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MODEL WITH CALCULABLE DIRAC NEUTRINO MASSES*

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Abstract

An electroweak model based on the gauge group $SU(2)_L \times U(1)_L \times U(1)_R$ is presented in which neutrinos are massless at the tree-level due to a discrete symmetry and acquire tiny finite masses at the one loop level due to the exchange two charged singlet Higgs (generalised Zee's mechanism) each carrying two units of lepton number. The anomalies are cancelled by adding vector like singlet fermions. These fermions are also responsible for giving masses to the conventional charged quarks and leptons by the see-saw mechanism.

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The phenomenal success that the standard model[1] is currently enjoying may well turn out to be representative of the true path chosen by Nature to describe the strong and the electroweak forces. One implication of this simple scheme is that the neutrinos are strictly massless. Strict masslessness of the neutrinos requires a symmetry principle which protects the neutrinos from acquiring mass. No such principle exists in the standard model. More generally, it is not even clear what the symmetry principle should be in order to maintain massless neutrinos in the standard model and its extensions. Neutrinos in the standard model are forced to be massless by requiring the absence of right-handed neutrino fields and/or by requiring the absence of a scalar triplet field. Mere simplicity of the standard model dictates this course of action.

Massless neutrinos are boring. Massive neutrinos provide a wealth of interesting results amongst which the two most widely discussed are (a) <u>the missing mass problem[2] in cosmology</u>: - neutrinos provide the right mass density to close the universe provided the masses of the three neutrino species satisfy the constraint $\sum_{i=e,\mu,e} m_{\nu_i} \leq 30$ eV.

(b) <u>Solar neutrino oscillations[3]</u>: - the observed deficit in the number of solar neutrinos from the sun can be understood if the electron neutrinos oscillate into the neutrinos of the other two flavours.

In the standard model neutrinos of such low mass can be accommodated at the expense of <u>ultra fine-tuning</u> of the tree level yukawa couplings. There are ways of eleviating the problem of fine tuning. One is to generate neutrino masses via the see-saw mechanism[4]. The neutrino masses are inversely proportional to the mass scale at which unification of the fundamental forces takes place. This being of order of the Planck scale, provides naturally the required suppression factor for tiny neutrino masses. The other method is to generate neutrino masses radiatively in conjunction with some additional symmetries that forbid tree-level neutrino masses[5]. These procedures of mass generation often result in the introduction of several new spin-0 and spin-$\frac{1}{2}$ fields. In some cases it is also necessary to extend the $SU(2)_L \times U(1)$ gauge structure. In what follows, a model of electroweak interactions based on the gauge group[6] $SU(2)_L \times U(1)_L \times U(1)_R$ is presented in which neutrinos are massless at the tree level due to a discrete symmetry and become massive four component Dirac particles from one loop finite quantum corrections due to the exchange of two charged singlet of higgs scalars that carry two units of lepton numbers and new fermions that transform as singlets under the weak isospin group $SU(2)_L$. The choice to describe the neutrinos as "Dirac" as opposed to "Majorana" particles is largely dictated by lepton number conservation which is observed to be conserved in Nature. It also puts all fermions on equal footing as far as the problem of mass is concerned. The weak isospin singlet fermions cancel the triangle anomalies of the theory and are also responsible for the masses of the conventional quarks and leptons through the generalised see-saw mechanism[7].

The gauge symmetry of the model is $G = SU(3)^C \times SU(2)_L \times U(1)_L \times U(1)_R$. Under G the conventional quarks and leptons of the three families transform as

$$
\begin{array}{llllll}
\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L , & \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L , & \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L & \sim & (3,\ 2,\ 1/3,\ 0) \\
U_{iR} , & c_{iR} , & t_{iR} & \sim & (3,\ 1,\ 0,\ 4/3) \\
d_{iR} , & s_{iR} , & b_{iR} & \sim & (3,\ 1,\ 0,\ -2/3) \\
\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & \sim & (1,\ 2,\ -1,\ 0) \\
\nu_{eR} , & \nu_{\mu R} , & \nu_{\tau R} & \sim & (1,\ 1,\ 0,\ 0) \\
e_R , & \mu_R , & \tau_R & \sim & (1,\ 1,\ 0,\ -2)
\end{array}
\qquad [1]
$$

