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Comment on the Calculations for Some Bound State Annihilation and Creation [1]

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Abstract

Respected to a process, involving annihilation or creation of a (quark-pair) bound state through two or more points of elementary interaction(s), we point

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out that the bound state effects in such a process need to be treated more precisely due to certain propagator(s) involved, rather than so crudely, to attribute the effects as a proportional factor of the wavefunction at origin of the bound state into the amplitude of the process finally.

When calculating a process involving one bound state annhilating into two or more elementary particles (or reverse), an approximation is made very common, that as final result, the amplitude of the concerning process will become proportional to the bound state wavefunction at origin in references[1,2,3]. However in this paper we will point out that this approximation is too crude, i.e., sometimes it may make a quite big deviation (sometimes bigger than fifty percent, even more, from which by more careful dealing with the involved bound state effects). In order to illustrate the fact, we will take $Z^0 \to \eta_b\gamma$ (or $\eta_c\gamma$) and $\Upsilon \to$ Higgs γ as examples, because the heavy quarkonia η_b, η_c and Υ etc. are investigated well and tested widely, i.e., we know the potential framework works very well for the heavy quark pair systems, moreover, the Z^0 decay processes may be accessible when accumulating enough Z^0 events in principal, and Z^0 is heavy enough for decaying to the heavy quarkonia, that provides enough phase space available i.e., the phase space is so open that the effects of the phase space cannot interrupt what we are interested in. As for the example $\Upsilon \to$ Higgs γ, although a very large rigion of Higgs mass has been ruled out by experiments now, here we take it just for illustrating the effects as it has been discussed a lot in references[2].

First of all, let us briefly describe the approximation in the literatures, not only in order to see how to make the approxitation but also to be able to apply the formulas to later discussions. As for our example $Z^0 \to \eta_b$ (or η_c) γ, there are two diagrams Fig.1 responsible for the decay and the general formula is the following

$$Am = \sum_q \int \frac{d^4k}{(2\pi)^4} TrO^q_\gamma(k) \cdot \Psi_P(k) \tag{1}$$

where P and k are the total and the relative momenta of the bound state respectively, $\Psi_P(k)$ is the B.S. wavefunction of 0^{-+} for the η_b or η_c, and

$$O^q\gamma(k) \;=\; \Bigg[(g_V^q \not\epsilon + g_A^q \not\epsilon\gamma_5)\frac{-\not P/2 - \not k - \not k_1 + m_q}{(P/2+k+k_1)^2 - m_q^2 + i\epsilon}\not e Q_q$$
$$+ \;\; Q_q \not e \frac{\not P/2 - \not k + \not k_1 + m_q^2}{(P/2-k+k_1)^2 - m_q^2 + i\epsilon}(g_V^q \not\epsilon + g_A^q \not\epsilon\gamma_5)\Bigg], \qquad (2)$$

here Q_q and m_q are the charge and the mass of q-type quark, ϵ_μ and e_μ are the polarization vectors of Z^0 and gamma respectively. For the lowest states ($L = 0$), the approximation

$$O^q_\gamma(k) \simeq O^q_\gamma(0) \qquad (3)$$

seems quite good i.e. to expend $O^q_\gamma(k)$ and to keep the lowest order term only, then

$$Am = (\cdots)\int \frac{d^4k}{(2\pi)^4}\Psi_P(k) = (\cdots)\psi(0),$$

that the amplitude proportional to the wavefunction at origin $\psi(0)$ is obtained. The approximation was thought available as the following 'reason'. Due to considering an S-wave bound state, in eq.(2) the linear or odd power terms in k under the integration eq.(1) will not contribute, and the k^2 and its higher order terms would be small due to the bound state being a nonrelativistic one i.e., the contribution being proportional to square of the velocity $\beta^2 \ll 1$ (in $c = 1$ unit) or higher, therefore the expension for the propagator(s) might be a good approximation in the case. However the problem is just from the expention of the propagator i.e. as for the integrated function, a product of O^q_γ and $\Psi_P(k)$ under the integration, the support of the wavefunction is around $k^2 = 0$ but that of O^q_γ around a certain k^2, sometimes quite big, and in addition, being of a nonrelativistic loose bound state, the wavefunction is not dropping fast enough that one may make the the expension O^q_γ at $k^2 = 0$ (the wavefunction's support is not dominant over to that of O^q_γ).

