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Oblique electroweak corrections and an extra gauge boson

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We develop an effective Lagrangian based framework for the inclusion of new heavy physics effects on gauge boson self energies. Various observables may be expressed in terms of the parameters S, T, and U. We then generalize this framework to include a new U(1) gauge boson. We treat the effects of mixing through kinetic terms with the Z and the photon as well as mass mixing with the Z. We show how the bulk of these effects produce effective shifts in the parameters S, T, and U.

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There has been recent interest[1-4] in the electroweak corrections due to heavy particles with a characteristic mass Λ greater than m_Z. If the heavy particles participate in SU(2)×U(1) symmetry breaking then they, when integrated out, will generate additional effective interactions involving the electroweak gauge bosons and the triplet of Goldstone bosons. These interactions are very conveniently described by a gauged chiral Lagrangian.

All parameters in this chiral Lagrangian are finite quantities renormalized at the Z mass scale. We consider terms in this effective theory at order p^2 and p^4 in the low energy expansion. The electroweak corrections induced at these orders are not suppressed by powers of m_Z/Λ, unlike terms of higher order in the energy expansion. Thus the chiral Lagrangian approach immediately focuses our attention on the finite parameters most important to electroweak corrections.

Of most immediate interest are "oblique"[5] corrections, those corrections entering through the gauge boson self-energies. When weak isospin violating effects are included we find that there are three independent terms in the chiral Lagrangian which contain correction terms quadratic in the gauge fields. We present a simple derivation of the relation between the coefficients of these terms, or equivalently the parameters S, T, and U, and various observables.

In this framework we are then able to easily treat the general mixing effects of an additional gauge boson X. We treat X-Z and X-A mixing in the kinetic terms as well as X-Z mixing in the mass terms. We show how some of these mixing effects are equivalent to shifts in S, T, and U. But additional effects in neutral current amplitudes will help distinguish the X boson from the effects of a heavy sector. This is true even in the case that the known fermions do not carry X charge. An interesting possibility for this latter case is vanishing X mass.

The kinetic mixing terms are often not considered. But they may be generated for example by a strongly interacting, electroweak symmetry breaking sector at scale Λ. In addition, the X boson may correspond to a U(1) gauge symmetry well above the scale Λ. If it does then it is intriguing to consider the term $X_{\mu\nu}B^{\mu\nu}$ where B_μ is the U(1) hypercharge boson. This gauge invariant, dimension four term could be produced by physics at arbitrarily high mass scales.

We start without the X boson. We assume CP invariance and we write down those terms at order p^2 and p^4 which contain terms quadratic in the gauge fields.

$$\begin{aligned} L_{VV} = \frac{F^2}{4} Tr\{\nabla_\mu U^\dagger \nabla^\mu U\} - {}^1/_2 Tr\{W_{\mu\nu}W^{\mu\nu}\} - {}^1/_2 Tr\{B_{\mu\nu}B^{\mu\nu}\} \\ + L_{10}\, gg' Tr\{U^\dagger B_{\mu\nu} U W^{\mu\nu}\} \\ + \Delta\rho\, F^2\, Tr\{\tau_3 U \nabla_\mu U^\dagger\} Tr\{\tau_3 U \nabla^\mu U^\dagger\} \\ + K\Big[Tr\{\tau_3 U \nabla_\mu \nabla_\nu U^\dagger\} Tr\{\tau_3 (\nabla^\mu \nabla^\nu U) U^\dagger\} \\ - {}^1/_2 g'^2 Tr\{B_{\mu\nu}B^{\mu\nu}\} - gg' Tr\{U^\dagger B_{\mu\nu} U W^{\mu\nu}\}\Big] \end{aligned} \tag{1}$$

$\nabla_\mu U \equiv \partial_\mu U - igUW_\mu + ig'B_\mu U$, $W \equiv W_a(x)\tau_a$, $B \equiv B(x)\tau_3$, $U \equiv exp(-2i\pi_a(x)\tau_a/F)$

$\pi_a(x)$ is the Goldstone boson triplet. The first term in isolation yields the tree order W and Z masses of the standard model. The coefficient L_{10} is named in analogy with the corresponding quantity in the Gasser and Leutwyler analysis of low energy QCD.[6] This analogy was used to estimate L_{10} in reference [1]. (For other estimates see [2-4]). The $\Delta\rho$ and K terms are present because of the explicit $SU(2)_R$ symmetry breaking expected in the underlying theory. $1+\Delta\rho$ is the heavy physics contribution, including that of a heavy top quark, to the usual ratio of the neutral to charged weak currents near zero momentum. The net effect of the K term on the gauge field self energies, when the Goldstone pole terms are included, is equivalent to the term $(K/4)g^2 W_{3\mu\nu} W_3^{\mu\nu}$. There are no other independent terms contributing to the gauge boson self-energies.

