamental representation. Some useful $SU(N)$ identities are:

$$
\begin{aligned}
\mathrm{tr}\,T^aT^b &= \frac{1}{2}\delta^{ab} \quad , \\
\mathrm{tr}\,T^aT^bT^c &= \frac{1}{4}\left[if^{abc} + d^{abc}\right] \quad , \\
\mathrm{tr}\,T^aT^bT^cT^d &= \frac{1}{4N}\delta^{ab}\delta^{cd} + \frac{1}{8}\left[d^{abe}d^{cde} - f^{abe}f^{cde} + i\left(f^{abe}d^{cde} + d^{abe}f^{cde}\right)\right] \quad .
\end{aligned} \tag{A.3}
$$

Despite the fact that we have imposed on Eq.(32) the following sourceless equations of motion

$$\partial^\mu F_{\mu\nu} = 0, \quad \partial^2 F_{\mu\nu} = 0 \quad , \tag{A.4}$$

and

$$D^\mu G_{\mu\nu} = \partial^\mu G_{\mu\nu} - ig[A^\mu, G_{\mu\nu}] = 0 \quad , \tag{A.5}$$

not all the covariant derivatives were eliminated. Operators that involved the covariant derivatives in Eq.(32) can always be expressed in terms of the field strengths of the gluons (modulo total derivatives), by imposing the Bianchi identity

$$D_\alpha G_{\beta\gamma} + D_\beta G_{\gamma\alpha} + D_\gamma G_{\alpha\beta} = 0 \quad , \tag{A.6}$$

and

$$D^2 G_{\alpha\beta} = 2ig[G^\lambda{}_\alpha, G_{\beta\lambda}] \quad . \tag{A.7}$$

Therefore we only need to classify those operators constructed solely out of the gluon field strengths. There are only three types of Lorentz invariant CP odd purely gluonic operators of dimension 8 one can write down:

$$\begin{aligned} A^{abcd} &= \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^c_{\alpha\beta} G^{d\alpha\beta} \quad , \\ B^{abcd} &= \epsilon^{\mu\nu\alpha\beta} G^a_{\lambda\mu} G^b_{\lambda\nu} G^c_{\rho\alpha} G^d_{\rho\beta} \quad , \\ C^{abcd} &= \tilde{G}^a_{\mu\nu} G^{b\nu\alpha} G^c_{\alpha\beta} G^{d\beta\mu} \quad . \end{aligned} \tag{A.8}$$

Note that the Lorentz indices are factorized in A^{abcd} but not in B^{abcd} and C^{abcd}. These operators have the following permutation symmetry:

$$\begin{aligned} A^{abcd} &= A^{bacd} = A^{abdc} \quad , \\ B^{abcd} &= -B^{bacd} = -B^{abdc} \quad , \\ C^{abcd} &= C^{adcb} \quad . \end{aligned} \tag{A.9}$$

Using the identity $\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha{}^{\lambda\mu\nu} = -\det(g^{\rho\sigma})$, $(\rho = \beta, \gamma, \delta;\ \sigma = \lambda, \mu, \nu)$, one can derive a linear relation among the A^{abcd} and C^{abcd},

$$C^{abcd} + C^{bacd} = \frac{1}{2} A^{abcd} \quad . \tag{A.10}$$

Then, with the help of Eqs. (A.9) and (A.10), we reorder indices in C several times to obtain

$$C^{abcd} - C^{bacd} = \frac{1}{2}\left[A^{adbc} - A^{bdac}\right] \quad . \tag{A.11}$$

Adding Eqs.(A.10) and (A.11), we derive

$$C^{abcd} = \frac{1}{4}\left[A^{adbc} - A^{bdac} + A^{abcd}\right] \quad . \tag{A.12}$$

Note that Eq.(A.2) implies

$$B^{abcd} = C^{abdc} - C^{abcd} \quad . \tag{A.13}$$

Eqs.(A.12,13) imply both B and C can be expressed in terms of A type operators which we shall use to define our independent sets of operators. General $SU(N)$ invariant operators are constructed by contracting A^{abcd} and B^{abcd} with the following invariant tensors

$$\delta^{ab}\delta^{cd},\ \delta^{ac}\delta^{bd},\ d^{abe}d^{cde},\ d^{ace}d^{bde} \quad . \tag{A.14}$$

