



## MASSES AND MIXINGS FOR MAJORANA AND DIRAC NEUTRINOS

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### ABSTRACT

Several theoretical issues pertinent to the existence of the 17 keV neutrino are discussed including neutrino types, mixings and bounds on its mass and lifetime arising from particle physics, cosmology and astrophysics. A partial list of various theoretical models that have been proposed is given and comments are made as to their acceptability.

The present highly accurate  $e^+e^-$  collider experiments at LEP have revealed<sup>1</sup> at the  $Z$  peak that there are just three flavors of light lefthanded doublet neutrinos under the  $SU(2)_L \times U(1)_Y$  electroweak group. But no information emerges concerning the number of heavy lefthanded doublet states or righthanded isosinglets. Turning this around, the key questions in neutrino physics concern the number of neutrino mass eigenstates, types of neutrinos, their masses and mixings. In particular, is the controversial 17 keV neutrino<sup>2</sup> one of them?

### 1. Neutrino Types and Mixings

To appreciate the types of neutrino states possible, it is convenient to draw analogies with the neutral pseudoscalar mesons  $\pi^0$ ,  $\eta$ ,  $K^0$  and  $\bar{K}^0$ . Majorana neutrinos are self-conjugate,  $\chi = \pm\chi^c$ , with broken lepton number and, in the absence of CP violation, are even or odd under CP parity as are  $\pi^0$  and  $\eta$  which have zero strangeness. In the simplest circumstance, these flavor states are also mass eigenstates. For Dirac neutrinos, the flavor state  $\nu$  with lepton number  $L = +1$  is distinct from  $\nu^c$  with  $L = -1$ , just as  $K^0$  and  $\bar{K}^0$  with strangeness  $S = +1$  and  $S = -1$ , respectively, are distinct hadronic states. The CP even and odd mass eigenstates then correspond to

$$\nu_{1,2} = \frac{1}{\sqrt{2}}(\nu \pm \nu^c) \quad (1)$$

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a pair of Majorana states analogous to  $K_1$  and  $K_2$ . If the masses of  $\nu_1$  and  $\nu_2$  are equal, the states are Dirac mass eigenstates, whereas if they differ due to small corrections they are called pseudo-Dirac; by analogy recall  $\delta m_K = 3.5 \times 10^{-6}$  eV.

The lefthanded component  $\nu_L$  of  $\nu_{1,2}$  in (1) is active in the weak interactions; however, the  $(\nu^c)_L \equiv (\nu_R)^c$  component is inert if it exists at all. If it does, the fields  $\nu_{1,2}$  participate in the weak interactions with only half the strength of a Majorana neutrino  $\chi_L$ . This has important implications for the leptonic CKM mixing matrix.

In the mass eigenbases with just 3 families, the weak charged-current Lagrangian has the form

$$\mathcal{L}_W^{(\pm)} = -\frac{g}{\sqrt{2}} \bar{\ell}_{iL} \gamma^\mu V_{i\alpha} \nu_{\alpha L} W_\mu + h.c. \quad (2)$$

with a  $3 \times n$  mixing matrix  $V$ , since there are just three known charged leptons but, in general,  $n$  neutrino mass eigenstates. From generalized unitarity

$$0 \leq \sum_{i=1}^3 |V_{i\alpha}|^2 \leq 1 \quad \text{for each } \alpha = 1, 2, \dots, n \quad (3a)$$

with the restriction

$$\sum_{\alpha=1}^n |V_{i\alpha}|^2 = 1 \quad \text{for each } i = 1, 2, 3 \quad (3b)$$

With just three light Majorana neutrinos present, the upper bound in (3a) will be nearly saturated for each of their columns, as the heavy Majorana neutrinos effectively decouple from the weak interactions. This is in keeping with the notion that the left-handed flavor states are mainly linear combinations of light neutrino mass eigenstates, while the singlet flavor states are mainly combinations of the massive neutrino states. In the case of each Dirac pair of Majorana mass eigenstates with an inert component, however, the sum in (3a) will be equal for the pair and bounded by 0.5.

## 2. Constraints on the 17 keV Neutrino

### 2.1. Particle Physics Constraints<sup>3</sup>

The upper limits on the mixing angles from the accelerator-based neutrino oscillation experiments, in terms of two-component oscillations, are

$$\begin{aligned} \nu_e \leftrightarrow \nu_\mu &: \quad \sin^2 2\theta_{12} < 0.0034 \\ \nu_e \leftrightarrow \nu_\tau &: \quad \sin^2 2\theta_{13} < 0.12 \\ \nu_\mu \leftrightarrow \nu_\tau &: \quad \sin^2 2\theta_{23} < 0.004 \end{aligned} \quad (4)$$

Since  $\sin^2 2\theta \sim 4|V_{1\alpha}|^2 \simeq 0.034$  from the Oxford experiment,<sup>2</sup> this implies that the 17 keV neutrino must be associated with the largest mass eigenstate in the  $\nu_\tau$  flavor state.

