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$K_L \to \pi^0\pi^0\gamma\gamma$ in Chiral Perturbation Theory

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Abstract

We calculate the rate and two-photon spectrum for the decay $K_L \to \pi^0\pi^0\gamma\gamma$ using chiral perturbation theory. We comment on the implications of the study of this decay.

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1 Introduction

Current experiments do a precision measurement of $K_L \to \pi^0\pi^0\pi^0$ as a calibration input to the measurement of ϵ'/ϵ. Because this is identified as six photons, as a byproduct of this measurement, one can also study the process $K_L \to \pi^0\pi^0\gamma\gamma$. This decay has the nice property of being calculable in leading order chiral perturbation theory. In this letter, we calculate the rate and discuss what may be learned from its experimental detection.

One can consider the two sets of radiative neutral kaon decays:

$$\begin{array}{ll} K_L \to \gamma\gamma & K_S \to \gamma\gamma \\ K_S \to \pi^0\gamma\gamma & K_L \to \pi^0\gamma\gamma \\ K_L \to \pi^0\pi^0\gamma\gamma & K_S \to \pi^0\pi^0\gamma\gamma \end{array} \tag{1}$$

Those on the right are finite one-loop calculations in chiral perturbation theory at lowest order (for $K_S \to \gamma\gamma$ see [1, 2] and for $K_L \to \pi^0\gamma\gamma$ see [3]), while those on the left should occur via tree diagrams using the anomalous two-photon coupling of the pion and eta. The decay $K_S \to \pi^0\gamma\gamma$ is, however, much harder to measure than the two K_L decays in the left hand column.

Unlike $K_L \to \gamma\gamma$, which vanishes in leading order chiral perturbation theory due to the Gell-Mann–Okubo formula, the decay $K_L \to \pi^0\pi^0\gamma\gamma$ can be predicted using the leading order Wess–Zumino term. In fact, if one works in the basis in which the kinetic energy takes its canonical form, only two diagrams contribute to this process, as shown in Figure 1. Although the Wess–Zumino term is a one-loop effect, its coefficient is determined by symmetry. Moreover, other one-loop diagrams will not contribute, as we explain in the following section. The calculation is therefore quite simple and can provide a clean test of chiral perturbation theory. A comparison between the measured and predicted rate will determine the accuracy of the leading order calculation. However, as we shall discuss below, one has to be careful to avoid the pion pole if one is to test the structure of the chiral perturbation theory vertices.

2 Calculation

In this section we give some details of our calculation. The necessary terms from the chiral lagrangian are

$$\mathcal{L} \;=\; \frac{f^2}{4}\,\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger) + \frac{\mu f^2}{2}\,\mathrm{tr}(\Sigma^\dagger M + \Sigma M)$$

$$+\frac{\lambda f^2}{4}\,\mathrm{tr}(h\partial_\mu\Sigma\partial^\mu\Sigma^\dagger+\text{h.c.}) \qquad (2)$$
$$+\mathcal{L}_{\mathrm{WZ}}+\cdots$$

where $\mathcal{L}_{\mathrm{WZ}}$ is the Wess–Zumino term. Here $\Sigma = \exp(2i\widetilde{\pi}/f)$ is the linearly transforming matrix exponential of the pion fields,

$$\widetilde{\pi} = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & \pi^0/\sqrt{2}-\eta/\sqrt{6} & K^0 \\ K^- & \overline{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}, \qquad (3)$$

and f is the tree level pion decay constant, with value 93 MeV. M is the matrix of quark masses, $M = \mathrm{diag}(m_u, m_d, m_s)$. We ignore isospin violation throughout, so we re-express M using $\mu M = \mathrm{diag}(m_\pi^2/2, m_\pi^2/2, m_K^2 - m_\pi^2/2)$. In the $\Delta S = 1$ terms, h is the matrix

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (4)$$

and the constant λ has the value 3.7×10^{-7} at tree level. We have fixed λ to reproduce the decay rate for $K_L \to 3\pi^0$. A combined fit at tree level in chiral perturbation theory to $K \to \pi\pi$ and $K \to \pi\pi\pi$ decays gives $\lambda = 3.8 \times 10^{-7}$, and this value is reduced by 30% in a one-loop fit [4]. Since we are ignoring CP violation, λ is real.