where i denotes the three colour degrees of the quarks. As it stands the model suffers from triangle anomalies. These anomalies are cancelled by adding fermions that transform as singlets under the weak isospin group $SU(2)_L$. Explicitly

$$
\begin{array}{llllll}
U_{iL} & , & C_{iL} & , & T_{iL} & \sim (3,\ 1,\ 0,\ 4/3) \\
U_{iR} & , & C_{iR} & , & T_{iR} & \sim (3,\ 1,\ 4/3,\ 0) \\
D_{iL} & , & S_{iL} & , & B_{iL} & \sim (3,\ 1,\ 0,\ -2/3) \\
D_{iR} & , & S_{iR} & , & B_{iR} & \sim (3,\ 1,\ -2/3,\ 0) \\
E_{L} & , & M_{L} & , & T_{L} & \sim (1,\ 1,\ 0,\ -2) \\
E_{R} & , & M_{R} & , & T_{R} & \sim (1,\ 1,\ -2,\ 0)
\end{array}
\qquad [2]
$$

Thus the model contains three sets of singlet fermions to cancel the anomalies of the three families of conventional quarks and leptons. The electric charge operator of the model is

$$Q = T^3_L + \frac{1}{2}\,(T^0_L + T^0_R) \qquad [3]$$

where T^3_L is the diagonal generator of $SU(2)_L$ and T^0_L , T^0_R are the generators of the abelian groups $U(1)_L$, $U(1)_R$.

The scalar sector of the model consists of a doublet, three neutral and two charged singlets[8]. These are

$$
\begin{array}{ll}
\phi & \sim (1,\ 2,\ -1,\ 0) \\
S^U & \sim (1,\ 1,\ 4/3,\ -4/3) \\
S^D & \sim (1,\ 1,\ -2/3,\ 2/3) \\
S^E & \sim (1,\ 1,\ -2,\ 2) \\
H^+_L & \sim (1,\ 1,\ 2,\ 0) \\
H^+_R & \sim (1,\ 1,\ 0,\ 2)
\end{array}
\qquad [4]
$$

Consistent with $SU(3)^C \times U(1)_{em}$ gauge invariance, the ground state of the system is determined by the following vacuum expectation values,

$$\langle\phi\rangle = \begin{pmatrix}\eta\\ 0\end{pmatrix} \quad , \quad \langle S^U\rangle = \sigma^U$$

$$\langle S^D\rangle = \sigma^D \quad , \quad \langle S^E\rangle = \sigma^E$$

$$\langle H_L^+\rangle = 0 \qquad \langle H_R^+\rangle = 0 \qquad [5]$$

Since no neutral bosons lighter than the Z^o of the standard model have been reported, the constraint on the vacuum expectation values is σ^U, σ^D, σ^E, $\gg \eta$ and the mode of descent is

$$G \xrightarrow{\sigma^{U,D,E}} SU(3)^C \times SU(2)_L \times U(1) \xrightarrow{\langle\phi\rangle} SU(3)^C \times U(1)_{em} . \qquad [6]$$

The spectrum of the gauge particles consists of the standard $W^{\pm}$, Z^o bosons and an additional neutral boson Z' with a mass lower bound of 140 GeV. The neutral current phenomenology which leads to this bound is adequately discussed in a previous work (ref 6).

Except for the neutrinos, all the fermions acquire masses through the see-saw mechanism. In order to avoid the problem of fine tuning, the tree-level neutrino masses are forbidden by a discrete symmetry $\nu_{\alpha R} \to -\nu_{\alpha R}$, $\alpha = e, \mu, \tau$. The yukawa couplings of the charged fermions with the scalars are given by

$$L_{yukawa} = \sum_{u,u'} y_{fu_{F}u'}\, \bar{f}^u_L \phi F^{u'}_R + y_{Fu_{F}u'}\, \bar{F}^u_R S^F\, F^{u'}_L + m_{Fu_{f}u'}\, \bar{F}^u_L f^{u'}_R$$

$$+ \sum_{x=d,e,d',e'} y_{fx_{F}x'}\, \bar{f}^x_L i\sigma_2 \phi^* F^{x'}_R + y_{Fx_{F}x'} \bar{F}^x_R S^F\, F^{x'}_L + m_{Fx_{f}x'} \bar{F}^x_L\, f^{x'}_R + h.c.$$