Contractions with other $SU(N)$ invariant tensors can be shown to be redundant due to the relation like

$$f^{abe}f^{cde} = \frac{2}{N}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}) + (d^{ace}d^{bde} - d^{ade}d^{bce}) \quad , \tag{A.15}$$

etc. Also contraction with the mixed invariant tensors, like $f^{abe}d^{cde}$ etc, gives null results. Thus, one obtains the following set of gauge invariant operators by contracting the invariant tensors (Eq.(A.14)) with A^{abcd}:

$$\begin{aligned}
\hat{O}_1 &= \tilde{G}^a_{\mu\nu} G^{a\mu\nu} G^b_{\alpha\beta} G^{b\alpha\beta} \quad , \\
\hat{O}_2 &= \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^a_{\alpha\beta} G^{b\alpha\beta} \quad , \\
\hat{O}_3 &= d^{abe} \tilde{G}^a_{\mu\nu} G^{b\mu\nu} d^{cde} G^c_{\alpha\beta} G^{d\alpha\beta} \quad , \\
\hat{O}_4 &= d^{abe} \tilde{G}^a_{\mu\nu} G^b_{\alpha\beta} d^{cde} G^{c\mu\nu} G^{d\alpha\beta} \quad .
\end{aligned} \tag{A.16}$$

Thus for $SU(N)$ there are at most four independent dimension 8 CP odd purely gluonic operators. To express the result of the threshold calculation Eq.(23) in terms of these four operators, some identities involving B type or C type operators may also be useful. For example, contracting both sides of Eq.(A.13) with $\delta^{ac}\delta^{bd}$ and $d^{ace}d^{bde}$ and using Eq.(A.12), we deduce

$$\begin{aligned}
\hat{O}_5 &= \epsilon^{\mu\nu\alpha\beta} G^a_{\lambda\mu} G^b_{\lambda\nu} G^a_{\rho\alpha} G^b_{\rho\beta} = \frac{1}{2}\left[\hat{O}_1 - \hat{O}_2\right] \quad , \\
\hat{O}_6 &= \epsilon^{\mu\nu\alpha\beta} d^{abe} G^a_{\lambda\mu} G^b_{\rho\alpha} d^{cde} G^c_{\lambda\nu} G^d_{\rho\beta} = \frac{1}{2}\left[\hat{O}_3 - \hat{O}_4\right] \quad .
\end{aligned} \tag{A.17}$$

Similarly, contracting Eq.(A.12) with $f^{abe}f^{cde}$ and using Eq.(A.15), we also deduce

$$\begin{aligned}
\hat{A} &= f^{abe} f^{cde} \tilde{G}^a_{\mu\nu} G^{c\nu\mu} G^b_{\alpha\beta} G^{d\beta\alpha} \quad , \\
&= -2 f^{abe} f^{cde} \tilde{G}^a_{\mu\nu} G^{b\nu\alpha} G^c_{\alpha\beta} G^{d\beta\mu} = \frac{2}{N}(\hat{O}_1 - \hat{O}_2) + (\hat{O}_3 - \hat{O}_4) \quad .
\end{aligned} \tag{A.18}$$

For $SU(3)$, we have one further constraint arise from the so–called Cayley–Burgoyne's identity [26],

$$\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} = 3\left[d^{abe}d^{cde} + d^{ace}d^{bde} + d^{ade}d^{bce}\right] \quad . \tag{A.19}$$

The identity can be proved by using the characteristic equation $A^3 - (\mathrm{tr}\, A^2)A - \det A = 0$ for any traceless 3×3 matrix. This matrix identity implies the trace identity $\mathrm{tr}\, A^4 = (\mathrm{tr}\, A^2)^2$. The traceless matrix can be expanded as $A = \sum \lambda^a T^a$.

