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MICROWAVE DISTORTIONS FROM COLLAPSING DOMAIN-WALL BUBBLES

MICHAEL S. TURNER AND RICHARD WATKINS

Department of Astronomy & Astrophysics
Enrico Fermi Institute
The University of Chicago

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory

LAWRENCE M. WIDROW

Department of Physics, Harvard University
Harvard-Smithsonian Center for Astrophysics

Canadian Institute for Theoretical Astrophysics
University of Toronto

ABSTRACT: It has been suggested that large-scale structure can be seeded by a post-recombination phase transition that produces soft domain walls. We find that oscillating domain-wall bubbles produce a distinctive signature on the microwave sky: hot and cold spots with amplitude characterized by $G\sigma H_0^{-1}$ (σ is the surface tension of the wall). These fluctuations are non-gaussian and offer a powerful probe of such models.

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Heavy walls have long been known to be a cosmological disaster (see. e.g., Zel'dovich et al., 1974; Vilenkin, 1984). However, very light (soft) domain walls have recently been invoked to explain large-scale structures in the Universe (Hill, Fry, and Schramm, 1988). The proponents of this scenario argue that soft domain walls can seed large-scale structure while producing only small distortions of the microwave background provided that the walls form after decoupling so that the energy density on the surface of last scattering can be arbitrarily smooth. This eliminates the Sachs–Wolfe (1967) distortions that usually provide the strongest constraint to models of structure formation. Distortions in the microwave background will, however, arise from three other sources: 1) the gravitational effects of infinite walls (Stebbins and Turner, 1989); 2) the gravitational effects of collapsing closed surfaces of domain wall (hereafter, vacuum bags or bubbles); and 3) the Rees–Sciama (1968) effect associated with matter that is accreting around vacuum bags (or walls).

The last of these effects is expected to give rise to very small temperature fluctuations, $\delta T/T \sim (v/c)^3 \lesssim 10^{-6}$, where v is the virial velocity of the bound clumps that form (Hill, Fry, and Schramm, 1988). The gravitational field of an infinite wall is nonlocal and difficult to treat; however, one expects that the microwave distortions associated with an infinite wall will be $\delta T/T \sim G\sigma\xi_{\rm corr}$, where $\xi_{\rm corr}$ is the correlation length for the wall network and σ is the wall surface tension (Stebbins and Turner, 1989). Today, one expects that ξ_{corr} is of order the Hubble radius H_0^{-1}, so $\delta T/T \sim G\sigma H_0^{-1} \simeq 3\times 10^{-6}\,(h^{-1}\sigma/{\rm MeV}^3)$ ($H_0 = 100h\,{\rm km\,sec^{-1}\,Mpc^{-1}}$ is the present value of the Hubble constant). These distortions arise on large angular scales ($\gtrsim 10°$) and current limits to the isotropy of the microwave background constrain σ to be less than about $100\,{\rm MeV}^3$ (Stebbins and Turner, 1989).

In this *Letter* we call attention to the fact that collapsing vacuum bubbles lead to a pattern of temperature fluctuations that is very distinctive and different from those arising from infinite domain walls or the Rees–Sciama effect. These distortions appear as small ($\lesssim 1°$) hot (or blue) and cold (or red) spots on the microwave sky. They arise when photons from the last scattering surface traverse a collapsing (or expanding) vacuum bubble because the gravitational blue shift that a photon experiences entering the bubble is not equal to the red shift it experiences exiting the bubble. Vacuum bubbles are formed during the phase transition and subsequent evolution of the domain-wall network (Vilenkin, 1984; Press, Ryden, and Spergel, 1989). Bubbles that are larger than the Hubble radius are conformally stretched with the expansion, whereas sub-Hubble-sized bubbles collapse relativistically due to surface tension. During the initial stages of collapse irregularly shaped bubbles become more spherical; in the final stages of collapse, the bubble walls move nearly at the speed of light and any remaining irregularities are frozen in (Widrow, 1989a). Recent numerical simulations (Press, Ryden, and Spergel, 1989) indicate that spherical, sub-Hubble-sized bubbles are a robust feature of a domain-wall network.

