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B–B^* Splitting: a Test of Heavy Quark Methods

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Abstract

We determine the one loop QCD matching between lattice and continuum theories of the chromomagnetic moment operator. The operator is responsible for breaking the degeneracy of B and B^* mesons at order $1/m$ in the static approximation.

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1. Introduction

The experimentally known B–B^* mass splitting [1] can provide a test of heavy quark methods [2]–[13]. At zeroth order in the $1/m$ expansion the B and B^* mesons are degenerate. At first order in the expansion, the degeneracy is broken solely by the chromomagnetic moment operator [8]. The coefficient of this operator in the continuum static effective theory has been determined to order α_S [8][13]. The matrix elements of operators in the static effective field theory must be evaluated using lattice gauge theory [7][8] or other nonperturbative methods. In this paper we perform the renormalization required for the lattice gauge theory determination of matrix elements of the chromomagnetic moment operator and relate B–B^* splitting to one such matrix element.

The paper is organized as follows. In section two, we review the determination of the coefficient of the chromomagnetic operator in the static effective field theory and relate B–B^* mass splitting to a particular matrix element of the operator. In section three we review the background field method on the lattice and make a choice of lattice operator. Then in the fourth section, we determine the coefficient of the operator to one loop. Our conclusions are in section five.

2. Continuum Result

At zeroth order in $1/m$, the static effective field theory is independent of the spin of the heavy quark. At first order in $1/m$ (after applying the zeroth order equations of motion) two dimension-five operators, $\mathcal{O}_{\mathbf{kin}}$ and $\mathcal{O}_{\mathbf{mag}}$, appear in the static effective field theory. In Minkowski space the Lagrangian is,

$$\mathcal{L} = b^\dagger i\mathcal{D}_0 b + Z_{\mathbf{kin}}\mathcal{O}_{\mathbf{kin}} + Z_{\mathbf{mag}}\mathcal{O}_{\mathbf{mag}}, \tag{2.1}$$

where $i\mathcal{D}_\mu = i\partial_\mu + gA_\mu$ is the gauge covariant derivative, and b and $b^\dagger$ are the bare heavy quark fields which satisfy canonical commutation relations. Only the chromomagnetic moment operator, $\mathcal{O}_{\mathbf{mag}}$, breaks the $SU(2)$ symmetry [6] which acts on the heavy quark field. Thus we do not need to reproduce the form of $\mathcal{O}_{\mathbf{kin}}$ here. $\mathcal{O}_{\mathbf{mag}}$ is,

$$\mathcal{O}_{\mathbf{mag}} = \frac{i}{2m}\, b^\dagger \epsilon_{ijk}\sigma_i \mathcal{D}_j \mathcal{D}_k b. \tag{2.2}$$

In reference [13], Z_{mag} was determined to full order α_S using the background field method (reviewed in reference [14]). This was done by matching the part of the background field effective action that is first order in the background field and has two external fermion lines. The full order α_S result for the coefficient, Z_{mag}, of the chromomagnetic moment operator in the static effective field theory is

$$Z^{(2)}_{\mathrm{mag}} = 2\frac{g^2}{16\pi^2}\left[C^{\mathrm{f}} - \frac{1}{2}C^{\mathrm{adj}} + \frac{1}{2}C^{\mathrm{adj}}\left(\ln\frac{\mu^2}{m^2}+3\right)\right]. \tag{2.3}$$

In this equation $C^{\mathrm{f}}\,\mathbf{1} = \sum_a T^{\mathrm{f}}_a T^{\mathrm{f}}_a$ and $C^{\mathrm{adj}}\,\mathbf{1} = \sum_a T^{\mathrm{adj}}_a T^{\mathrm{adj}}_a$, with values 4/3 and 3 respectively for gauge group $SU(3)$. These results apply to any matrix element of the chromomagnetic moment operator.

In the static limit all four of the B and B^* states are degenerate because they are part of an $SU(2)\times SU(2)$ multiplet. The first factor is the $SU(2)$ discussed above which acts on the heavy quark field. The second factor acts on all other spinors and vectors and on the arguments of fields. The chromomagnetic moment operator is invariant only under the ordinary rotation group, which is the diagonal subgroup of these two $SU(2)$'s.

