



## Heavy Meson Decay Constants: $1/m$ Corrections

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### **Abstract**

The coefficients of the  $1/m$ -suppressed dimension-five operators in the static effective field theory Lagrangian have been calculated to one loop order. In this paper, we calculate to order  $\alpha_S$  the coefficients of the  $1/m$ -suppressed operators in the expansion of the vector and axial vector currents containing one light and one heavy quark field. The matrix element of the time component of the axial vector current determines the decay constants of pseudoscalar heavy-light mesons.

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## 1. Introduction

In a meson composed of one quark much heavier than the QCD scale and one much lighter, one expects that the heavy quark does not react much to the cloud of virtual light quarks and gluons which surround it. Apart from calculable short-distance corrections, one expects that the behavior of operators which create a heavy quark that is nearly at rest becomes independent of mass in the limit that the heavy quark mass becomes large. An approximation incorporating these expectations is the static approximation, which is the zeroth order approximation in a systematic expansion in the inverse of the mass,  $m$ , of a heavy quark (see reference [1] and references therein).

The conceptually clearest and computationally most efficient way to formulate the  $1/m$  expansion is in terms of an effective field theory action [2]–[7]. Once formulated this way, it is clear [2][4] that the static effective theory is conceptually very similar to the nonrelativistic effective field theory already developed for QED by Caswell and Lepage [8]. In the effective field theory formulation of the static approximation, corrections to scattering matrix elements can be systematically included by adding operators of dimension greater than four to the action. Matrix elements of operators also have a systematic expansion. Corrections come from both the  $1/m$  corrections to the effective field theory action, and  $1/m$  corrections to the operators in the effective theory [9][10].

A phenomenologically important application is the determination of decay constants of heavy-light mesons. The decay constants are defined by the matrix element between a heavy meson state and the vacuum of a weak interaction current, and the static approximation provides one way of determining these matrix elements numerically using lattice gauge theory [2][11]. The coefficients of the  $1/m$ -suppressed operators in the effective field theory expansion of this current which determines the corrections to heavy-light meson decay constants have been calculated in the leading logarithmic approximation in the papers of reference [10]. In this paper, we perform a full order  $\alpha_S$  calculation of these coefficients. Furthermore, we reduce the number of hadronic matrix elements which are needed to determine the  $1/m$  corrections to pseudoscalar heavy meson decay constants to a linear combination of two time-ordered products and a linear combination of three local operators.

## 2. Static Effective Field Theory

The degrees of freedom of a heavy quark that is nearly at rest are described by a two-component field that is a doublet under rotations,  $\varphi$ . Only two invariants of dimension four or less can be built from this field. They are  $\varphi^\dagger i\partial^0\varphi$  (from which the power counting dimension of the field is determined) and  $\varphi^\dagger\varphi$ . In Minkowski space, at zeroth order in the  $1/m$  expansion, the static effective field theory action is therefore [2][4]

$$\mathcal{L} = Z\varphi^\dagger i\mathcal{D}^0\varphi - Z\delta m\varphi^\dagger\varphi, \quad (2.1)$$

where the derivative  $i\partial^\mu$  has been replaced by the gauge-covariant derivative,  $i\mathcal{D}^\mu = i\partial^\mu + gA^\mu$ .  $Z$  is the wave function renormalization of the heavy quark field, and  $\delta m$  is the mass counterterm. A generalization of this Lagrangian valid for heavy quarks with four-velocity near an arbitrary four-velocity  $U_\mu$  is  $\mathcal{L} = Z\varphi^\dagger U_\mu i\mathcal{D}^\mu\varphi$ . This is the heavy quark piece of the Lagrangian (8) of reference [6]. For the purposes of this paper the use of a Lagrangian valid for arbitrary velocity is not necessary.