To begin we calculate the frequency shift for a photon traversing a collapsing, spherical bubble in Minkowski space. We neglect the expansion of the Universe, the small effect of matter falling in upon the vacuum bubble, and the bubble's self gravity. This should be a reasonable approximation for the vacuum bubbles of interest—those with radius much smaller than the Hubble radius and much greater than their Schwarzschild radius. A spherical bubble is characterized by its mass M; the maximum radius of the bubble is $R_{\rm max} \equiv \sqrt{M/4\pi\sigma}$. Space-time is Schwarzschild outside the bubble and flat inside. (We neglect the finite thickness of the bubble wall.) Consider a photon that originates far from a bubble and reaches its surface when the bubble radius is $R(t_{\rm in})$. The photon exits sometime later when the bubble radius $R(t_{\rm out}) < R(t_{\rm in})$ is smaller. The photon is blue shifted by an amount $GM/R(t_{\rm in})$ in reaching the bubble wall, is unaffected as it traverses the inside of the bubble, and is red shifted by an amount $GM/R(t_{\rm out})$ as it travels far from the bubble. The net shift in the photon frequency is

$$\frac{\Delta\nu}{\nu} = \left(\frac{R_{\rm max}}{R(t_{\rm in})} - \frac{R_{\rm max}}{R(t_{\rm out})}\right)\Delta_0 \equiv \mathcal{A}\Delta_0, \tag{1}$$

which results in a temperature shift $\delta T/T = \Delta\nu/\nu$; here $\Delta_0 \equiv GM/R_{\rm max} = 4\pi G\sigma R_{\rm max}$. The quantity Δ_0 corresponds to the characteristic microwave distortion for a domain-wall network with correlation length $R_{\rm max}$; the dimensionless amplitude $\mathcal{A}$ represents the enhancement due to the fact that the bubble is collapsing and can be very much smaller than its initial size when the photon exits.

For simplicity, we consider an infinitely thin, spherical bubble that collapses once. (We leave discussion of the post-collapse fate of the bubble for later.) The equation of motion for a collapsing bubble in Minkowski space is (Ipser and Sikivie, 1984)

$$\ddot{R} = -2\frac{1-\dot{R}^2}{R}, \tag{2}$$

where an overdot denotes d/dt. Integrating Eq. (2) it follows that

$$t = R_{\rm max}\int_1^{R(t)/R_{\rm max}} \frac{dw}{\left[1-w^4\right]^{1/2}}, \tag{3}$$

where we have assumed that the bubble starts its collapse at time $t=0$. The speed of the bubble's surface ($=\dot{R}$) starts from zero and quickly approaches the speed of light. The total time for the vacuum bag to collapse is $\tau \simeq 1.3 R_{\rm max}$.

A photon traversing a collapsing vacuum bubble is characterized by its impact parameter b, the closest distance it passes from the center of the bubble, and d, the perpendicular distance from the photon to the plane of the bubble's center at time $t = 0$. At time t, the distance of the photon from the center of the bubble is $R_\gamma(t) = \sqrt{b^2 + (d-t)^2}$. By equating $R_\gamma(t)$ to the position of the bubble's surface, we can solve for $R(t_{\rm in})$ and $R(t_{\rm out})$ and determine $\delta T/T$. For photons that enter the bubble prior to time $t = 0$ we take $R(t_{\rm in}) = R_{\rm max}$, avoiding discussion of the evolution of the bubble prior to its collapse. In Fig. 1 we show the microwave distortion profiles for spots with different values of $d/R_{\rm max}$.

We can use the fact that the bubble surface quickly approaches the speed of light to find a simple analytic form for $\mathcal{A}$ as a function of d for photons that pierce the center of the bubble (i.e., with $b = 0$); for these photons: $-R_{\rm max} < d < \tau$. If we assume that $R(t_{\rm in}) \simeq R_{\rm max}$ and that the radius of the bubble at time t is given by $(\tau - t)$, it follows that

$$\mathcal{A}(d, b=0) = \frac{2R_{\rm max}}{\tau - d} - 1 \ . \tag{4}$$

This agrees well with our numerical results, except for the region around $d = -R_{\rm max}$. Since results for this regime are suspect anyway, we will be content to use this analytic form in the rest of our analysis.