Note that the K term is higher order in the energy expansion compared to the $\Delta\rho$ term, and that both terms require isospin violating physics. Thus the typical technicolor contributions to K are expected to be suppressed relative to contributions to $\Delta\rho$ by $(m_Z/\Lambda)^2$. (But this will not necessarily be true for the X boson contributions.)

We should stress that we are concerned only with possible new physics contributions to various electroweak parameters. Expressions for actual physical quantities must also include the light physics corrections of the standard model. These effects are by assumption small compared to the new physics and they will not be not addressed in this paper.

In the following we will adopt a more attractive notation.[2]

$$S \equiv -16\pi L_{10}\,,\ T \equiv \Delta\rho/\alpha,\quad U \equiv -16\pi K \tag{2}$$

The terms quadratic in gauge field with two derivatives are the following.

$$-4L_{\rm kin} = [W_1{}^{\mu\upsilon}]^2 + [W_2{}^{\mu\upsilon}]^2 + \\ + (1 + \frac{\alpha}{4s_\theta{}^2}U)[W_3{}^{\mu\upsilon}]^2 + (1 + \frac{\alpha}{4c_\theta{}^2}V)[B^{\mu\upsilon}]^2 + \frac{\alpha}{2s_\theta c_\theta}S\, B_{\mu\upsilon}W_3^{\mu\upsilon} \\ W^{\mu\upsilon} = \partial^\mu W^\upsilon - \partial^\upsilon W^\mu \qquad \text{etc.} \tag{3}$$

Note that we have introduced the V term even though it may be absorbed by a redefinition of the g' coupling. V will drop out of observables and it is retained only for later convenience. Two other parameters g and F characterize the charged W mass and couplings, and these parameters are not affected by S, T, or U.

We may make the conventional transformation to the mass eigenstate basis.

$$W_{3\mu} = c_\theta Z'_\mu + s_\theta A'_\mu \qquad B_\mu = -s_\theta Z'_\mu + c_\theta A'_\mu \tag{4}$$

$$c_\theta \equiv \frac{g}{\sqrt{g^2 + g'^2}} \qquad s_\theta \equiv \frac{g'}{\sqrt{g^2 + g'^2}} \tag{5}$$

In this primed basis the kinetic terms for the neutral fields and mass term are:

$$L_{\rm AZ} = -\frac{1}{4}[1-2\Delta_Z][Z'^{\mu\upsilon}]^2 - \frac{1}{4}[1-2\Delta_A][A'^{\mu\upsilon}]^2 + \frac{1}{2}\Delta_{AZ}A'_{\mu\upsilon}Z'^{\mu\upsilon} \\ + \frac{1}{2}m_Z^2\,(1-\alpha T)\,Z'_\mu Z'^\mu \tag{6}$$

where

$$\Delta_{AZ} = -\frac{1}{4}\frac{(c_\theta{}^2 - s_\theta{}^2)S\alpha}{c_\theta s_\theta} - \frac{1}{4}\frac{Uc_\theta\alpha}{s_\theta} + \frac{1}{4}\frac{Vs_\theta\alpha}{c_\theta} \tag{7}$$

$$\Delta_A = -\frac{1}{4}S\alpha - \frac{1}{8}U\alpha - \frac{1}{8}V\alpha \tag{8}$$

$$\Delta_Z = \frac{1}{4}S\alpha - \frac{1}{8}\frac{Uc_\theta{}^2\alpha}{s_\theta{}^2} - \frac{1}{8}\frac{Vs_\theta{}^2\alpha}{c_\theta{}^2} \tag{9}$$