There are two dimension-five operators which incorporate the  $1/m$  corrections to the static effective theory Lagrangian, the nonrelativistic kinetic energy and the chromomagnetic moment operator:

$$O_{kin} = \frac{1}{2m}\varphi^\dagger \mathcal{D}^i \mathcal{D}^i \varphi, \quad O_{mag} = \frac{i}{2m}\varphi^\dagger \epsilon_{ijk} \mathcal{D}^i \mathcal{D}^j \sigma_k \varphi. \quad (2.2)$$

In the Minkowski space Lagrangian, these operators appear with coefficient  $Z Z_{kin}$  and  $Z Z_{mag}$  respectively. Their normalization has been chosen so that  $Z_{kin}$  and  $Z_{mag}$  are both unity at tree level.  $Z_{kin}$  and  $Z_{mag}$  have been determined at one loop order [7] with the result that

$$Z_{kin} = 1, \quad Z_{mag} = 1 + \frac{\alpha_S}{3\pi} \left( \frac{9}{4} \ln \frac{\mu^2}{m^2} + \frac{13}{2} \right). \quad (2.3)$$

The operators  $O_{kin}$  and  $O_{mag}$  are the same as the operators that appear in the nonrelativistic effective field theory. The essential difference between the static and nonrelativistic theories is that in the former, these operators are treated perturbatively; they do not modify the propagators of the heavy quark fields. Within the framework of the nonrelativistic effective theory, Lepage and Luo [2] determined the anomalous dimension of the chromomagnetic moment operator. Their calculation and the calculation of anomalous dimensions in reference [10] are in agreement with equation (2.3).

### 3. Currents to Order $1/m$ at Tree Level

At zeroth order in the  $1/m$  expansion and at tree level, the current corresponding to  $\bar{q}\gamma^0 b$ , where  $b$  is the heavy quark field in the full theory, is  $\bar{q} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi$ . The matrix appearing between the four-component light quark field,  $\bar{q}$ , and the two-component heavy quark field,  $\varphi$ , is four-by-two. The form and coefficient of the effective field theory operators are determined by matching matrix elements between an incoming heavy quark and an outgoing light quark or an incoming light antiquark. In a Dirac basis, the four-by-two matrix is easily expressible in terms of two-by-two blocks. The current corresponding to  $\bar{q}\gamma^i b$  is  $-\bar{q} \begin{pmatrix} 0 \\ \sigma_i \end{pmatrix} \varphi$ .

There are three  $1/m$ -suppressed operators which can appear in the expansion of  $\bar{q}\gamma^0 b$ . They are

$$\begin{aligned} \mathcal{O}^{(1)} &= \frac{m_l}{m} \bar{q} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi, & \mathcal{O}^{(2)} &= \frac{1}{m} \bar{q} \begin{pmatrix} 0 \\ \sigma_j \end{pmatrix} i\mathcal{D}^j \varphi, \\ \text{and} & & \mathcal{O}^{(3)} &= \frac{1}{m} \bar{q} (-i\overleftarrow{\mathcal{D}}^j) \begin{pmatrix} 0 \\ \sigma_j \end{pmatrix} \varphi. \end{aligned} \tag{3.1}$$

Any  $1/m$ -suppressed operators with these rotational and discrete symmetry transformations can be expressed in terms of these three operators to this order in  $1/m$  by using the equations of motion. It is neither necessary nor possible to determine the coefficients of additional operators which vanish by virtue of the equations of motion using the above matching procedure between on-shell states [12].

The effective field theory currents are determined by matching matrix elements between an incoming heavy quark and an outgoing light quark or incoming light antiquark. To determine the coefficients of the operators in (3.1) it is necessary to expand the full theory amplitude to order  $1/m$ . The tree level matrix element of the time component of the current between a heavy quark with spinor  $u$  and an outgoing light quark with spinor  $u'$  is  $\bar{u}'\gamma^0 u$ , which (ignoring a factor of  $\sqrt{2m}$  associated with the normalization of states) can be expanded to

$$\bar{u}' \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{p^j}{2m} \begin{pmatrix} 0 \\ \sigma_j \end{pmatrix} \right] U \tag{3.2}$$

to first order in  $1/m$ .  $U$  is the two-component spinor associated with the heavy quark, and  $p$  is its momentum. The first factor in brackets gives the zeroth order contribution noted above, and the second factor is reproduced in the effective theory by including  $-\frac{1}{2}\mathcal{O}^{(2)}$  in the  $1/m$  expansion of the current.