We do not need a possible term $\mathrm{tr}(hM\Sigma^\dagger)+\text{h.c.}$, which just renormalizes the usual mass term. We can always rotate this term away at order $\Delta S = 1$. Such a rotation leaves the usual kinetic and mass terms invariant, and can only produce higher order weak interactions from the $\Delta S = 1$ term.

We have included photon couplings only in the Wess–Zumino term. Photons coupled by covariant derivatives will not contribute to this decay when CP is conserved. This is seen from the following operator analysis. The K_L and π^0 are both CP odd. To construct an operator which is even under CP requires that the two photon field strengths are contracted with an ϵ tensor since any number of derivatives will not change the CP of the operator. An ϵ tensor cannot be generated with only scalar loops, so it must have been present as a fundamental vertex. Therefore, at leading order, only the Wess–Zumino coupling of the photons is required. At this order, there are no other independent derivative operators coupled with an ϵ tensor.

It is most convenient to work in a basis where the kinetic energy and mass terms of the pion fields are diagonalized. The transformations of the fields which do this to first order in weak interactions are given by Ecker, Pich and de Rafael in reference [5].

This reduces the number of diagrams we have to calculate to just two, where a pion or eta couples to the final photon pair (see Figure 1). In the shifted basis the interaction terms needed for the decay $K_L \to \pi^0\pi^0\gamma\gamma$ are,

$$\frac{\lambda}{12\sqrt{3}f^2}\Big[3\eta K_L\partial_\mu\pi^0\partial^\mu\pi^0 - 6\,\eta\partial_\mu K_L\pi^0\partial^\mu\pi^0 + 2\,\partial_\mu\eta K_L\pi^0\partial^\mu\pi^0 + \partial_\mu\eta\partial^\mu K_L\pi^0\pi^0 \\ - \sqrt{3}\,K_L\pi^0\partial_\mu\pi^0\partial^\mu\pi^0 + \sqrt{3}\,\partial_\mu K_L\pi^0\pi^0\partial^\mu\pi^0\Big], \tag{5}$$

from the $\mathrm{tr}(h\partial_\mu\Sigma\partial^\mu\Sigma^\dagger + \mathrm{h.c.})$ term,

$$\frac{\lambda}{6\sqrt{3}f^2}\Big[-\eta K_L\partial_\mu\pi^0\partial^\mu\pi^0 + \eta\partial_\mu K_L\pi^0\partial^\mu\pi^0 + \partial_\mu\eta K_L\pi^0\partial^\mu\pi^0 - \partial_\mu\eta\partial^\mu K_L\pi^0\pi^0\Big], \tag{6}$$

from shifting the fields in the $\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)$ term, and

$$\frac{\lambda}{24f^2}\Big[\sqrt{3}\,\eta K_L\pi^0\pi^0\,m_\eta^2 + K_L\pi^0\pi^0\pi^0\,m_\pi^2\Big], \tag{7}$$

from shifting the fields in the $\mathrm{tr}(M\Sigma + M\Sigma^\dagger)$ term.

Finally, we need the anomalous couplings of the π^0 and η to two photons from $\mathcal{L}_{WZ}$, given by,

$$\frac{e^2}{32\pi^2 f}\,\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\left(\pi^0 + \frac{\eta}{\sqrt{3}}\right). \tag{8}$$

The Wess–Zumino term does not contain vertices which can cause $K_L \to \pi^0\pi^0\gamma\gamma$ directly.