Now let us show the point stated above. As the potential framework works very well for heavy quarkonium problems, hence we adopt it as a better working framework and in order to relate to the concerning problem, the first thing one needs to do is to rewrite the formula eq.(1) under the instantaneous approximation. It is becuase the amplitude is in B.S. formulism, as a matter of fact, the Schrödinger solution (the start point of the potential framwork) is directly related to that of the B.S. equation under the instantaneous approximation[4] and it is easy to establish the relation of the solutions of the two equations i.e. the Schrödinger one with a suitable potential and the B.S. one with a corresponding kernel under the instantous approximation. They have the following relation in the center mass frame of the bound state (we note here that without emphasis the formulas always in the CMS of the bound state from now on):

$$\Psi_P(k) = \frac{(1+\gamma_0)\left(P_0 - 2\sqrt{\vec{k}^2+m_q^2}\right)(A\gamma_5 + B\,\not{\epsilon})}{2\sqrt{2}\left(p_{10} - \sqrt{\vec{k}^2+m_q^2}+i\epsilon\right)\left(p_{20} - \sqrt{\vec{k}^2+m_q^2}+i\epsilon\right)} \cdot \Phi\left(\vec{k}\right),$$

here $\Psi_P(k)$ is the B.S. wavefunction, but $\Phi(\vec{k})$ is the Schrödinger one, and p_1 and p_2 are the momenta of the quarks in the bound state respectively (see Fig.1), and we have $P = p_1 + p_2$ as well as $k = \frac{1}{2}(p_1 - p_2)$. However as for the parameters, $A = 1, B = 0$ corresponds to the 0^{-+} state; $A = 0, B = 1$ to the 1^{--} state for later use in the $\Upsilon \to$ Higgs γ example. After a straitfoward calculation we obtain the amplitude

$$Am = \sum_q \frac{2}{\sqrt{3M}} Q_q \cdot g_V \varepsilon^{0\mu\nu\rho} P_0 \epsilon_\mu e_\nu B_\rho, \tag{4}$$

here the definition of B_ρ is

$$\begin{aligned} B_\rho \;=\; & \int \frac{d^4k}{(2\pi)^4}\Bigg\{\frac{M - 2\sqrt{\vec{k}^2+m_q^2}}{\left(k_0 + M/2 - \sqrt{\vec{k}^2+m_q^2}+i\epsilon\right)\left(k_0 - M/2 + \sqrt{\vec{k}^2+m_q^2}-i\epsilon\right)} \\ & \cdot \left[\frac{(k_1+k)_\rho}{(P/2+k+k_1)^2 - m_q^2 + i\epsilon} + \frac{(k_1-k)_\rho}{(P/2-k+k_1)^2 - m_q^2 + i\epsilon}\right]\Bigg\}\Phi(\vec{k}). \end{aligned} \tag{5}$$

It is easy to see that the components of B_ρ, which come from the itegration on the terms proportoinal to k_ρ in the numerators of the second factor (in the square braket) and are perpendicular to the photon momentum $k_{1\,\rho}$, contribute zero to the decay amplitude due to the symmetry of $\Phi(\vec{k})$ in S-wave. Namely

$$B_\rho = k_{1\,\rho} \int_0^\infty dkWF(k) \cdot U(k),$$

and $WF(k)$ is the radius part of the wavefunction $\Phi(\vec{k})$. One may also see once more if the dependence on k in the denorminators of the second factor (in the square bracket) could be ignored then B_ρ, so the amplitude, would be proportional to the Schrödinger wavefunction at origin of the quarkonium.