From Eq. (4) we calculate the probability that a spot produced by a bubble of initial size $R_{\rm max}$ has a temperature shift greater than $(GM\sigma/R_{\rm max})\mathcal{A}_0$ at its center:

$$P(\mathcal{A} > \mathcal{A}_0) = \frac{1}{\mathcal{A}_0 + 1}. \tag{5}$$

To determine the distribution of spots on the sky as a function of angular size and temperature fluctuation amplitude we consider a spatially-flat, matter-dominated Universe with cosmological scale factor $a(t) = (1+z)^{-1} = (t/t_0)^{2/3}$, where z is the red shift corresponding to time t and $t_0 = 2H_0^{-1}/3$ is the present age of the Universe. Based upon the "scaling" solution found in the numerical simulations of Press, Ryden, and Spergel (1989), we assume that both the number of vacuum bags that form in a Hubble volume in a Hubble time and the size of the typical bubble formed (relative to the Hubble radius) are constant. Quantitatively, we write $dn = \alpha dt/t^4$, where dn is the number density of bubbles of initial size $R_{\rm max} = \beta t$ and lifetime $\tau = \gamma\beta t$ that form between time t and $t + dt$. The quantities α, β, and γ are dimensionless parameters; based on the simulations of Press, Ryden, and Spergel (1989) α and β are expected to be $\mathcal{O}(0.1 - 1)$, while γ is expected to be slightly larger that unity. The number of spots on the sky due to bubbles that form at time t is $dN = Vdn$, where V is the volume of space containing the bubbles that lie in our past light cone. It is easy to see that V is a spherical shell centered about us whose comoving radius $r(t) = 2H_0^{-1}[1-(1+z)^{-1/2}]$ with thickness $(1+\gamma)\beta t$, so that

$$dN = 54\pi\alpha\beta(1+\gamma)\frac{2+z-2(1+z)^{1/2}}{1+z}dz. \tag{6}$$

In Eq. (1) we defined $\mathcal{A}$ as the frequency shift relative to the quantity $GM/R_{\rm max}$. However, $R_{\rm max}$ is epoch dependent. It is convenient to define the scaled quantity $\tilde{\mathcal{A}} \equiv \mathcal{A}t/t_0$ so that the temperature shift due to any bubble is measured relative to the characteristic amplitude $8\pi\beta G\sigma H_0^{-1}/3$,

$$\frac{\delta T}{T} \equiv \left(\frac{8\pi}{3}\beta G\sigma H_0^{-1}\right)\tilde{\mathcal{A}} = 2.64\times10^{-4}\,h^{-1}\,\beta\left(\frac{\sigma}{10\,{\rm MeV}^3}\right)\tilde{\mathcal{A}}. \tag{7}$$

In terms of $\tilde{\mathcal{A}}$, $P(\mathcal{A} > \mathcal{A}_0)$ is: $P(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0) = 1/\left[1+(1+z)^{3/2}\tilde{\mathcal{A}}_0\right]$. The total number of spots on the sky with amplitude $\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0$ is given by

$$\begin{aligned}
N(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0) &= \int_0^{z_F} P(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0)dN \\
&= 18\pi\alpha\beta(1+\gamma)\Bigg\{\left(\tilde{\mathcal{A}}_0{}^{-2/3} + 2\tilde{\mathcal{A}}_0{}^{-1/3}\right)\ln\left[\frac{(\tilde{\mathcal{A}}_0{}^{1/3}+1)^2(\tilde{\mathcal{A}}_0{}^{2/3} - a_F^{1/2}\tilde{\mathcal{A}}_0{}^{1/3} + a_F)}{(\tilde{\mathcal{A}}_0{}^{2/3} - \tilde{\mathcal{A}}_0{}^{1/3}+1)(\tilde{\mathcal{A}}_0{}^{1/3} + a_F^{1/2})^2}\right] \\
&+ 2\sqrt{3}\left(\tilde{\mathcal{A}}_0{}^{-2/3} - 2\tilde{\mathcal{A}}_0{}^{-1/3}\right)\left[\tan^{-1}\left(\frac{2\tilde{\mathcal{A}}_0{}^{1/3} - a_F^{1/2}}{\sqrt{3a_F}}\right) - \tan^{-1}\left(\frac{2\tilde{\mathcal{A}}_0{}^{1/3}-1}{\sqrt{3}}\right)\right] \\
&+ 2\ln\left(\frac{1+\tilde{\mathcal{A}}_0}{a_F^{3/2}+\tilde{\mathcal{A}}_0}\right)\Bigg\},
\end{aligned} \tag{8}$$

where z_F and $a_F = (1+z_F)^{-1}$ are the red shift and scale factor at the time of the phase transition. (Hill, Fry, and Schramm (1989) argue that z_F should be be between 100 and 1000.) In the limit $\tilde{\mathcal{A}}_0 \gg 1$, $N(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0) \to 36\pi\alpha\beta(1+\gamma)/\tilde{\mathcal{A}}_0$; that is, the number of spots decreases only as the inverse of the spot amplitude, indicating that these microwave distortions are non-gaussian both because of their distinctive shape and the distribution of spot amplitudes.