With these interactions we calculate the differential decay rate for $K_L(k) \to \pi^0(p_1)\,\pi^0(p_2)\,\gamma(q_1)\,\gamma(q_2)$ to be

$$\frac{d\Gamma}{dx\,dz} = P\,z^2|A(z)|^2\,\lambda^{1/2}(1,z,x)\,(1-4d/x)^{1/2}, \tag{9}$$

where $x = (p_1+p_2)^2/m_K^2$ and $z = (q_1+q_2)^2/m_K^2$ are the invariant mass-squared of the pion pair and photon pair, in units of m_K^2. The variables x and z lie within the region given by,

$$4d < x < (1-\sqrt{z}\,)^2, \qquad 0 < z < (1-2r)^2, \tag{10}$$

where $r = m_\pi/m_K$ and $d = r^2$. The function λ is,

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab+bc+ca), \tag{11}$$

and the overall constant P is,

$$P = \frac{m_K^7\lambda^2\alpha^2}{288\,(4\pi)^7\,f^6} = 5.9\times 10^{-21}\,\mathrm{MeV}\,. \tag{12}$$

In units of the total K_L width, $P = 4.9 \times 10^{-7}$. This will be multiplied by the dimensionless integral over x and z to determine the true rate.

In P, we have incorporated two factors of $1/2$ to account for two pairs of identical final state particles. This is valid so long as the invariant mass of the photon pair is sufficiently far from the support of the Breit–Wigner distribution of a pion. Since we always integrate the photon pair over invariant masses which differ from the pion mass by many times its width, this formula is correct.

The function $A(z)$ is,

$$A(z) = \frac{3 - z + d}{z - d + ir\Gamma_\pi/m_K} + \frac{3z + d - 1}{3(z - d_\eta)}, \tag{13}$$

with $d_\eta = m_\eta^2/m_K^2$. Note that we used the Gell-Mann–Okubo relation to replace m_η^2 by $(4m_K^2 - m_\pi^2)/3$ [6]. We have included the pion width, $\Gamma_\pi = 7.8\,\text{eV}$, in $A(z)$, since $z = d$ is within the allowed range. However, in practice we will want to make a cut in the photon pair invariant mass around the neutral pion mass. Given current photon energy resolutions [7] this cut will be of order a few to tens of MeV so we can ignore the finite pion width when calculating the rate with a cut. Although in principle, the amplitude could have depended on an additional angular variable, it does not. Furthermore, x dependence occurs only in kinematic factors and not in the amplitude.

By performing the integral over x, we obtain the distribution, $d\Gamma/[AAdz$, of the photon pair invariant mass-squared. This is shown in Figure 2, where one converts to the physical value by multiplying by the constant P given in Equation 12.

We also calculate the decay rate with a cut of width $2\,\delta m$ in the mass of the photon pair, around m_π. This is shown in Figure 3 (again, multiply by P to get the rate). For small values of the cut width, δm (but much larger than the pion width), the rate for photon pairs outside the cut band has the form,

$$\Gamma_{\text{cut}} = \frac{3.4\,P}{(\delta m/\,\text{MeV})}. \tag{14}$$

This approximation works to about 10% for δm up to 3 MeV. It is clear that small values of δm do not tell us much about the structure of the amplitude.

3 Discussion

The overall rate is clearly strongly dominated by the pion pole. If one makes too small a cut near the pion mass, one probes only the tail of the pion resonance. This is made manifest by the excellent agreement between the exact results in Figure 3 and the analytic result of Equation 14 describing the rate as a function of the cut for a cut around the pion pole less than a few MeV. The rate in this regime is very sensitive to the precise value taken for the cut.

Moreover, the result for small cuts will be quite sensitive to the experimental resolution, since mismeasurement of the mass could artificially enhance the rate owing to the steeply falling distribution as a function of pion pair invariant mass.

The most interesting measurement would be one with as large a cut as possible. Near the pion mass, the z dependence of the rate is determined solely by the pion denominator factor, $1/(z-d)$. Further from the pole, the distribution should be sensitive to the z dependence of the $\Delta S = 1$ amplitude and the anomaly vertex, as well as the second diagram with the η intermediate state.

To test the sensitivity to the perturbation theory result, we compare three curves in Figures 2 and 3. The solid curve gives the exact result. The dot-dashed curve gives the result with the z dependence retained only in the Breit–Wigner denominator and in phase space (that is, we assume constant vertices determined by their value at the pion pole). Finally, we give a dotted curve with the z dependence exact everywhere except for a constant $\Delta S = 1$ vertex. These curves all agree at the pion pole.