We wish to relate the B–B^* mass splitting to a matrix element of the chromomagnetic moment operator which can be measured on the lattice. Define E_0 and Δ by

$$H|B^*\rangle = (E_0+\frac{1}{4}\Delta)|B^*\rangle, \quad H|B\rangle = (E_0-\frac{3}{4}\Delta)|B\rangle. \tag{2.4}$$

E_0 is the center of gravity of the four states and Δ is the splitting. At zeroth order in the $1/m$ expansion, the B and B^* states can be classified by their heavy quark spin and the remainder of the total angular momentum:

$$|B^*\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad |B\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}, \tag{2.5}$$

where we have written down just one B^* state with $J_z = 0$.

With these definitions and using a nonrelativistic normalization of states, an expression for Δ valid to first order in the $1/m$ expansion is

$$\Delta(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}') = 2\,\mathrm{Re}\,\langle\mathbf{p}'\downarrow\uparrow|\,H\,|\mathbf{p}\uparrow\downarrow\rangle. \tag{2.6}$$

The part of H contributing to this matrix element is $-\int d^3x\, Z_{\mathrm{mag}} \mathcal{O}_{\mathrm{mag}}$, where Z_{mag} is the coefficient of $\mathcal{O}_{\mathrm{mag}}$ in the static effective field theory. An additional factor of Z, the wave function renormalization of the heavy quark, would appear here and in each term of the Lagrangian, Eq. (2.1), had we not written them in terms of heavy quark fields satisfying canonical commutation relations. Inserting this into the above expression,

$$\Delta = -2\,\mathrm{Re}\,\langle\downarrow\uparrow|\, Z_{\mathrm{mag}} \mathcal{O}_{\mathrm{mag}}\, |\uparrow\downarrow\rangle. \tag{2.7}$$

We thus have an expression for Δ in terms of a matrix element which we can determine using lattice gauge theory.

3. Lattice Operators

Here we write down the background field lattice action for the gauge fields [15] and then give our choice of discretization of the chromomagnetic moment operator. The Wilson plaquette action is,

$$S_{\mathrm{W}} = \frac{-1}{2g^2} \sum_{\substack{n \\ \mu,\nu}} \mathrm{tr}\left(U_{\mu\nu}(n) + U_{\mu\nu}^\dagger(n) \right). \tag{3.1}$$

For the plaquette labelled by site n and directions μ and ν, we have, in the background field formulation,

$$U_{\mu\nu}(n) = U_\mu(n) U_\nu(n+e_\mu) U_\mu^\dagger(n+e_\nu) U_\nu^\dagger(n), \tag{3.2}$$

where,

$$U_\mu(n) = V_\mu(n) U_{\mathrm{c}\,\mu}(n), \qquad V_\mu(n) = e^{iga\, Q_\mu(n)}, \qquad U_{\mathrm{c}\,\mu}(n) = e^{iga\, A_\mu(n)}. \tag{3.3}$$

Here $Q_\mu(n)$ is the quantum field and $A_\mu(n)$ is the background field. The lattice spacing is denoted by a.

In background field Feynman gauge, the gauge fixing term is

$$S_{\mathrm{gf}} = a^4 \sum_{\substack{n \\ \mu,\nu}} \mathrm{tr}\left(D_\mu^- Q_\mu(n) D_\nu^- Q_\nu(n) \right), \tag{3.4}$$

where D^-_μ is a background field lattice covariant derivative,

$$a\,D^-_\mu Q_\nu(n) = U^{-1}_{c\,\mu}(n-e_\mu)Q_\nu(n-e_\mu)U_{c\,\mu}(n-e_\mu) - Q_\nu(n). \tag{3.5}$$

We also need a discretization of the chromomagnetic moment operator which in the continuum is given by Eq. (2.2). In the Euclidean lattice theory we choose,

$$\mathcal{O}^{\rm latt}_{\rm mag} = \frac{ig}{4m} b^\dagger(n)\sigma_i\epsilon_{ijk}F^{\rm latt}_{jk}(n)b(n), \tag{3.6}$$

with,

$$a^2 g\,F^{\rm latt}_{jk}(n) = \frac{1}{8i}\left[U_{jk}(n) + U_{k,-j}(n) + U_{-j,-k}(n) + U_{-k,j}(n) - \text{h.c.}\right]. \tag{3.7}$$

This is gauge invariant, satisfies lattice cubic rotational invariance and has $\mathcal{O}_{\rm mag}$ as its naive continuum limit. With the phase conventions for the heavy quark part of the lattice action used in reference [16], and the above definition for $\mathcal{O}^{\rm latt}_{\rm mag}$, the coefficient of the operator in the Euclidean action is one at tree level. (Since the naive continuum limit of the conventional lattice covariant derivative is $\partial_\mu + igA_\mu$ rather than $\partial_\mu - igA_\mu$, there is an apparent sign disagreement with the continuum.)