Now let us calculate B_ρ with care. To match the instantous approximation we first calculate out the integration on k_0 of B_ρ in its complex plane with the method of residual theorem instead of the Feynman integration technique. It is interesting that due to the fact, two poles meet togather occationally when the following condition is satisfied (in the CMS of the quarkonium),

$$\sqrt{\vec{k}^2 + m_q^2} + \sqrt{(\vec{k}+\vec{k_1})^2 + m_q^2} = k_{20}$$

with given momenta k_1 and k_2, we obtain a complex value for B_ρ i.e. it has different result in $|\vec{k}|$ upper half plane from that in lower half plane. In order to see the effects which we are illustrating, we plot the the real part and the imaginary part seperately as well as the wavefunction in Fig.2. The deviation of the crude approximation from the one carefully treated is shown clearly. Indeed, the numerical result obtained by a straightforward calculayion is bigger than that by the approximation proportional to the wavefunction at origin[1] i.e. more than 1.5 times that of the approximation.

Now let us see the decay $\Upsilon \rightarrow$Higgs γ to illustrate the factor. It contains two diagrams similar to Fig.1, but the bound state in the initial state instead. The 4-momenta of the bound state Υ, the photon and the Higgs are P, k_1 and k_2, the polarization vectors of the photon and Υ are e_μ and ϵ_ν respecively. The corresponding amplitude is

$$Am = 2 \cdot 2^{1/4} Q_b \sqrt{3 M_\Upsilon G_F} \cdot B_i \Big[(k_1 \cdot P)(e \cdot \epsilon) - (k_1 \cdot e)(P \cdot \epsilon) \Big] \tag{6}$$

here Q_b is the charge of b-quark, M_Υ is the mass of Υ and the definition of B_i is

$$B_i = \int_0^\infty dk L(k, M_h) \cdot WF(k).$$

Note: in the above formula, only the one dimesional integration is left (the integrations of the angles have been executed already thus the radius part $WF(k)$ instead of the $\Phi(\vec{k})$) and some factor depending on M_h is absorbed into the factor $L(k, M_h)$, which dictates the propagator effect. The deviation of the approximation eq.(3) from the better treatment is various as the Higgs mass varies. To show the factors we plot the curves of $L(k, M_h)$ and $WF(k)$ as well as the approximation one L_{app} in Fig.3 and the ratio R in decay rate i.e. the approximation decay rate to the careful calculated one as the following:

$M_h(GeV)$	2.0	4.0	6.0	8.0
R	0.60	0.57	0.49	0.33

The results are similar to those of ref.[3], although they use the infinite momentum system to calculate and they call the deviation from the wavefunction at origin approach as relativistic one(s). However, based on our illustration at least one factor is clarified up i.e. to calculate the processes, involving annihilation or creation of a bound state through two or more points of elementary interaction(s), or say, the processes involving elementary propagator(s) and a bound state, the involved propagator(s) need to be treated carefully, and one can not attribute them to a constant or a nonrelativistic expension. The deviation of the crude approximation eq.(3) sometimes is to make the result big such as the decay $\Upsilon \rightarrow$ Higgs γ and sometimes small such as the decays $Z^0 \rightarrow \eta_c\gamma$ and $Z^0 \rightarrow \eta_b\gamma$, varying with the concrete process.

Due to clarifying up the effect, more processes are under reexamining.

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Figure Captions

Fig. 1: The Feynman diagrams. k_1, k_2, P are the momenta of gamma, Z^0 and the quarkonium η_c or η_b for the $Z^0 \rightarrow \eta_c$ (or η_b) γ decay, but they are the momenta of gamma, Higgs and the quarkonium Υ, for the $\Upsilon \rightarrow$Higgs γ decay respectively,

Fig. 2: The behavior of the wavefunction $WF(k)$ and the factor $U(k)$ from the propagator. Fig.2a for the $Z^0 \rightarrow \eta_c\gamma$ case; Fig.2b for the $Z^0 \rightarrow \eta_c\gamma$ case. $U_{re}(k)$– the real part; $U_{im}(k)$– the imaginary part; U_{app}– the one under the approximation eq.(3).

Fig. 3: The behavior of the wavefunction $WF(k)$ and the factor $L(k, M_h)$ from the propagator with different Higgs mass M_h. L_{app} is the one under the approximation eq.(3).