We see that the first two results can be quite different. For a cutoff δm greater than 25 MeV, the rates-with-cut differ by more than 50%. This is misleading: the dotted curve, where we have retained the z^2 dependence in the differential decay rate (Equation 9), is virtually indistinguishable from the exact result. This is because the pion pole term has very weak z dependence in the numerator and dominates. The factor of z^2 is guaranteed by gauge invariance of the anomalous two photon coupling. We conclude that it is in fact virtually impossible to test the z dependence of the lowest order vertex.

So any deviation from our result would indicate the presence of higher order operators in the chiral lagrangian. Part of their effect is absorbed in order to normalize the $K_L \to \pi^0\pi^0\pi^0$ rate correctly. The effect of higher order operators could be visible however in a rate-with-cut or z dependence different from our calculation. Moreover a nontrivial dependence on x or on the angle between the planes of the photons and pions would be evidence of higher order contributions. With sufficiently many events, one could hope to have an independent probe of the four derivative $\Delta S = 1$ terms which were studied in Ref. [8] as well as the higher order contributions which contribute to $K_L \to \gamma\gamma$ [2, 9]. Furthermore, when sufficiently far from the pole, higher order operators could be present which contribute directly to the process we have computed. For a sufficiently large cut, the presence of higher order operators would be reflected both in the overall rate and the detailed distribution of the photon pair invariant mass. On the basis of chiral perturbation theory, we expect these to be of the order of 30%. Any greater contribution would be an interesting indication of the breakdown of chiral perturbation theory, which is also presently being tested for in the decay mode $K_L \to \pi^0\gamma\gamma$ [10].

In contrast, if higher order effects are not important, we have seen that the photon pair mass distribution is relatively model independent, once the $\Delta S = 1$ and anomaly vertices are normalised by the rates for $K_L \to 3\pi^0$ and $\pi^0 \to \gamma\gamma$, and we remember to include the z^2 factor required by gauge invariance. In this case the $K_L \to \pi^0\pi^0\gamma\gamma$ decay could be used as a check on the experimental acceptance variation as the photon pair mass varies, given that one should reproduce the solid curve of the distribution in Figure 2.

Unfortunately however, the rate is probably too small to perform a precision comparison with our prediction. The current ϵ'/ϵ experiment at Fermilab, E731, has a single event sensitivity of about 4×10^{-8}, assuming equal detection efficiencies for $K_L \to 3\pi^0$ and $K_L \to \pi^0\pi^0\gamma\gamma$ [7], so it may see this process at the expected branching ratio, of order 10^{-8} to 10^{-7} depending on the cut (δm between 10 and 40 MeV). The upcoming rare decay experiment E799(I) at Fermilab should do better. However, as we have emphasized above, the photon pair distribution away from the pion pole is of most interest and this requires many detected events.

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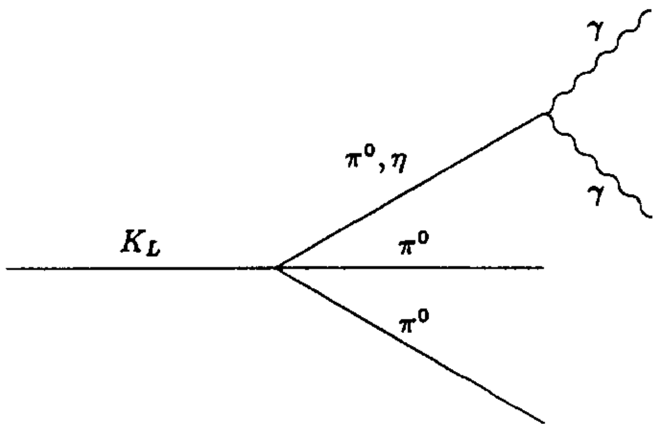