4. One Loop Calculations

Following the procedure reviewed in section 2, we calculate the part of the background field gauge generating functional with one external background field, an incoming heavy quark, an outgoing heavy quark and one insertion of the chromomagnetic moment operator. We expand the amplitude up to terms linear in k, the momentum inserted by the background gauge field. The tree level diagram is shown in Figure 1.

In the continuum static theory we work in background field Feynman gauge, regulate in the ultraviolet with $\overline{\rm MS}$ and use a gluon mass λ to regulate infrared divergences. Then we find that the one-loop 1PI graphs give a contribution that is a factor of

$$\frac{g^2}{16\pi^2}\left[-2C^{\rm adj}\ln\frac{\mu^2}{\lambda^2} - 2\left(C^{\rm f} - \tfrac{1}{2}C^{\rm adj}\right)\ln\frac{\mu^2}{\lambda^2}\right] \tag{4.1}$$

times the tree level vertex. We also need to add in the heavy quark wavefunction renormalization in this scheme which is at order g^2,

$$Z^{(2)} = \frac{g^2}{16\pi^2}\, 2C^{\mathrm{f}} \ln \frac{\mu^2}{\lambda^2}. \tag{4.2}$$

Obtaining vertices from the lattice action and chromomagnetic moment operator in Section 3 the one-loop 1PI diagrams on the lattice are obtained and shown in Figure 2. The diagrams in Figure 2(d) and 2(e) do not contribute in background field Feynman gauge since the propagating gluon is required to carry a spatial index by the chromomagnetic moment operator and a time index by the coupling to the heavy quark. The diagrams in Figure 2(f) have no terms linear in k_i and a possible term linear in k_0 in fact vanishes. Hence we need calculate only the diagrams in Figures 2(a), 2(b) and 2(c). For this we need the lattice Feynman rules for the chromomagnetic moment operator of equation (3.6) with one background field, two quantum fields or one background field and two quantum fields. On the lattice, the vertex appearing in Figure 1 has Feynman rule

$$\frac{-i}{4m} \sum_{ij} \sigma_i \epsilon_{ijn}\, g \frac{T^a}{2} (e^{-ik_j a} - e^{ik_j a})(1 + e^{ik_n a}). \tag{4.3}$$

Expanded to first order in k this becomes,

$$\frac{-1}{2m} \sum_{ij} \sigma_i \epsilon_{ijn} k_j\, g T^a. \tag{4.4}$$

The vertices with two quantum fields or two quantum and one background field are considerably more complicated so we do not quote their Feynman rules here. For example the two-quantum one-background field part of the lattice chromomagnetic moment operator contains 160 terms. Note that this vertex is peculiar to the lattice, hence so is the diagram in Figure 2(a) which contains it.

The result of the computation is that the one loop graphs give the tree vertex of equation (4.4) multiplied by the following factor:

$$\begin{aligned} \frac{g^2}{16\pi^2} \Big\{ & - 4\pi^2 C^{\mathrm{f}} - D_a C^{\mathrm{adj}} \\ & - (D_b - 2 \ln \lambda^2 a^2) C^{\mathrm{adj}} \\ & - (D_c - 2 \ln \lambda^2 a^2) \left(C^{\mathrm{f}} - \tfrac{1}{2} C^{\mathrm{adj}} \right) \Big\}. \end{aligned} \tag{4.5}$$

The three lines inside the curly braces come from diagrams 2(a), 2(b) and 2(c) respectively. The quantities D_i are evaluated numerically using the Monte Carlo integration routine VEGAS [17] and take the values,

$$D_a = 3.55 \qquad D_b = 1.30 \qquad D_c = 4.53. \tag{4.6}$$

Errors are at most order one in the last decimal place. Analytical expressions for these numerical constants are listed in the appendix.

The background field lattice heavy quark wavefunction renormalization in Feynman gauge at order g^2 is [16],

$$Z^{\mathrm{latt}\,(2)} = \frac{g^2}{16\pi^2} C^{\mathrm{f}} \left[e - 2\ln \lambda^2 a^2 \right], \tag{4.7}$$

where the constant e has the value 24.48. An expression for e is given in the appendix.