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(There are many references, but for each item we will only list those related to the illustrating examples taken in this paper.)

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FIG.1

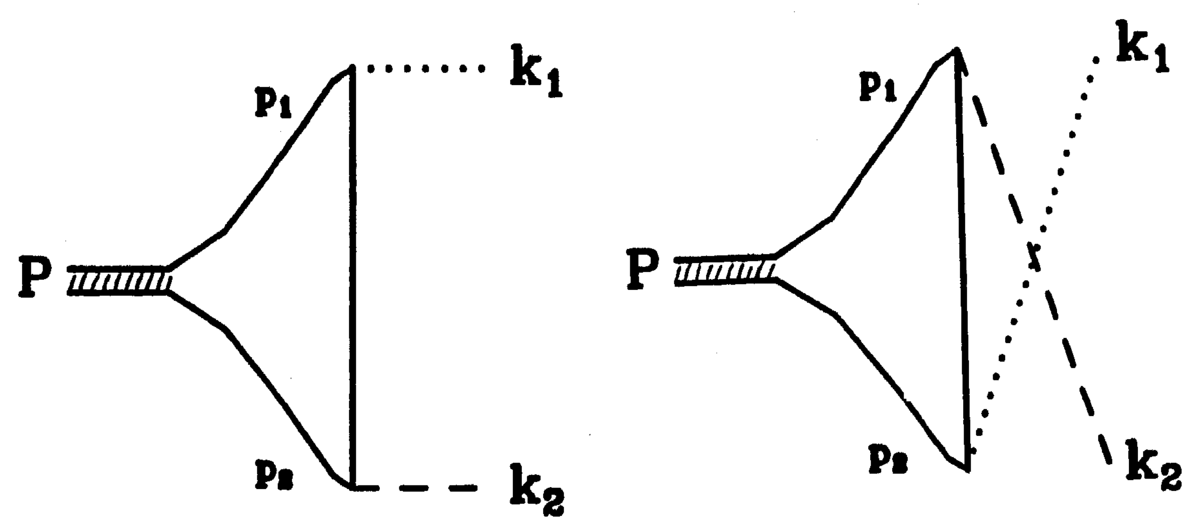


FIG.2a

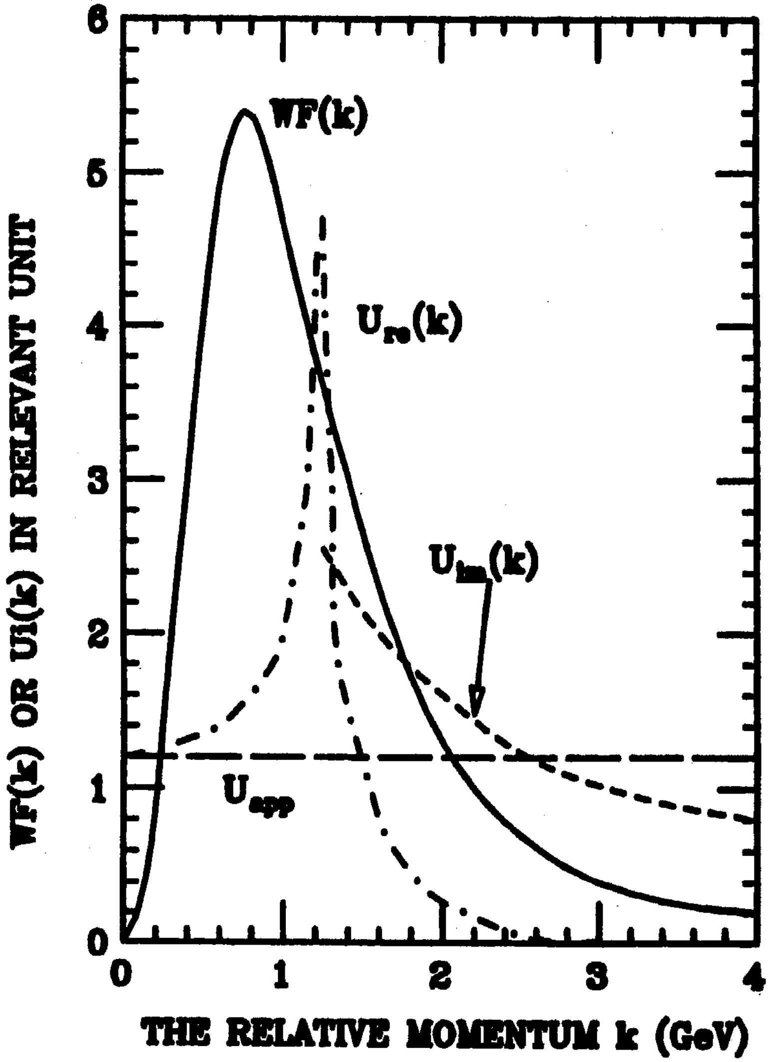

WF(k) OR Ui(k) IN RELEVANT UNIT
WF(k)
U_re(k)
U_im(k)
U_app
0
1
2
3
4
5
6
THE RELATIVE MOMENTUM k (GeV)