Figure 1 Tree level diagram for $K_L \to \pi^0\pi^0\gamma\gamma$

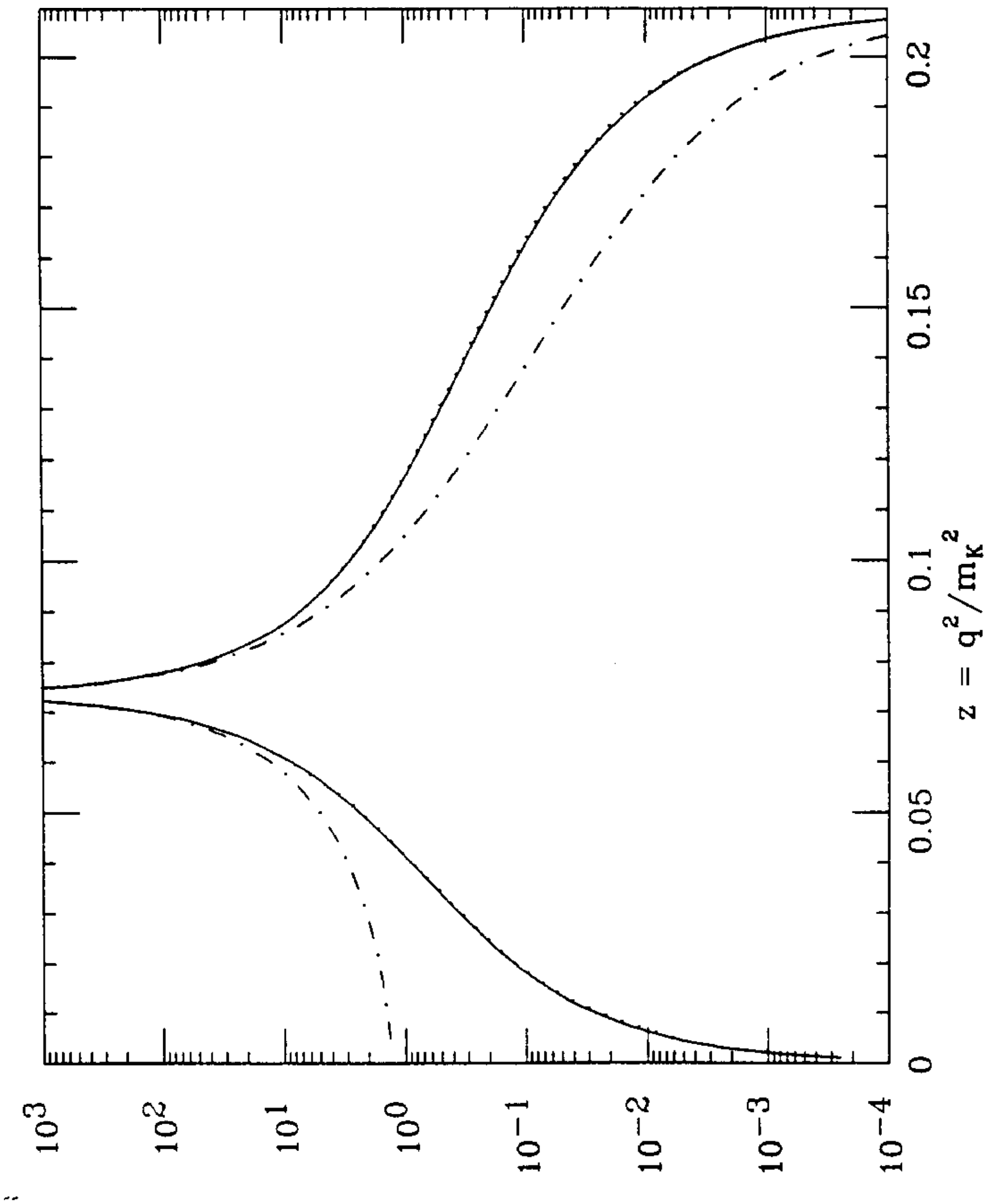

Figure 2 Photon pair z distribution in $K_L \to \pi^0\pi^0\gamma\gamma$. Multiply by P from Equation 12 to obtain $d\Gamma/dz$. Solid curve is full result, dot dashed curve has z dependence from pion resonance and phase space, dotted curve has all z dependence save for a constant $\Delta S = 1$ vertex.