A strict bound on Majorana neutrino contributions also arises from the neutrinoless double beta decay searches. The helicity mismatch and present nonobservation of these  $(\beta\beta)_{0\nu}$  decay modes place the following limit on the effective neutrino mass

$$\langle m_\nu \rangle \simeq \left| \sum_{\alpha=1}^n \eta_\alpha m_\alpha (V_{1\alpha})^2 \right| < 1.8 \text{ eV} \quad (5)$$

With  $m_3 = 17$  keV and  $V_{13} \sim \sin \theta \simeq 0.092$ , a 17 keV Majorana neutrino can only escape this bound if some other Majorana neutrino, with opposite CP parity  $\eta$  and appropriate mass and mixings, nearly cancels its contribution.

## 2.2. Cosmological Constraints<sup>4</sup>

To evade the upper bound,  $\sum_i m_{\nu_i} \lesssim 40$  eV, on the total mass of light stable doublet neutrinos arising from the relic neutrino contribution to the present density of the universe, the 17 keV neutrino must annihilate rapidly, which is difficult, or decay rapidly by massless Majoron emission,  $\nu_\tau \rightarrow \nu_{e,\mu} + X$ , with a lifetime less than  $10^7$  sec. from structure formation considerations. On the other hand, the massless Majoron contributes an additional  $\delta N_\nu = 4/7$  to the number of light neutrino species present at the time of primordial nucleosynthesis, whereas the present bound from  ${}^4\text{He}$  abundance is  $N_\nu \leq 3.4$ . This bound is also violated by a 17 keV pseudo-Dirac neutrino, unless  $\delta m_D < 10^{-7}$  eV to prevent inert-active oscillations which would lead to  $\delta N_\nu = 1$ .

## 2.3. Astrophysical Constraints<sup>4</sup>

The 17 keV lifetime bound is lowered to less than  $3 \times 10^4$  sec. by the absence of secondary neutrino pulses from the SN1987A explosion, while to avoid excess cooling of the SN1987A explosion, the lifetime must be greater than  $10^{-4}$  sec. Moreover, the same excess cooling restriction places an extremely conservative upper bound<sup>5</sup> of 28 keV on the mass of a Dirac or pseudo-Dirac neutrino with an inert righthanded component. Extra production processes not considered in obtaining this bound are expected<sup>6</sup> to reduce this number below 10 keV.

## 3. Proposed Theoretical Models

We now consider models of the neutrino mass matrix which have been proposed in the weak flavor basis. In general, we write

$$M_N = \frac{\overline{\nu_{iL}}}{(N_{jR})^c} \begin{pmatrix} (\nu_{iL})^c & N_{jR} \\ \mathbf{L}_M & \mathbf{M}_D \\ \mathbf{M}_D^T & \mathbf{R}_M \end{pmatrix} \quad (6)$$

where  $\mathbf{L}_M$  is the lefthanded Majorana matrix,  $\mathbf{M}_D$  the Dirac submatrix and  $\mathbf{R}_M$  the righthanded Majorana matrix with  $i = 1 \rightarrow 3$  for the 3 lefthanded flavor states and  $j = 0 \rightarrow (n - 3)$  for the righthanded singlet flavor states.

### 3.1. $3 \times 3$ Matrix with only $\mathbf{L}_M$ present

One model<sup>7</sup> has a conserved  $L = L_e - L_\mu + L_\tau$  global symmetry leading to one zero mass neutrino and a Dirac pair of 17 keV neutrinos with all components active. This is by far the most economical model, but it can not accommodate the MSW resonant matter  $\nu_e \rightarrow \nu_\mu$  conversion effect, and the triplet Majorons required have been ruled out by the observed  $Z$  width.<sup>1</sup>

### 3.2. $4 \times 4$ Matrix with $\mathbf{L}_M = \mathbf{R}_M = 0$

This model<sup>8</sup> prior to radiative corrections leads to a pair of massless neutrinos and a 17 keV Dirac neutrino pair with inert components, which conflicts with the supernova cooling constraint.

### 3.3. $5 \times 5$ Matrix with $\mathbf{L}_M = 0$

Here a model<sup>9</sup> accommodates the MSW effect and leads to one massless and four massive Majorana neutrinos with the 17 keV neutrino identified mostly with  $\nu_\tau$ , while the fourth neutrino has a mass near 4 MeV and the fifth near 1 TeV. The 4 MeV mass is too high to avoid the  $(\beta\beta)_{0\nu}$  bound and too low to escape the  $\delta N_\nu \leq 0.4$  bound from primordial nucleosynthesis.

### 3.4. $6 \times 6$ Matrix with $\mathbf{L}_M = 0$ and $\mathbf{R}_M$ rank 2

This form finally allows a  $3 \times 3$  Dirac submatrix similar to that for quarks. A rank 2  $\mathbf{R}_M$  leads<sup>10</sup> to a pseudo-Dirac pair of 17 keV neutrinos. To avoid the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation bound in (4), many new Higgs fields must be introduced so  $\mathbf{M}_D$  is very different from the up quark matrix;<sup>11</sup> moreover,  $\delta m_D \sim 10^{-3}$  eV, too large by a factor of  $10^4$  to escape the nucleosynthesis constraint.

## 4. Summary

Experimentally the 17 keV neutrino is still a viable object, though unambiguous confirmation has not been achieved. Theoretically, so many constraints from particle physics, cosmology and astrophysics must be applied that no acceptable theoretical model has yet been written down. It appears, moreover, that all models involving a 17 keV Dirac neutrino with an inert component are ruled out.

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