An additional transformation is necessary to obtain standard kinetic terms. This transformation is uniquely defined if we are to remain in a mass eigenstate basis. We treat the Δ's as small quantities; our results henceforth are true to lowest order in Δ's. The transformation is

$$Z'_\mu = [1 + \Delta_Z]Z_\mu \tag{10}$$

$$A'_\mu = [1 + \Delta_A]A_\mu + \Delta_{AZ}Z_\mu \tag{11}$$

In the unprimed basis the kinetic terms have conventional form. The Z mass is

$$m_Z = (1 + \tilde{\Delta}_Z)\, m_Z^o \tag{12}$$

where m_Z^o is the Z mass in the absence of corrections, and

$$\tilde{\Delta}_Z = \tfrac{1}{4} S\alpha - \tfrac{1}{2} T\alpha - \tfrac{1}{8}\frac{U c_\theta^2 \alpha}{s_\theta^2} - \tfrac{1}{8}\frac{V s_\theta^2 \alpha}{c_\theta^2} \tag{13}$$

The quantities Δ_A, Δ_Z, Δ_{AZ}, and $\tilde{\Delta}_Z$ and their analogs will play a central role in our discussion of the X boson. For completeness we will derive the Z and A couplings to matter (following the notation of ref. [5,7]) in terms of these quantities.

The relevant combination of the neutral gauge field is:

$$g W_{3\mu} I_3 + g' B_\mu Y = \frac{e}{s_\theta c_\theta} Z'_\mu [I_3 - Q s_\theta^2] + e A'_\mu Q \qquad e \equiv s_\theta g \tag{14}$$

By transforming to unprimed fields and defining e_* we may rewrite this as:

$$[1 + \Delta_Z - \Delta_A]\frac{[I_3 - Q s_\theta^2] Z_\mu e_*}{s_\theta c_\theta} + \Delta_{AZ} Q Z_\mu e_* + A_\mu Q e_* \tag{15}$$

$$e_* = [1 + \Delta_A] e \tag{16}$$

The middle term in (15) may be absorbed by defining another weak mixing angle.

$$s_* = \sin(\theta_*) \qquad c_* = \cos(\theta_*) \tag{17}$$

$$s_*^2 - s_\theta^2 = -c_\theta s_\theta \Delta_{AZ} \tag{18}$$

(15) then takes the form

$$\frac{\sqrt{Z_*}\, e_*}{s_* c_*} Z_\mu [I_3 - Q s_*^2] + e_* A_\mu Q \tag{19}$$

where Z_* is

$$Z_* = 1 + 2\,[\Delta_Z - \Delta_A] - \frac{[c_\theta^2 - s_\theta^2]\Delta_{AZ}}{c_\theta s_\theta}\ . \tag{20}$$

It is convenient to define yet another weak mixing angle[7] in terms of quantities which are well measured. ($\alpha_*^{-1} = 128.8 \pm 0.1$)

$$s_Z = \sin(\theta_Z) \qquad c_Z = \cos(\theta_Z) \tag{21}$$

$$s_Z^2 c_Z^2 \equiv \frac{\pi \alpha_*}{\sqrt{2}\, G_F m_Z^2} \tag{22}$$

On the other hand

$$s_\theta^2 c_\theta^2 = \frac{\pi \alpha}{\sqrt{2}\, G_F m_Z^{o2}} \tag{23}$$

By computing $s_Z^2 - s_\theta^2$ and using the previous result for $s_*^2 - s_\theta^2$ we find

$$s_*^2 - s_Z^2 = -c_\theta s_\theta \Delta_{AZ} - 2\frac{c_\theta^2 s_\theta^2}{c_\theta^2 - s_\theta^2}[\Delta_A - \tilde{\Delta}_Z] \tag{24}$$

Introducing s_Z into (19) yields

$$\frac{\sqrt{Z_Z}e_*}{s_Z c_Z} Z_\mu [I_3 - Q s_*^2] + e_* A_\mu Q \tag{25}$$

where

$$Z_Z = 1 + 2(\Delta_Z - \tilde{\Delta}_Z) \tag{26}$$

Another observable is $\dfrac{m_W^2}{m_Z^2 c_Z^2}$. Noting that $\dfrac{m_W^2}{m_Z^{o2} c_\theta^2} = 1$ we obtain

$$\frac{m_W^2}{c_Z^2 m_Z^2} = 1 + 2\frac{s_\theta^2 \Delta_A - c_\theta^2 \tilde{\Delta}_Z}{c_\theta^2 - s_\theta^2} \tag{27}$$

We now combine the above results. Any of the definitions of c and s may be used on the RH side of these equations.