Eq.(A.19) then follows by expanding both sides of the trace identity in λ's with complete symmetrization. Contracting Eq.(A.19) with A^{abcd} one deduces

$$\hat{O}_4 = \frac{1}{6}\hat{O}_1 + \frac{1}{3}\hat{O}_2 - \frac{1}{2}\hat{O}_3 \quad . \tag{A.20}$$

Thus for $SU(3)$, $\hat{A} = \frac{1}{2}\hat{O}_1 - \hat{O}_2 + \frac{3}{2}\hat{O}_3$. Since $d^{abc} = 0$ for $SU(2)$, only $\hat{O}_1$ and $\hat{O}_2$ exist. Finally for $U(1)$, only $O_{8,9}$ survives. This completes the proof.

By using the above formulas and dropping total derivatives, the following trace identities are deduced for $SU(N)$:

I. Purely gluonic:

$$\begin{aligned}
g^2 \mathrm{tr}\, \gamma_5 (\sigma \cdot G)^2 &= 4iO_\theta \quad , \\
g^2 \mathrm{tr}\, \gamma_5 (\sigma \cdot G) D^2 \sigma \cdot G &= g^3\, \mathrm{tr}\, \gamma_5 (\sigma \cdot G)^3 = -24 i O_6 \quad , \\
\mathrm{tr}\, \gamma_5 G_{\mu\nu} G^{\mu\nu} (\sigma \cdot G)^2 &= i\left[\frac{2}{N}\hat{O}_1 + \hat{O}_3\right] \quad , \\
\mathrm{tr}\, \gamma_5 G_{\mu\nu} (\sigma \cdot G) G^{\mu\nu} \sigma \cdot G &= i\left[\frac{2}{N}\hat{O}_1 + \hat{O}_3 - 2\hat{A}\right] \quad , \\
\mathrm{tr}\, \gamma_5 (D^2 \sigma \cdot G) D^2 \sigma \cdot G &= 8 i g^2 \hat{A} \quad , \\
\mathrm{tr}\, \gamma_5 [G_{\mu\nu}, (D^\mu \sigma \cdot G)] D^\nu \sigma \cdot G &= 2 g \hat{A} \quad , \\
\mathrm{tr}\, \gamma_5 (D_\mu D_\nu \sigma \cdot G) D^\nu D^\mu \sigma \cdot G &= 10 i g^2 \hat{A} \quad , \\
\mathrm{tr}\, \gamma_5 (D_\mu D_\nu \sigma \cdot G) D^\mu D^\nu \sigma \cdot G &= 12 i g^2 \hat{A} \quad , \\
\mathrm{tr}\, \gamma_5 (\sigma \cdot G)^2 D^2 \sigma \cdot G &= -2 \mathrm{tr}\, \gamma_5 \sigma \cdot G (D_\mu \sigma \cdot G) D^\mu \sigma \cdot G = 8 i g \hat{A} \quad , \\
\mathrm{tr}\, \gamma_5 (\sigma \cdot G)^4 &= 4i\left[\frac{2}{N}\hat{O}_1 + \hat{O}_3 + 2\hat{A}\right] \quad .
\end{aligned} \tag{A.21}$$