Now we determine the distribution of spots as a function of both amplitude and angular size. A vacuum bubble of physical radius R formed at time t subtends an angle $\theta = 2R/a(t)r(t)$ on the sky. Taking R to be the size of the bubble when the photons exit—which determines the spot size—then the angular size is related to $\tilde{\mathcal{A}}$ and z by, $\theta(z,\tilde{\mathcal{A}}) \simeq$

$2\beta/3\tilde{\mathcal{A}}(1+z)^2[1-(1+z)^{-1/2}]$; as expected, for fixed z the brighter spots have smaller angular size. If a spot formed at red shift z is to have an angular size $\theta > \theta_0$, then necessarily $\tilde{\mathcal{A}} < \tilde{\mathcal{A}}_{\rm max}(z)$, where $\tilde{\mathcal{A}}_{\rm max}(z) = 2\beta/3\theta_0(1+z)^2[1-(1+z)^{-1/2}]$. Therefore, the number of spots with $\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0$ and $\theta > \theta_0$ is

$$N(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0, \theta > \theta_0) = \int_0^{\min(z_c, z_F)} \left[P(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0) - P(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_{\rm max})\right] dN, \tag{9}$$

where $\tilde{\mathcal{A}}_{\rm max}(z_c) = \tilde{\mathcal{A}}_0$ defines z_c. The first term is given in Eq. (5), while the second is

$$\int_0^z P(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_c) dN = 108\pi\alpha\beta(\gamma+1)\left[\frac{1}{2a} - \frac{(k+2)}{\sqrt{a}}\right.$$

$$\left. +(k^2+k+1)\ln\left(\frac{1+\sqrt{a}(k-1)}{\sqrt{a}k}\right) + \frac{1}{k-1}\ln\left(\frac{1+\sqrt{a}(k-1)}{k}\right) + k + \frac{3}{2}\right], \tag{10}$$

where $k = 2\beta/3\theta_0$. A contour plot of $N(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0, \theta > \theta_0)$ is shown in Fig 2.

Thus far we have considered only the collapse phase of a bubble. The fate of the bubble once it has collapsed depends on the thickness of the wall, the underlying particle physics model, and on the asphericity of the bubble. In models where the scalar potential is given by $V(\phi) = -m^2\phi^2 + \lambda\phi^4$, the energy of the bubble is dissipated during the initial collapse in an outgoing pulse of scalar particles. On the other hand, sine-Gordon vacuum bubbles "bounce" (Widrow, 1989b): The collapsing bubble passes through itself, the bubble expands to a smaller maximum radius (because some of its energy is radiated in scalar particles), and so on. Photons traversing either an outgoing pulse of radiation or an expanding bubble experience a net blue shift. We therefore expect hot as well as cold spots on the sky, though their shapes and amplitudes may be different. The number of hot and cold spots should be about equal unless collapse of a bubble to a black hole is common; if most bubbles suffer this fate (on the first bounce) blue spots will be rare.

Recent experiments have set strong limits to the isotropy of the microwave background. RELICT has mapped 75% of the sky and finds no fluctuations with $\delta T/T \gtrsim 6\times10^{-5}$ on angular scales $\gtrsim 3°$ (Klypin et al., 1988; Starobinskii, 1990). Preliminary COBE results (Smoot, 1990) provide the weaker limit of 3×10^{-4} on scales of a few degrees. By demanding that there be less than one spot of size 3° and amplitude 6×10^{-5} one can place a limit to the surface tension σ: For the plausible values $\alpha = 0.5$, $\beta = 0.2$, and $\gamma = 1$, it is $\sigma \lesssim 10\,\mathrm{MeV}^3$. In the original model of Hill, Fry, and Schramm (1989) $\sigma \sim 30\,\mathrm{MeV}^3$.

Cosmic textures (Turok, 1989), recently been proposed as another topological mechanism for seeding the large-scale structure, also give rise to hot and cold spots on the sky, though in this case there is a characteristic temperature shift that is set by particle physics (Turok and Spergel, 1990). The distribution of spots as a function of amplitude is therefore very different than that for vacuum bubbles, and one should be able to easily differentiate between the two models if spot-like distortions on the sky are detected.

In sum, a domain-wall network formed in a post-recombination phase transition gives rise to microwave fluctuations through the gravitational effects of infinite walls, the Rees–Sciama effect for accreting matter, and the collapse of vacuum bubbles. The distortions that result from collapsing bubbles are much larger and more important than those from the Rees–Sciama effect and are of comparable or larger amplitude, but smaller angular size, than those from infinite walls. They have a characteristic signature that should permit stringent tests of the late-time phase-transition scenario soon.