Now we match the 1PI functions in the continuum and lattice static theories with the result that the order g^2 contribution to the renormalization constant $Z_{\mathrm{mag}}^{\mathrm{latt}}$ is the difference of the one loop graphs in the continuum and on the lattice, equation (4.1) minus equation (4.5), plus the difference of the heavy quark wavefunction renormalizations, equation (4.2) minus equation (4.7):

$$\begin{aligned} Z_{\mathrm{mag}}^{\mathrm{latt}\,(2)} = \frac{g^2}{16\pi^2} \Big\{ & - C^{\mathrm{adj}} \ln \mu^2 a^2 + C^{\mathrm{f}} (D_c - e + 4\pi^2) \\ & + C^{\mathrm{adj}} \left(D_a + D_b - \tfrac{1}{2} D_c \right) \Big\}. \end{aligned} \tag{4.8}$$

The result is independent of the gluon mass λ which was introduced as an infrared regulator. We have obtained the same result with a hard momentum space infrared cutoff.

5. Conclusion

To obtain the coefficient of the lattice chromomagnetic moment operator, we take the product of Z_{mag} and $Z_{\mathrm{mag}}^{\mathrm{latt}}$. From Equation (2.3) using a strong coupling with $\Lambda_{\overline{\mathrm{MS}}}^{(4)}$ of 200 MeV, a b-quark mass of 5 GeV and a scale μ of 2 GeV, we find that to order g^2, $Z_{\mathrm{mag}} = 1.06$. There is a significant uncertainty in the numerical value of the second factor due to the uncertainty in the value of α_S in the matching to the lattice regulated theory. Taking, for example, a value of α_S at an inverse lattice spacing of 2 GeV that is 1.8 times the bare lattice value of $1/4\pi$ [18], we find that to order g^2, $Z^{\mathrm{latt}}_{\mathrm{mag}} = 1.39$. With these values we find that $Z_{\mathrm{mag}} Z^{\mathrm{latt}}_{\mathrm{mag}} = 1.47$ to order g^2, a large correction to the tree level result.

Calculating the matrix element of $\mathcal{O}_{\mathrm{mag}}$ in Eq. (2.7) on the lattice will give a quantitative prediction of B–B^* splitting. Unlike many hadronic spectra calculations performed using lattice gauge theory, the determination of this splitting from the matrix element of a hadronic operator strongly parallels the approach already being used to determine weak matrix elements. It will provide a significant test of the heavy quark effective theory approach to determining the properties of heavy-light systems, such as the B meson decay constant and mixing parameter.

More generally, we expect $1/m$ corrections to be important in D and B meson systems. The renormalization of the discretized chromomagnetic moment operator performed here is necessary for corrections to a variety of phenomenologically relevant quantities. One might expect that in B mesons $1/m$ corrections to quantities like f_B would be of order 5% and in D mesons they could be of order 15%. In this case, they could account for part of the discrepancy between the D and B meson decay constants which have been measured by two different lattice gauge theory methods (see reference [19] for a recent review).

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Appendix. Numerically Evaluated Constants

In this appendix, we express the numerically evaluated factors in the renormalization constants. Following reference [20] we introduce the following notation:

$$\begin{aligned}\Delta_1 &= \textstyle\sum_\mu \sin^2 \frac{q_\mu}{2}, \\ \Delta_4 &= \textstyle\sum_\mu \sin^2 q_\mu.\end{aligned} \tag{A.1}$$

The sums on μ run from 1 to 4. Let $\Delta_1^{(3)}$ be identical to Δ_1 except with q_4 set to zero. The quantities D_a, D_b, D_c and e are given by

$$\begin{aligned}D_a &= \frac{3}{4\pi^2}\int d^4q\,\frac{1}{4\Delta_1} - \frac{3\pi^2}{2}, \\ D_b &= D_c - \frac{2}{9}D_a - \frac{1}{3\pi^2}\int d^4q\,\frac{\Delta_4}{64\Delta_1^2} - \frac{\pi^2}{6}, \\ D_c &= -2 + \frac{1}{\pi^2}\int d^4q\,\left[2\left(\frac{1}{16\Delta_1^2} - \theta(1-q^2)\frac{1}{q^4}\right) - \frac{1}{8}\frac{1}{4\Delta_1}\right], \\ e &= D_c + \frac{1}{\pi}\int d^3q\,\frac{1}{4\Delta_1^{(3)}}.\end{aligned} \tag{A.2}$$

Each integration variable is in the range $[-\pi, \pi]$.