$$s_*^2 - s_Z^2 = \frac{1}{4}\frac{\alpha}{c^2 - s^2}(S - 4c^2 s^2 T) \tag{28}$$

$$\frac{m_W^2}{c_Z^2 m_Z^2} = 1 + \frac{1}{4}\frac{\alpha}{(c^2 - s^2)s^2}(-2s^2 S + 4c^2 s^2 T + [c^2 - s^2]U) \tag{29}$$

$$Z_Z = 1 + \alpha T \equiv \rho \tag{30}$$

The corrections to various observables which depend on the Z coupling to fermions are most conveniently determined by using (25) in the form:

$$\left(2^{5/4} m_Z \sqrt{Z_Z G_F}\right) Z_\mu [I_3 - Q s_*^2] + e_* A_\mu Q \tag{31}$$

s_* is measured directly via the polarization and forward-backward Z asymmetries. We note that U dependence only appears in the m_W formula. ((28) and (29) without the U term were given in ref. [1] and [2].)

We now finally introduce the X boson. With a mass well below Λ it will introduce a number of new terms in the SU(2)×U(1) invariant chiral Lagrangian. But we need only consider the new mixing contributions to quadratic gauge field terms and we choose to work in the A-Z-X basis. We again attach primes to the fields to indicate nonstandard kinetic terms; there are two kinetic mixing terms and one mass mixing term.

$$L_{AZX} = -\frac{1}{4}A'_{\mu\nu}A'^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} - \frac{1}{4}X'_{\mu\nu}X'^{\mu\nu} + \frac{1}{2}m_Z^2 Z'_{\mu}Z'^{\mu} + \frac{1}{2}m_X^2 X'_{\mu}X'^{\mu}$$
$$+ x m_Z^2 Z'_{\mu}X'^{\mu} - y\frac{1}{2}Z'_{\mu\nu}X'^{\mu\nu} - w\frac{1}{2}A'_{\mu\nu}X'^{\mu\nu} \tag{32}$$

We find that the following transformation to unprimed fields recovers conventional kinetic terms while maintaining a diagonal mass matrix. This is true to second order in the small quantities x, y, and w.

$$\begin{pmatrix} X'_{\mu} \\ Z'_{\mu} \\ A'_{\mu} \end{pmatrix} = \begin{pmatrix} 1+\Delta_X^X & \Delta_{XZ}^X & \Delta_{XA}^X \\ \Delta_{ZX}^X & 1+\Delta_Z^X & \Delta_{ZA}^X \\ \Delta_{AX}^X & \Delta_{AZ}^X & 1+\Delta_A^X \end{pmatrix} \begin{pmatrix} X_{\mu} \\ Z_{\mu} \\ A_{\mu} \end{pmatrix} \tag{33}$$

$$\Delta_X^X = \frac{1}{2}w^2 + \frac{1}{2}\frac{(m_X^2 - 2m_Z^2)m_X^2\, y^2 + 2m_Z^4\, xy - m_Z^4\, x^2}{(m_X^2 - m_Z^2)^2} \quad \Delta_{XZ}^X = \frac{(y-x)m_Z^2}{m_X^2 - m_Z^2} \quad \Delta_{XA}^X = 0$$

$$\Delta_{ZX}^X = \frac{m_Z^2\, x - m_X^2\, y}{m_X^2 - m_Z^2} \quad \Delta_Z^X = \frac{1}{2}\frac{([m_Z^2 - 2m_X^2]\, y^2 + 2m_X^2\, xy - m_Z^2\, x^2)m_Z^2}{(m_X^2 - m_Z^2)^2} \quad \Delta_{ZA}^X = 0$$

$$\Delta_{AX}^X = -w \qquad \Delta_{AZ}^X = \frac{(x-y)\,w m_Z^2}{m_X^2 - m_Z^2} \qquad \Delta_A^X = 0 \tag{34}$$

The X and Z masses are shifted by the following amounts.

$$m_X = m_{X'}\left(1 + \tilde{\Delta}_X^X\right) \qquad m_Z = m_{Z'}\left(1 + \tilde{\Delta}_Z^X\right) \tag{35}$$

$$\tilde{\Delta}_X^X = \frac{1}{2}w^2 + \frac{1}{2}\frac{[m_Z^2 x - m_X^2 y]^2}{[m_X^2 - m_Z^2]m_X^2} \qquad \tilde{\Delta}_Z^X = -\frac{1}{2}\frac{[x-y]^2 m_Z^2}{m_X^2 - m_Z^2} \tag{36}$$

These results apply as long as the Δ's are small, ie. as long as m_X not too close to m_Z. (And we treat the case $m_X = 0$ separately below.)