Similarly, there are five  $1/m$ -suppressed operators which can appear in the expansion of  $\bar{q}\gamma^i b$ . They are

$$\begin{aligned} \mathcal{O}^{(4)} &= \frac{m_l}{m} \bar{q} \begin{pmatrix} 0 \\ \sigma_i \end{pmatrix} \varphi, & \mathcal{O}^{(5)} &= \frac{1}{m} \bar{q} \begin{pmatrix} 1 \\ 0 \end{pmatrix} i\mathcal{D}^i \varphi, & \mathcal{O}^{(6)} &= \frac{1}{m} \bar{q} (-i\overleftarrow{\mathcal{D}}^i) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi, \\ \mathcal{O}^{(7)} &= \frac{1}{m} \bar{q} \begin{pmatrix} \sigma_i \sigma_j \\ 0 \end{pmatrix} i\mathcal{D}^j \varphi, & \text{and} & & \mathcal{O}^{(8)} &= \frac{1}{m} \bar{q} (-i\overleftarrow{\mathcal{D}}^j) \begin{pmatrix} \sigma_j \sigma_i \\ 0 \end{pmatrix} \varphi. \end{aligned} \quad (3.3)$$

Expanding the tree level amplitude involving the full theory current, one finds that the current in the effective theory must contain  $\frac{1}{2} \mathcal{O}^{(7)}$  at order  $1/m$ .

#### 4. Currents at One Loop

In this section the coefficients of the operators in the expansion of the vector and axial vector currents are determined to one loop order. This calculation for the part of the current that appears at zeroth order in the  $1/m$  expansion was performed in references [4] and [13]. The result is that the coefficient of  $\bar{q} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi$  in  $\bar{q}\gamma_0 b$  is

$$1 + \frac{\alpha_S}{3\pi} \left( -2 - \frac{3}{2} \ln \frac{\mu^2}{m^2} \right). \quad (4.1)$$

The coefficient of the logarithm is in agreement with the calculations of references [3] and [14]. The coefficient of  $-\bar{q} \begin{pmatrix} 0 \\ \sigma_i \end{pmatrix} \varphi$  in  $\bar{q}\gamma^i b$  is,

$$1 + \frac{\alpha_S}{3\pi} \left( -4 - \frac{3}{2} \ln \frac{\mu^2}{m^2} \right). \quad (4.2)$$

The difference of the amplitudes in the two theories is a function of the three-momentum of the heavy quark,  $p^i$ , the three-momentum of the light quark,  $q^i$ , and the mass of the light quark,  $m_l$  (and of the renormalization point). As stated in the previous section, the procedure for determining the tree level coefficients was to expand the tree level amplitudes to order  $1/m$ . In order to determine the coefficients of the operators in (3.1) and (3.3) to order  $\alpha_S$ , the difference of the one-loop amplitudes used to obtain (4.1) and (4.2) must similarly be recalculated and expanded retaining terms of first order in  $p^i/m$ ,  $q^i/m$  and  $m_l/m$ .

The amplitudes matched to obtain (4.1) and (4.2) were logarithmically infrared divergent, and an infrared regulator was introduced to make them finite. Once we expand these amplitudes to first order in  $1/m$  they will be linearly and logarithmically infrared divergent. Of course, the effective theory is constructed to

reproduce the low energy behavior of the full theory, and the infrared divergences in the matching of corresponding amplitudes in the two theories will cancel. Indeed, if we use dimensional regularization to regulate the infrared as well as the ultraviolet divergences, we are even free to apply  $\overline{\text{MS}}$  to the infrared poles so long as we apply the same procedure to these poles in the effective theory.