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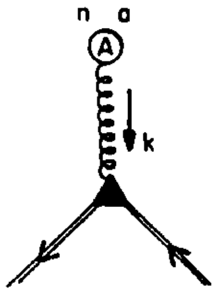

Fig. 1: Tree level diagram

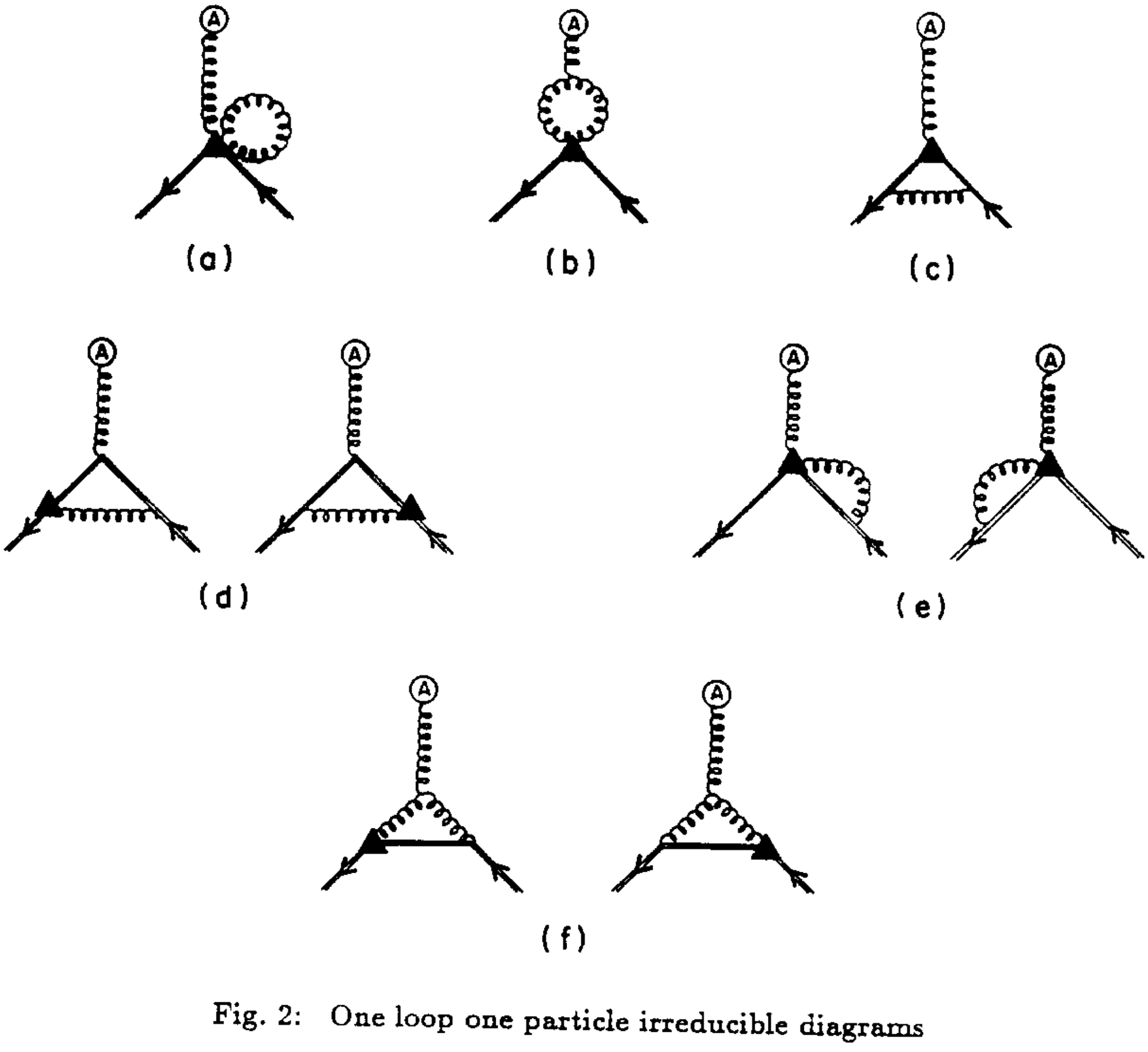

Fig. 2: One loop one particle irreducible diagrams