FIG.2b

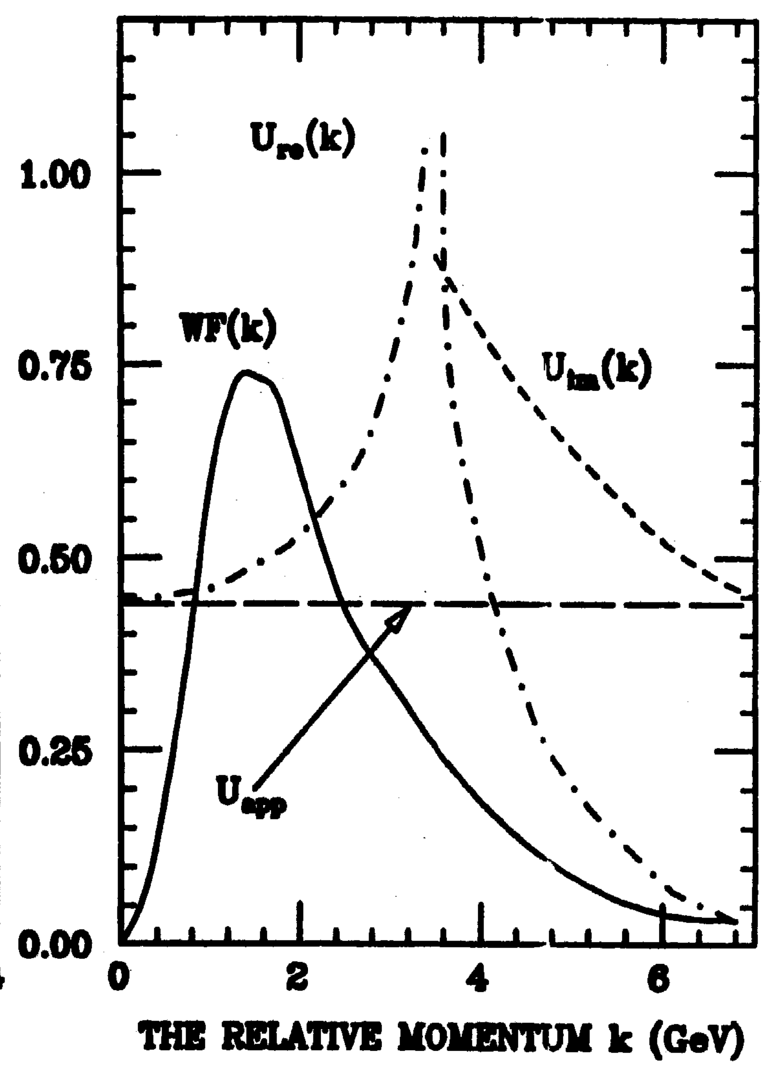
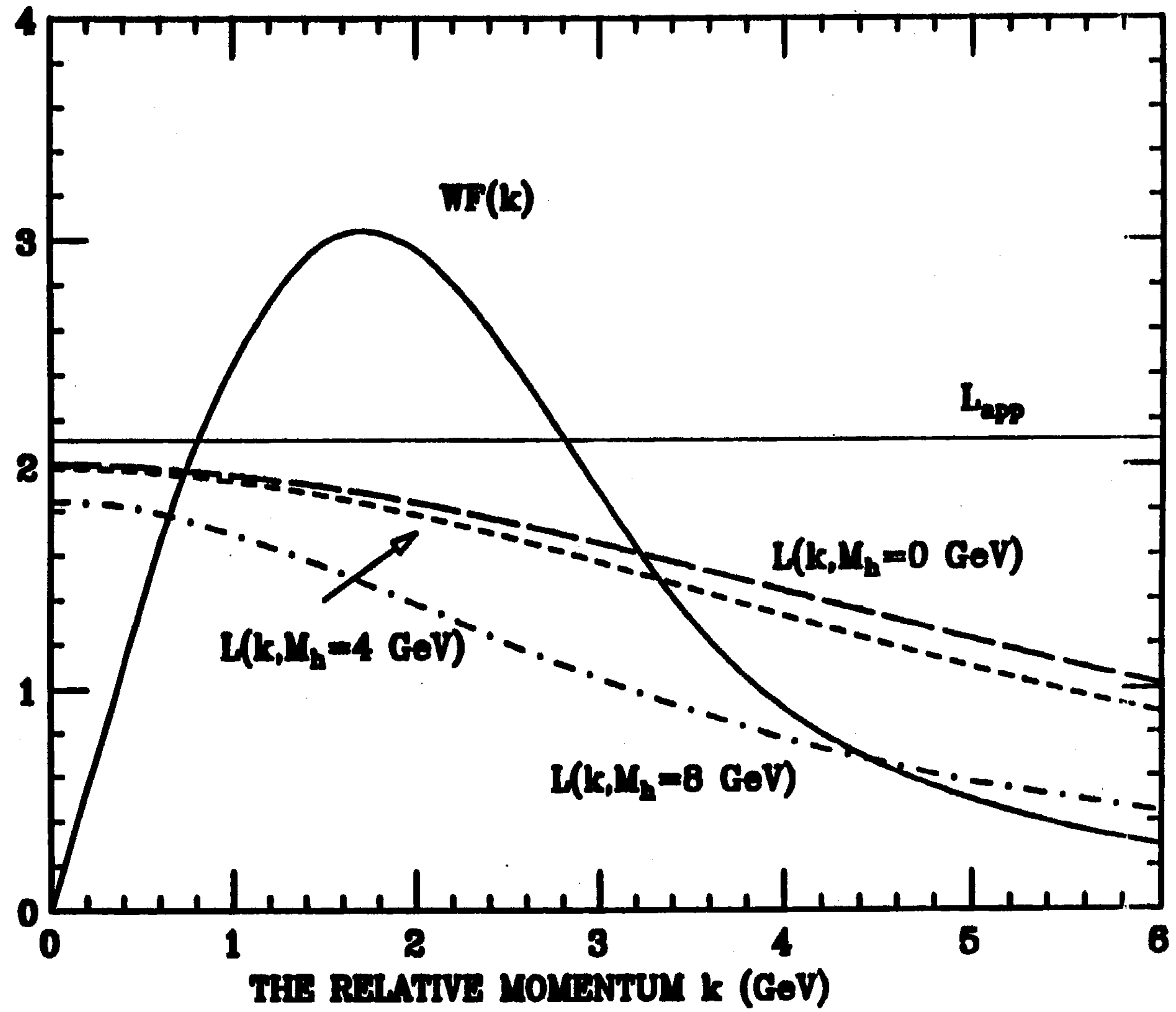

0.00
0.25
0.50
0.75
1.00
U_re(k)
WF(k)
U_im(k)
U_app
0
2
4
6
THE RELATIVE MOMENTUM k (GeV)

FIG.3



WF(k)
L_{app}
L(k,M_h=0 GeV)
L(k,M_h=4 GeV)
L(k,M_h=8 GeV)
THE RELATIVE MOMENTUM k (GeV)
0
1
2
3
4
5
6