[7]

After spontaneous symmetry breaking the mass matrix for the up-type quark sector (u,c,t U,G,T) is given by

$$M^U = \begin{pmatrix} 0 & Y^u\eta \\ M^u & Y^U\sigma^U \end{pmatrix} \qquad [8]$$

where M^u , Y^u , Y^U are 3 x 3 sub matrices. The mass-matrices for the down-type quarks sector (d, s, b, D, S, B) and electron-type leptons (e, μ,τ, E, M, Γ) are similar. The mass matrix M^U is not Hermitian in general. It can be block diagonalized by a biunitary transformation.

$$\bar{M}_U = V_L\, M_U\, V_R^+ \qquad [9]$$

For the see-saw mechanism to work the elements of M^U are taken to satisfy the constraints $M^u << Y^U\,\sigma^U$, $\eta y^u << Y^U\sigma^U$ and $M^u \sim \eta\, Y^u$. The block diagonalised mass matrix takes the form

$$\bar{M}_U \simeq \begin{pmatrix} -\frac{\eta}{\sigma U}\, Y^u\, (Y^U)^{-1}\, M^u & 0 \\ 0 & \sigma^U\, M^U \end{pmatrix} \qquad [10]$$

and the transformation matrices block diagonalising the mass matrix are

$$V_A^u \simeq \begin{vmatrix} 1 - 1/2\, \gamma_A^u\, \gamma_A^{u+} & -\gamma_A^u \\ \gamma_A^{u+} & 1 - 1/2\, \gamma_A^{u+}\, \gamma_A^u \end{vmatrix} \qquad [11]$$

A = L,R, where $\gamma_L \simeq \frac{\eta}{\sigma^U}\, Y^u\, (Y^U)^{-1}$, $\gamma_R \simeq \frac{1}{\sigma^U}\, (Y^U)^{-1}\, M^u$.

The elements of γ_L, γ_R are of order 10^{-2}. The transformation matrices V_L, V_R are unitary to third order in $\gamma_L\, \gamma_R$. The matrix $\bar{M}^U$ is diagonalised further i.e. $M^U_{diag} = \Lambda_L\, M^U\, \Lambda_R^+$, by employing two unitary transformations Λ_L, Λ_R

$$\Lambda_{L,R} = \begin{pmatrix} k^u_{L,R} & 0 \\ 0 & K^U_{L,R} \end{pmatrix} \qquad [12]$$

where $k^u_{L,R}$, $K^U_{L,R}$ are 3 x 3 unitary matrices. The conventional quark masses are see-saw masses

$m^u = \frac{\eta}{\sigma^U} \frac{(M^u)_{11}(Y^u)_{11}}{(y^U)_{11}}$, etc. and $m^U \simeq \sigma^U \; (y^U)_{11}$ etc. The small

mixings elements of γ_L, γ_R lead to flavour changing neutral currents. All constraints are satisfied provided the mixing elements of γ_L, γ_R are of order 10^{-2} or less. This can readily be achieved by taking the singlet fermion masses in the range of a few hundred GeV to 1 TeV. The main advantage of generating masses of the conventional fermions via the see-saw mechanism is that the fine tuning of the yukawa couplings, which is present in the standard electroweak model discussion of fermion masses is kept to a minimum. We now discuss neutrino masses.

The yukawa interactions due to the singlets are[8]

$$L_i = \frac{1}{2}\, g^{(L)}_{\alpha\beta}\; \psi^{\alpha T}_L\; c^{-1} i\sigma_2\; \psi^{\beta}_L\; H_L + g^{(R)}_{\alpha\beta}\; e^{\alpha T}_R\; C^{-1}\; \nu_R^{\;\beta}\; H_R + (h.c) \qquad [13]$$

where ψ^{α}_L (α = 1,2,3) denotes the e, μ, τ left-handed doublets and e^{α}_R , ν^{α}_R denote the corresponding right handed singlets.