We may describe some of these effects as effective shifts in S, T, and U. We note that the transformation within the Z-A subspace has the same form as in (10-11) since $\Delta_{ZA} = 0$. We thus invert our previous expressions for Δ_A, Δ_Z, Δ_{AZ}, and $\widetilde{\Delta}_Z$ ((7-9) and (13)) and solve for S, T, U (and V).

$$S = \frac{4cs}{\alpha}([s^2 - c^2]\Delta_{AZ} - 2cs\Delta_A + 2cs\Delta_Z) \tag{37}$$

$$T = \frac{2}{\alpha}(\Delta_Z - \widetilde{\Delta}_Z) \tag{38}$$

$$U = -8\frac{s^2}{\alpha}(cs\Delta_{AZ} + s^2\Delta_A + c^2\Delta_Z) \tag{39}$$

We may replace the Δ's in these expressions by $\overset{X}{\Delta}_A$, $\overset{X}{\Delta}_Z$, $\overset{X}{\Delta}_{AZ}$, and $\overset{\sim X}{\Delta}_Z$ and thus obtain the effective S, T, and U as functions of the parameters x, y, and w.

These shifts in S, T, and U are sufficient to describe how the X boson modifies the W mass and the Z asymmetries. We note that the shift in U is not necessarily small compared to the shifts in S and T, although it may be small for some ranges of parameters.

Consider next the quantity $\overset{X}{\Delta}_{XZ}$. This will modify the Z couplings to those fermions which carry X charge. If such fermions are lighter than $\frac{1}{2}m_Z$ then this will affect the Z width. This effect is linear in $x-y$, whereas all other affects we discuss are quadratic in x, y, and w. Then the Z width potentially puts a strong constraint on $x-y$. But this constraint is model dependent since it depends on the X charges of fermions.

It is of interest to consider the case that the known fermions do not carry X charge. Then the effects of an "invisible" X on the W mass, the Z asymmetries, and the partial and total Z widths are completely described by the shifts in S, T, and U.

On the other hand, for an invisible X the quantities $\overset{X}{\Delta}_{ZX}$, and $\overset{X}{\Delta}_{AX}$ are important. These imply that the X boson picks up small induced couplings to fermions proportional to their Z charge and electric charge respectively. Then X exchange will modify the standard model neutral current amplitudes. The latter takes the form

$$Z_Z\, I_3^{\text{lepton}}(I_3 - s_*^2 Q) \tag{40}$$

for neutrino and anti-neutrino deep inelastic scattering. The atomic parity violating amplitude also originates from this form, since in that case the dominant contribution arises from the axial piece of I_3^{electron}. X boson exchange contributes two additional terms to these neutral current amplitudes as long as m_X is sufficiently above the relevant energies.

$$\Delta_{ZX}{}^2\left(\frac{m_Z}{m_X}\right)^2(I_3^{\text{lepton}}[I_3 - s^2Q]) + sc\Delta_{ZX}\Delta_{AX}\left(\frac{m_Z}{m_X}\right)^2(I_3^{\text{lepton}}Q) \qquad (41)$$

These terms may thought of as effective shifts in Z_Z and s_*

$$\delta Z_Z\big|_{\text{NC}} = \Delta_{ZX}{}^2\left(\frac{m_Z}{m_X}\right)^2 \qquad (42)$$

$$\delta s_*^2\big|_{\text{NC}} = -\,sc\Delta_{ZX}\Delta_{AX}\left(\frac{m_Z}{m_X}\right)^2 \qquad (43)$$

A measurement of some observable may be translated into a band of allowed region on a S-T-U plot. (42-43) imply additional shifts in the bands from neutral current measurements. For example, we find:

$$\alpha\Delta T_{\nu\text{N}} = -0.79\ \delta s_*^2\big|_{\text{NC}} + 0.74\ \delta Z_Z\big|_{\text{NC}} \qquad (44)$$

$$\alpha\Delta T_{\bar{\nu}\text{N}} = \delta Z_Z\big|_{\text{NC}} \qquad (45)$$

$$\alpha\Delta S_{\text{atomicPV}} = 2.16\ \delta s_*^2\big|_{\text{NC}} + 0.71\ \delta Z_Z\big|_{\text{NC}} \qquad (46)$$

(We have use the table of results in ref. [8].) Thus all bands intersect on a S-T-U plot except for those bands from these three measurements (and perhaps other neutral current measurements). (44-46) gives the amount by which these bands are shifted from the point of intersection of the other bands.