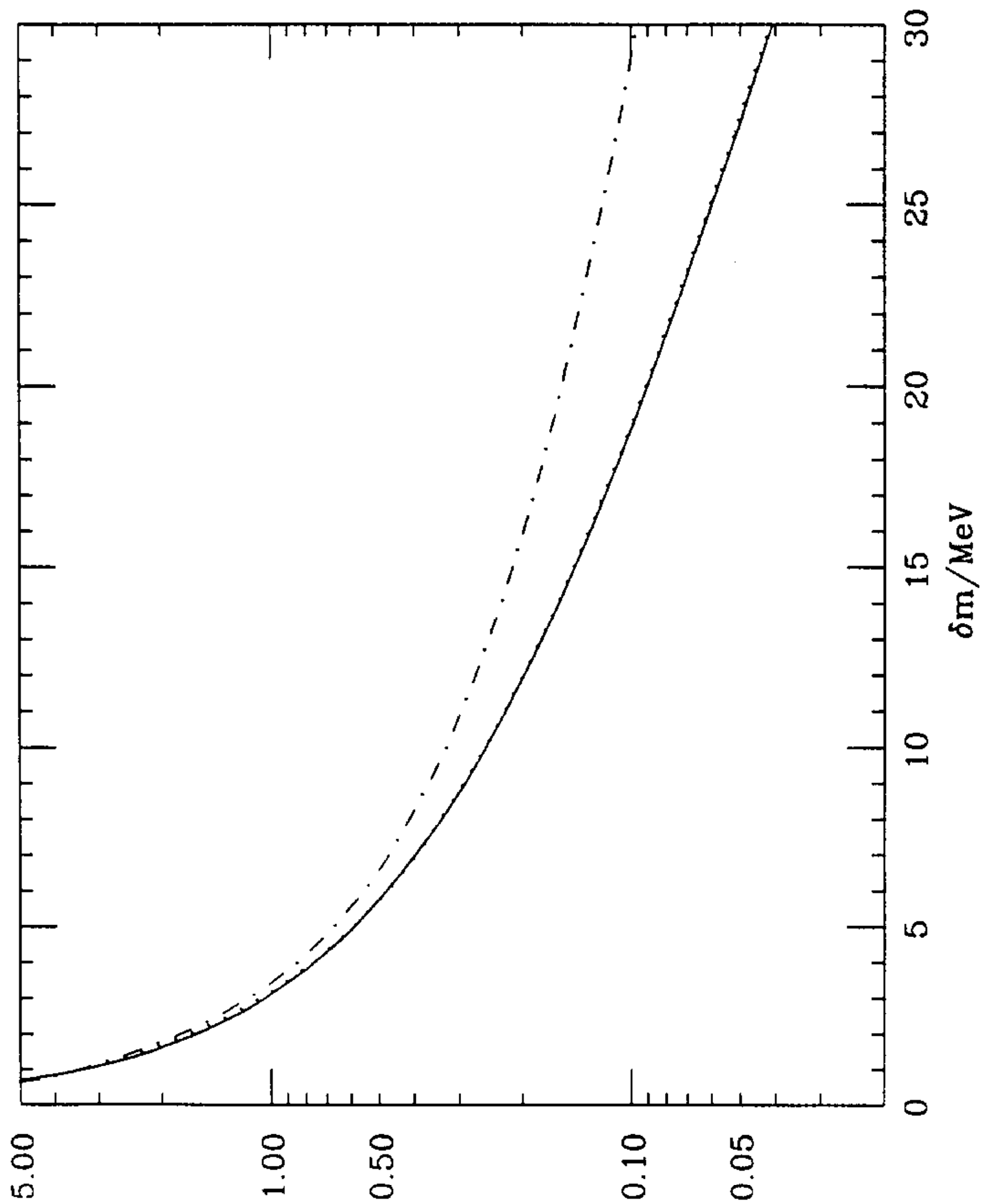

Figure 3 Decay rate Γ_{cut} for $K_L \to \pi^0\pi^0\gamma\gamma$ with a band of width $2\delta m$ in the photon pair invariant mass cut out around the pion pole. Solid curve is full result, dot dashed curve has z dependence from pion resonance and phase space, dotted curve has all z dependence save for a constant $\Delta S = 1$ vertex.