Performing the expansion in the effective theory with dimensional regularization as both the ultraviolet and infrared regulator leaves no remaining scale in the integrand, and this results in the remarkable simplification that all one-loop integrals in the effective theory vanish. This general argument allows us to postpone further evaluation of the graphs that appear in the effective theory to the following section, where it will be necessary to separate the ultraviolet and infrared divergences.

The expansion of the full theory amplitude for the matrix element of the vector current between an incoming light quark and an outgoing heavy quark can be expressed in terms of the following basis to first order in  $1/m$ :

$$\begin{aligned}
A_1 &= \bar{u}' \gamma^\alpha u, & A_2 &= \frac{1}{m} p^\alpha \bar{u}' u, & A_3 &= \frac{1}{m} q^\alpha \bar{u}' u, & A_4 &= \frac{m_l}{m} \bar{u}' \gamma^\alpha u, \\
A_5 &= \frac{m_l}{m^2} p^\alpha \bar{u}' u, & A_6 &= \frac{p \cdot q}{m^2} \bar{u}' \gamma^\alpha u, & \text{and} & & A_7 &= \frac{p \cdot q}{m^3} p^\alpha \bar{u}' u.
\end{aligned} \tag{4.3}$$

Since  $p^0 = m + \mathcal{O}(1/m)$ , the terms in the basis with factors of  $p^\alpha$  when  $\alpha = 0$  or  $p \cdot q$ , or both, are not as suppressed as they appear; we have not computed the amplitude to third order in the  $1/m$  expansion.

Denoting the coefficient of  $A_i$  as  $a_i$ , etc., we find that the one-loop amplitude is given by

$$\begin{aligned}
a_1 &= 1 + \frac{1}{2} \frac{\alpha_S}{3\pi} (-4 - 3 \ln \frac{\mu^2}{m^2}) + \frac{\alpha_S}{3\pi} (-2), & a_2 &= \frac{\alpha_S}{3\pi} 2, \\
a_3 &= \frac{\alpha_S}{3\pi} (\ln \frac{\mu^2}{m^2} + 2), & a_4 &= \frac{\alpha_S}{3\pi} (-2), & a_5 &= \frac{\alpha_S}{3\pi} (-3 \ln \frac{\mu^2}{m^2} - 2), \\
a_6 &= \frac{\alpha_S}{3\pi} (-3 \ln \frac{\mu^2}{m^2} - 2), & \text{and} & & a_7 &= \frac{\alpha_S}{3\pi} (2 \ln \frac{\mu^2}{m^2} + 6).
\end{aligned} \tag{4.4}$$

The second term in  $a_1$  results from the wave function renormalization of the heavy quark and the third results from the vertex correction graph.

Expanding the amplitudes in (4.3) in exactly the same fashion as in equation (3.2) we find that the order  $1/m$  contributions to the effective theory current for

$\bar{q}\gamma^0 b$  are

$$(a_3 + a_4 + a_5 + a_6 + a_7)\mathcal{O}^{(1)} + \frac{a_2 - a_1}{2}\mathcal{O}^{(2)} - (a_3 + a_6 + a_7)\mathcal{O}^{(3)}, \quad (4.5)$$

and that the  $1/m$  contributions to the effective theory current for  $\bar{q}\gamma^i b$  are

$$(a_6 - a_4)\mathcal{O}^{(4)} + a_2\mathcal{O}^{(5)} + a_3\mathcal{O}^{(6)} + \frac{a_1}{2}\mathcal{O}^{(7)} + a_6\mathcal{O}^{(8)}. \quad (4.6)$$

The results for  $\bar{q}\gamma_5\gamma^0 b$  and  $\bar{q}\gamma_5\gamma^i b$  are obtained simply by exchanging the upper and lower two-by-two blocks in the four-by-two matrices and sending  $m_l$  to minus itself where it appears in (3.1), (3.3) and (4.3).