II. Mixed photon–gluon:

$$
\begin{aligned}
e_Q g^3 \mathrm{tr}\, \gamma_5 G_{\mu\nu} G^{\mu\nu} \sigma \cdot G \sigma \cdot F &= 6iO_{8,4} \quad , \\
e_Q g^3 \mathrm{tr}\, \gamma_5 F_{\mu\nu} G^{\mu\nu} (\sigma \cdot G)^2 &= 6iO_{8,5} \quad , \\
e_Q g^3 \mathrm{tr}\, \gamma_5 (\sigma \cdot G)^3 \sigma \cdot F &= 12i \left[O_{8,4} + O_{8,5} \right] \quad , \\
e_Q g^3 \mathrm{tr}\, \gamma_5 \sigma \cdot F \sigma \cdot G \sigma \cdot F \sigma \cdot G &= -16i \left[O_{8,6} + O_{8,7} - 4O_{8,8} \right] \quad , \\
e_Q^2 g^2 \mathrm{tr}\, \gamma_5 F_{\mu\nu} G^{\mu\nu} \sigma \cdot F \sigma \cdot G &= 8iO_{8,8} \quad , \\
e_Q^2 g^2 \mathrm{tr}\, \gamma_5 F_{\mu\nu} F^{\mu\nu} (\sigma \cdot G)^2 &= 8iO_{8,7} \quad , \\
e_Q^2 g^2 \mathrm{tr}\, \gamma_5 G_{\mu\nu} G^{\mu\nu} (\sigma \cdot F)^2 &= 8iO_{8,6} \quad , \\
e_Q^2 g^2 \mathrm{tr}\, \gamma_5 (\sigma \cdot F)^2 (\sigma \cdot G)^2 &= 16i \left[O_{8,6} + O_{8,7} \right] \quad .
\end{aligned}
\tag{A.22}
$$

III. Purely photonic:

$$
\begin{aligned}
e_Q^4 \mathrm{tr}\, \gamma_5 F_{\mu\nu} F^{\mu\nu} (\sigma \cdot F)^2 &= 96iO_{8,9} \quad , \\
e_Q^4 \mathrm{tr}\, \gamma_5 (\sigma \cdot F)^4 &= 384iO_{8,9} \quad .
\end{aligned}
\tag{A.23}
$$

Using Eqs.(A.21–A.23) with $N = 3$, one can readily obtain Eqs.(36) and (37) from Eqs.(32) and (33).

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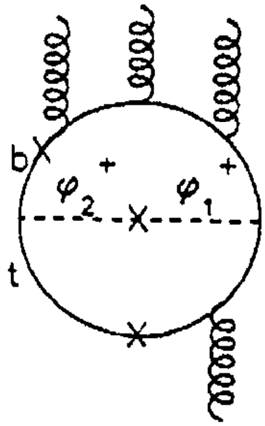
Figure Captions

*Fig.*1 Feynman diagrams due to the CP violating interactions of the heavy particles which contribute to the dimension 8 gluonic operators.

*Fig.*2 Feynman diagrams for the dimension 8 gluonic operators due to the effective heavy quark vertices which arise from shrinking the short distance interactions.



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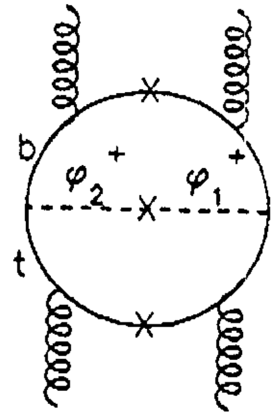

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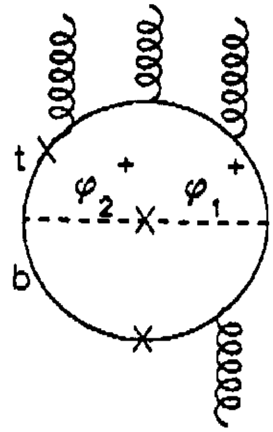

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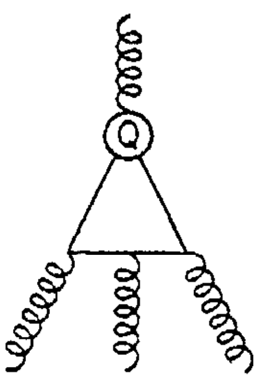

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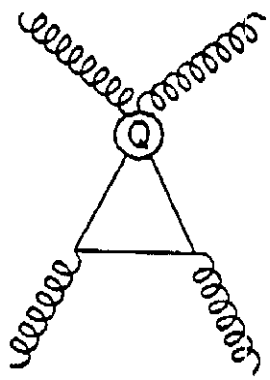

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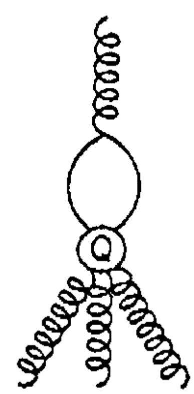

(2c)