



NEXT-TO-LEADING ORDER CALCULATIONS IN QCD

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1. Introduction

The detailed understanding of QCD processes, such as multijet production, and QCD-associated processes such as production of W s or Z s in association with jets, is important to our ability to find new physics at present and future hadron colliders such as the Tevatron, the LHC, and the SSC. An example can be found even in the realm of (as yet unknown) standard-model physics: in searching for the top quark in the single lepton + missing energy + jets mode, one must demand four distinct identified jets in order to produce an acceptable signal-to-background ratio.¹

An important step in the theoretical side of such an understanding is the calculation of the perturbative QCD predictions for such processes at next-to-leading order. The leading-order QCD prediction for these various processes is given by tree graphs alone. There are several flaws in a leading-order calculation, which are partially rectified in a next-to-leading order one. At leading order, the matrix element has no dependence on the renormalization scale μ , but as one conventionally uses the one-loop running coupling constant, the latter does depend on μ . As a result, computed cross-sections and distributions have a strong dependence on the choice of this unphysical parameter. In a next-to-leading order calculation, the virtual corrections to the lowest-order hard-scattering matrix elements do have a dependence on μ , which helps compensate the dependence in the coupling constant. For certain observables, the next-to-leading result can be quite insensitive to the choice of μ over a relatively wide range.

Another problem with leading-order calculations is their incorrect dependence on the experimental resolution parameters such as the jet cone size $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ and the minimum jet transverse energy E_T . Indeed, in the leading processes, the lowest-order results have no dependence — the two-jet or inclusive jet cross-sections are independent of the jet cone size, and the $W + 0$ jet cross-section is independent of the minimum transverse energy. This is corrected in a next-to-leading order calculation through the presence of real radiation inside the jet cone (that is, the possibility of a two outgoing partons forming a single jet), or through the presence of real radiation outside any jet cones.

From a more theoretical point of view, one knows that the presence of infrared divergences in leading-order calculations of multijet cross-sections result in a perturbative expansion for these processes not in powers of the strong coupling constant α_s , alone, but in powers of the strong coupling constant times logarithms in the jet resolution parameters, $\ln \Delta R$ or $\ln E_T^2/Q^2$, where Q^2 is some characteristic hard scale in the

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process. If the cuts or resolution parameters are too small, these logarithms will become large and spoil the applicability of perturbation theory. In a leading-order calculation, there is no warning of this breakdown, whereas in a next-to-leading order calculations, these dangerous logarithms are calculated explicitly.

2. Next-to-Leading Order Calculations

As suggested above, several ingredients enter into a next-to-leading order calculation of an n -jet process: one needs the virtual corrections to the tree-level diagrams for the relevant $2 \rightarrow n$ or $2 \rightarrow n + \{W, Z\}$ subprocesses; the tree-level processes producing $n + 1$ partons in the final-state, integrated over the phase space for one unobserved parton; and a set of next-to-leading order structure functions, which are available from several groups.^{2,3} The technical difficulty of such calculations results from the presence of separate infrared divergences in the virtual and soft or collinear integrations, divergences which cancel only when forming an experimentally observable quantity.

Recently, new technologies have emerged which promise to simplify the calculation of such corrections in QCD. Zvi Bern and one of the authors have presented⁴ a string-based technique for performing some of the virtual corrections. The traditional techniques for performing the soft and collinear integrations in processes with gluons are quite cumbersome as well. We report here on the extension, to hadron collider calculations, of new techniques developed by two of the authors for simplifying the integrations over real unobserved radiation.⁵

The key point in the simplification of the soft integrations is the use of the color decomposition of tree-level amplitudes,⁶ in which one writes the tree-level amplitude as a sum over color factors times kinematic factors or *partial amplitudes*,

$$\mathcal{A}_n(\{k_i, \lambda_i, a_i\}) = \sum_{\sigma \in S_n/Z_n} \text{Tr}(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}}) A_n(k_1, \lambda_1; \dots; k_n, \lambda_n)$$

and the observation⁷ that the partial amplitudes (unlike the full amplitude or squared cross-section) have a simple behavior in the soft limit,

$$|A_n(k_1, \lambda_1; \dots; k_n, \lambda_n)| \rightarrow_{k_j \rightarrow 0} s(k_{j-1}, k_j, k_{j+1}) |A_{n-1}(k_1, \lambda_1; \dots)|$$

where the soft factor is reminiscent of the one in QED,

$$s(k_{j-1}, k_j, k_{j+1}) = \sqrt{\frac{k_{j-1} \cdot k_{j+1}}{k_{j-1} \cdot k_j k_j \cdot k_{j+1}}}$$

(The examples here are drawn from pure glue amplitudes, but the discussion extends in a straightforward way to amplitudes containing quarks.) A similar factorization of course occurs in collinear limits (as it does for the squared cross-section as a whole).

In the squared amplitude, one does not have to consider the entire amplitude when performing the integration over the soft (and collinear) regions; it is sufficient

to consider the relevant soft and collinear factors (squared), and integrate them over $(4 - \epsilon)$ -dimensional phase space. When added to the virtual corrections in a process not containing colored partons in the initial state, the resulting poles in ϵ cancel the poles in the virtual contributions, leaving a differential cross-section which is finite as $\epsilon \rightarrow 0$. The remainder of the integrations — over the hard invariants — can be performed in 4 dimensions.

One must integrate the soft and collinear factors from their singularities out to the jet cone boundary. It is convenient to split up this region of integration into two parts, delineated by a minimum invariant mass s_{\min} . The soft and collinear integrations are performed analytically out to the ‘theoretical’ cone of s_{\min} , and from that cone out to the actual quasi-experimental cone numerically. This allows one to handle a more-or-less arbitrary cone shape.

For hadron-hadron scattering, one must also handle the question of ‘initial-state’ collinear singularities; the one-loop evolution of the parton distribution functions yields additional poles in ϵ , which are cancelled by the poles in a ‘crossing’ function, representing the difference between collinear emission of a parton in the initial and final states.

It is worth noting that these integrated soft, collinear, and crossing functions are *universal*, in that they apply to any process; thus once they have been performed, implementing a numerical program for a new process requires only the computation of the virtual corrections. Furthermore, the real radiative matrix elements are needed only in 4 dimensions, not in $(4 - \epsilon)$ dimensions.

It is desirable to keep the matrix elements fully differential, and with these building blocks, it is easy to do so.

We have used these techniques to construct programs for $W, Z + 0, 1$ jets and for two-jet quantities; results will be reported in forthcoming publications.

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