## 5. Anomalous Dimensions

In the one-loop corrections to the weak decay matrix element, there are corrections proportional to  $\log ma$ , where  $a^{-1}$  is the scale at which the hadronic matrix element is evaluated. If the matrix elements are computed using lattice gauge theory, then  $a$  is the lattice spacing, which at the present time is typically about  $(2 \text{ GeV})^{-1}$ . In this case the logarithm is not large for either the bottom or charm quarks, and it would be inconsistent to ignore the non-logarithmic terms. However, if one imagines that there is some method for evaluating the hadronic matrix elements at a scale nearer the QCD scale, then it is useful to sum the leading logarithms using the renormalization group in the effective theory [9] as was done in reference [10]. In that work, the mixing of twelve weak current operators carrying a four-vector index with a set of time-ordered products involving three flavor-conserving operators was considered. In this section, we derive the mixing of the local operators in equations (3.1) and (3.3) with the time-ordered products involving the dimension-five operators in equation (2.2), which give the  $1/m$  corrections to heavy meson decay constants, and make a comparison with the results of reference [10].

The matching calculation of the previous section was performed using dimensional regularization to treat the infrared as well as the ultraviolet divergences, which greatly simplifies the calculations, but obscures the high energy behavior of the diagrams. Thus, to obtain the anomalous dimensions, we need to recalculate the amplitudes in the full theory and the effective theory which were used to determine the coefficients of the  $1/m$ -suppressed operators using a different infrared regulator. We have chosen to use a gluon mass, which is acceptable since all the diagrams we have to calculate are QED-like.

In the effective theory, there are two types of diagrams which need to be computed: there are diagrams for the QCD corrections to the time-ordered products of  $O_{kin}$  and  $O_{mag}$  with the zeroth order term of the current, and there are diagrams for the QCD corrections to the local operators which appear in the expansion of the current [10]. Using the method for handling the non-covariant poles presented in reference [4] the computation of the various diagrams is straightforward. The results from these one-loop computations are inserted into the renormalization group equation to obtain the anomalous dimensions of the operators which can then be used extend the range of validity of the perturbative results.

A byproduct of a full order  $\alpha_S$  calculation of the effective theory diagrams using a gluon mass as an infrared regulator is that it permits a check of the matching performed in the preceding section. The computation of the full theory diagram with the gluon mass regulator and decomposition into the amplitude basis (4.3) is straightforward. The results of this matching reproduce the linear combinations of local operators given in equations (4.5) and (4.6).

We first review the anomalous dimensions of the two dimension-five operators in the Lagrangian. They run independently of each other because they have different transformation properties under the spin symmetry acting only on the heavy quark field [2][15]. The anomalous dimension matrix for  $Z_{kin}$  and  $Z_{mag}$  is thus contained in equation (2.3), and the renormalization group for these coefficients is

$$\left\{ \mu \frac{d}{d\mu} + \gamma_{fc} \right\} \begin{pmatrix} Z_{kin} \\ Z_{mag} \end{pmatrix} = 0, \quad \text{where} \quad \gamma_{fc} = \frac{\alpha_S}{3\pi} \begin{pmatrix} 0 & 0 \\ 0 & -9/2 \end{pmatrix}. \quad (5.1)$$

Assemble the coefficients of the weak decay operators into a row vector,  $C(\mu)$ , and denote the coefficient of the zeroth order in  $1/m$  weak decay operator by  $c_0(\mu)$ . The coefficient  $c_0(\mu)$  was given to one-loop order in equations (4.1) and (4.2) for the cases of the rotational scalar and rotational vector operators respectively. In either case, at this order it satisfies  $(\mu \frac{d}{d\mu} + \gamma)c_0=0$ , where  $\gamma = \frac{\alpha_S}{3\pi} 3$ . The running of  $C$  in the effective theory is given by [10]

$$\mu \frac{d}{d\mu} C + \gamma_w^T C + \gamma_{mix}^T c_0 \begin{pmatrix} Z_{kin} \\ Z_{mag} \end{pmatrix} = 0. \quad (5.2)$$

Here  $\gamma_w$  is the anomalous dimension matrix of the weak decay operators, and  $\gamma_{mix}$  is the matrix which mixes the time-ordered products with them.