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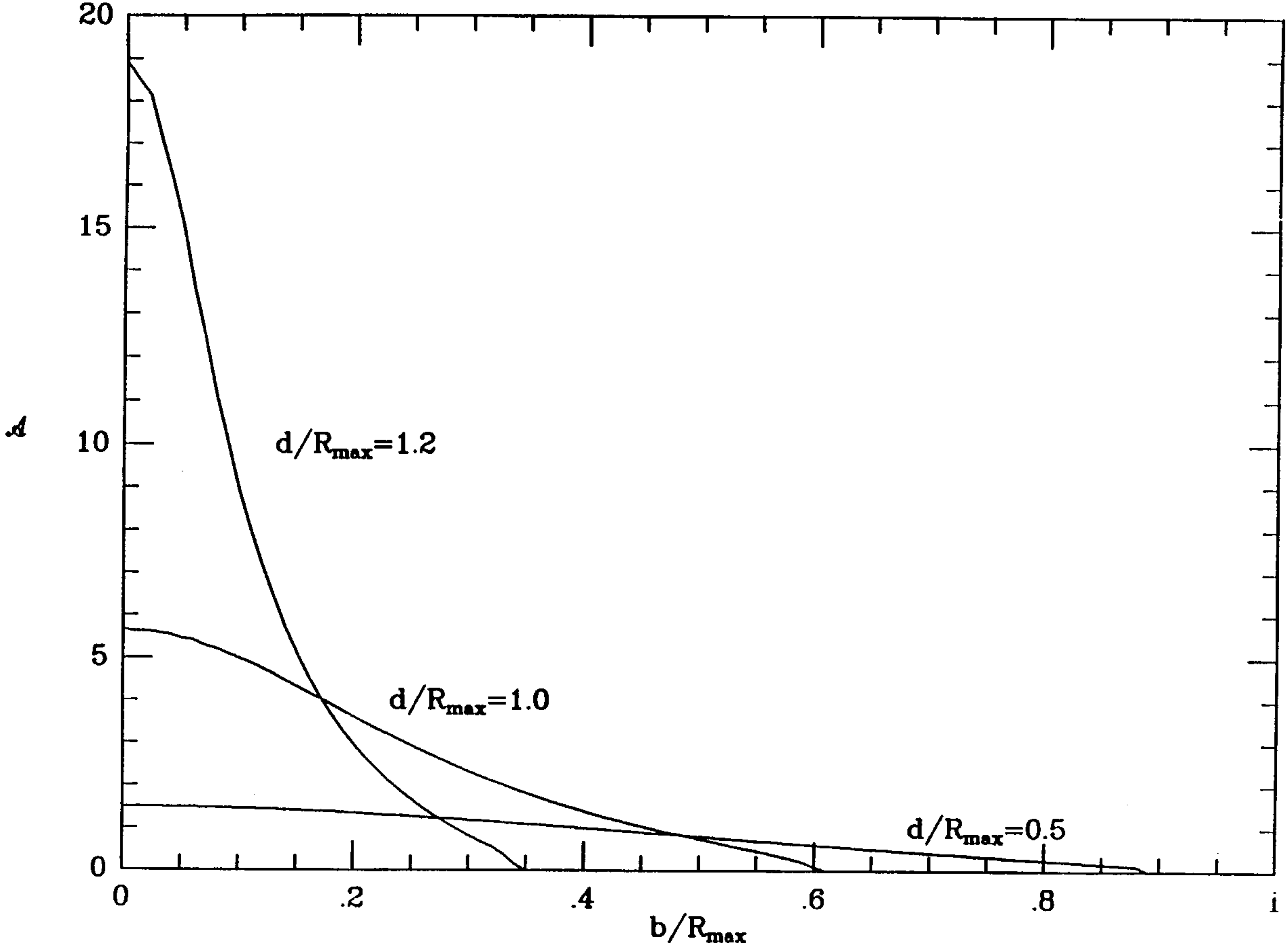
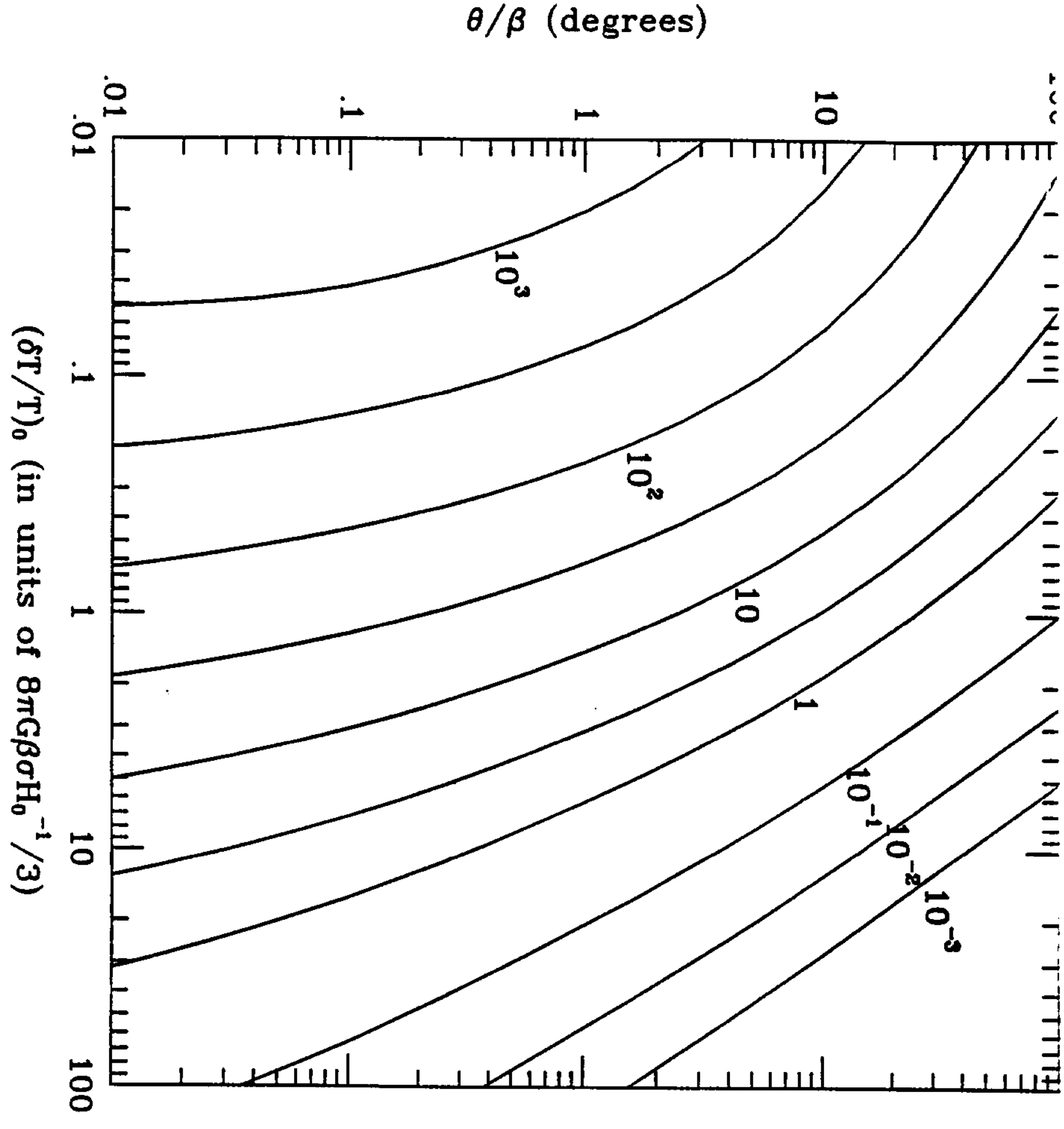
FIGURE CAPTIONS

Fig. 1: Profiles of the distortion amplitude $\mathcal{A}$ versus impact parameter b for spots with various values of $d/R_{\max}$.

Fig. 2: Contours of the number of spots on the sky, in units of $\alpha\beta(1+\gamma)$, with central amplitude $\delta T/T$ greater than $(\delta T/T)_0$ (in units of $8\pi G\beta H_0^{-1}\sigma/3$) and angular size greater than θ_0 (in units of β degrees): $N(\tilde{\mathcal{A}} > \tilde{\mathcal{A}}_0, \theta > \theta_0)/\alpha\beta(1+\gamma)$ for $z_F = 100$.



- FIG 1 -



θ/β (degrees)
(δT/T)₀ (in units of 8πGβσH₀⁻¹/3)
.01
.1
1
10
100
10³
10²
10
1
10⁻¹
10⁻²
10⁻³

- FIG 2 -