With these formulae, the reader may check for himself how various quantities are shifted for various values of x, y, w and m_X. As mentioned in the introduction, a case of interest occurs if all mixing originates in a $X_{\mu\nu}B^{\mu\nu}$ term. Then $y = -\frac{s}{c}w$ and $x = 0$ and we obtain:

$$\alpha S = 4\frac{(c^2 - r^2)s^2w^2}{(r^2-1)^2} \qquad \alpha T = -\frac{r^2s^2w^2}{(r^2-1)^2c^2} \qquad \alpha U = 4\frac{s^4w^2}{(r^2-1)^2}$$

$$\delta Z_Z\big|_{\text{NC}} = \frac{r^2s^2w^2}{(r^2-1)^2c^2} \qquad \delta s_*^2\big|_{\text{NC}} = \frac{s^2w^2}{r^2-1} \qquad (47)$$

For $r \equiv \frac{m_X}{m_Z} > 1$ this gives negative S, and U is relatively suppressed. But the additional shifts in the neutral current amplitudes are quite substantial.

It is also amusing to consider the case $m_X = 0$. In this case the mass eigenstate basis does not completely determine the definition of the fields. But it is appropriate to define the photon field as the field radiated by normal matter. This requires an additional orthogonal rotation of the fields and so the above formulas do not apply for $m_X = 0$. We instead arrive at the following.

$$\Delta_Z = \frac{1}{2}y^2 \,,\ \Delta_A = \frac{1}{2}w^2 \,,\ \Delta_{AZ} = y\,w \,,\ \tilde{\Delta}_Z = \frac{1}{2}y^2 \,,$$
$$\Delta_{XZ} = -\,y \,,\ \Delta_{XA} = -\,w \,,\ \Delta_{AX} = \Delta_{ZX} = \Delta_{ZA} = 0. \tag{48}$$

We see that the induced X couplings to matter now vanish and X exchange becomes harmless. Inserting these results into the formulas for S, T, and U we obtain

$$\alpha S = 4([y^2 - w^2]cs + [s^2 - c^2]wy)cs \qquad T = 0$$
$$\alpha U = -4(s^2w^2 + c^2y^2 + 2cswy)s^2 \tag{49}$$

S and U now vanish when $y = -\frac{s}{c}w$. On the other hand for $y = 0$, S is negative and $U = \frac{s}{c}S$. In this case the net effect for all quantities, including neutral current amplitudes, is a common shift toward negative S. This may be of interest if the slight tendency[8-10] in the present data towards negative S is confirmed.

Finally we note that a technicolor theory with a technicolor gauge group containing a U(1) factor is by definition an example of an invisible gauge boson. The mixing we have been discussing may then arise through loop effects involving technifermions. For example two technielectrons with opposite X charge and masses m_1 and m_2 yields[11], up to a sign

$$w = \frac{e g_X}{6\pi^2}\ln\left(\frac{m_2}{m_1}\right) \tag{50}$$

Thus with the X coupling g_X large and with technicolor and techniflavor factors inserted, such contributions to w may well be significant (of order 0.1). And we note that the violation of weak isospin is not required for this mixing.

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References

1) B. Holdom and J. Terning, Phys. Lett. **B247** (1990) 88.

2) M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65** (1990) 964.

3) M. Golden and L. Randall, Fermilab-Pub-90/83-T (1990).

4) A. Dobado, D. Espriu, and M. Herrero, CERN-TH.5785/90 (1990).

5) D. Kennedy and B. W. Lynn, Nucl. Phys. **B322** (1989) 1; B. W. Lynn, M. E. Peskin, and R. G. Stuart, in Physics at LEP (CERN Report No. CERN 86-02, 1986)

6) J. Gasser and H. Leutwyler, Annals of Physics **155** (1984) 142;
J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 93.

7) M. E. Peskin, SLAC-PUB-5210 (1990), lectures presented at the 17th SLAC Summer Institute, 1989.

8) W. J. Marciano and J. L. Rosner, Phys. Rev. Lett. **65** (1990) 2963.

9) D. C. Kennedy and P. Langacker, Phys. Rev. Lett. **65** (1990) 2967.

10) G. Altarelli and R. Barbieri, CERN-TH.5863/90 (1990).

11) B. Holdom, Phys. Lett. **166B** (1990) 196.