The anomalous dimension matrices,  $\gamma_w$ , include a contribution to compensate for the running of  $m_l$  which appears explicitly in  $\mathcal{O}^{(1)}$  and  $\mathcal{O}^{(4)}$ . The running mass satisfies  $(\mu \frac{d}{d\mu} + \gamma_m)m_l=0$ , where  $\gamma_m = \frac{\alpha_S}{3\pi}6$ . The anomalous dimension matrices are then

$$\gamma_w = \frac{\alpha_S}{3\pi} \begin{pmatrix} -3 & 0 & 0 \\ -6 & 3 & 0 \\ -6 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \gamma_{mix} = \frac{\alpha_S}{3\pi} \begin{pmatrix} 6 & 0 & -4 \\ -3 & 0 & 4 \end{pmatrix} \quad (5.3)$$

for the rotational scalar operators, and

$$\gamma_w = \frac{\alpha_S}{3\pi} \begin{pmatrix} -3 & 0 & 0 & 0 & 0 \\ 2 & 3 & -3 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ -2 & 0 & -6 & 3 & 2 \\ 6 & 0 & 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \gamma_{mix} = \frac{\alpha_S}{3\pi} \begin{pmatrix} 6 & 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix} \quad (5.4)$$

for the rotational vectors.

For comparison with the results of reference [10], one makes a linear transformation on their three flavor-conserving operators so that the basis is  $O_{kin}$ ,  $O_{mag}$ , and an operator which is proportional to the zeroth order heavy quark equations of motion and therefore vanishes to this order in  $1/m$  [12]. Similarly, one makes a linear transformation on their twelve weak current operators, changing the basis to one in which there are three currents whose zeroth components are the rotational scalars in (3.1) and whose vector components vanish, five operators whose vector components are the rotational vectors in (3.3) and whose zeroth component vanishes, and four currents which are proportional to the light or heavy quark equations of motion. When the linear transformations are applied to equations (8), (9) and (10) of reference [10], the anomalous dimensions agree with equations (5.1), (5.3) and (5.4).

## 6. Conclusions

The coefficients of the  $1/m$ -suppressed operators in the effective field theory expansion of the vector and axial vector currents involving a heavy quark and a light quark have been determined to one loop order. The  $1/m$  corrections to heavy meson decay constants are determined by the matrix elements of a linear combination of two time-ordered products, and from a linear combination of three local operators given in equation (4.5).

We expect that  $1/m$  corrections are of practical importance in  $D$  and  $B$  meson systems. They nicely account for the ratio of  $D^*-D$  to  $B^*-B$  splitting. Solely on dimensional grounds, one might expect that in  $B$  mesons  $1/m$  corrections to quantities like  $f_B$  would be about 5% and in  $D$  mesons they could be of order 15%. In this case, they could account for a significant part of the discrepancy between  $D$  and  $B$  meson decay constants which have been measured by two different lattice gauge theory methods (see reference [16] for a recent review). On the other hand, one might expect the corrections to be smaller, suppressed by a factor of the velocity of the light quark, in a picture based on the non-relativistic quark model.

A promising method by which the matrix elements of the  $1/m$ -suppressed operators can be evaluated is lattice gauge theory. For this purpose the full order  $\alpha_S$  matching of the full theory to the static effective theory performed here is consistent and necessary, since the logarithms of the mass of the heavy quark over the scale at which these matrix elements will be evaluated are not large. A lattice determination of these matrix elements would require a choice of discretization of the dimension-five operators appearing in the time-ordered products and of the three local operators in equation (3.1). Further perturbative computations which are relevant to this program are the computation of the matching of these discretized operators to their continuum counterparts, and, for the eventual precision determination of heavy meson decay constants, that part of a two-loop computation in the continuum effective theory which would allow one to extend this full order  $\alpha_S$  computation into a renormalization group improved next-to-leading-logarithmic computation.

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