The crucial components responsible for generating tiny and finite neutrino masses is the interaction term between H_L, H_R and S^E in the scalar potential V,

$$V = \;.\;.\;.\;.\;.\; + \lambda\, H_L^{\;*}\, H_R\, S^E + \;.\;.\;.\;.\;.\; + h.c. \qquad [14]$$

when S^E develops vacuum expectation value σ^E , H_L , H_R are no longer eigenstates.

The physical eigenstates are

$$H_1 = H_L \cos\omega + H_R \sin\omega$$

$$H_2 = H_R \cos\omega - H_L \sin\omega \qquad [15]$$

with masses m_1 , m_2 given by

$$m_1^2 = m_{11} \cos^2\omega + m_{22} \sin^2\omega + 2m_{12} \sin\omega \cos\omega$$

$$m_2^2 = m_{22} \cos^2\omega + m_{11} \sin^2\omega - 2\, m_{12} \sin\omega \cos\omega \qquad [16]$$

The mixing angle ω is given by $\tan 2\omega = 2\, m_{12}/(m_{11} - m_{22})$ where

$$M_{11} = M_L^2 + \lambda_L^U \sigma_U^2 + \lambda_L^D \sigma_D^2 + \lambda_L^E \sigma_E^2$$

$$M_{22} = M_R^2 + \lambda_L^U \sigma_U^2 + \lambda_L^D \sigma_D^2 + \lambda_L^E \sigma_E^2$$

$$M_{12} = \lambda\, \sigma_E \qquad [17]$$

The neutrinos acquire radiative masses from the two one loop diagrams (Fig. 1). Although the inidividual diagrams are divergent, their sum is finite, giving the mass of the i-th neutrino to be

$$m_{\nu_i} = \sum_j \sin 2\omega \quad \frac{(g_{ij}\, f_{ji} + f_{ij}\, g_{ji})}{64\pi^2} \left(I_j(1) - I_j(2)\right)$$

where

$$I_j^{(\sigma)} = m_j \left(\frac{m_{H\sigma}^2}{M_{H\sigma}^2 - M_j^2} \right) \ell n \frac{M_{H\sigma}^2}{M_j^2} \qquad [18]$$

In deriving the above expressions the intergenerational mixings between the various charged leptons are taken to be negligibly small. For simplicity the yukawa couplings $g_{\alpha\beta}^{(L)}$ and $g_{\alpha\beta}^{(R)}$ are taken to be real. If we adapt the conservative viewpoint that the couplings $g^{(L)}_{\alpha\beta}$ and $g^{(R)}_{\alpha\beta}$ are roughly of the same order of magnitude, the masses of ν_e and ν_μ are expected to dominate as they are proportional to m_τ . The bounds on the yukawa couplings follow from the leptonic processes mediated by the charged higgs H_1 and H_2 which we now discuss. Throughout the discussion we will work in the zero momentum transfer approximation which is justified since the lower bound on the masses of the changed scalars from Tristan [9] is 26.5 GeV. In this approximation the current cross current interation Lagrangian for muon decay, after Fierz reshuffling of fields, is given by

$$L = \frac{g^{(L)\,2}_{12}}{2} \left(\frac{\cos^2 \omega}{M^2_{H_1}} + \frac{\sin^2 \omega}{M^2_{H_2}} \right) \bar{\nu}_{\mu L}\gamma^\mu \mu_L \; \bar{e}_L \gamma_\mu \nu_{eL}$$

$$+ \frac{g^{(L)}_{12}\, g^{(R)}_{12}}{2} \sin 2\omega \left(\frac{1}{M^2_{H_1}} - \frac{1}{M^2_{H_2}} \right) \left[\bar{e}_R \,\nu_{eL} \cdot \bar{\nu}_{\mu R}\, \mu_L - \frac{1}{4}\, \bar{e}_R\, \sigma^{\mu\nu} \nu_{eL} \cdot \bar{\nu}_{\mu R} \sigma_{\mu\nu} \mu_L \right.$$

$$\frac{g^{(R)\,2}_{12}}{2} \left(\frac{\cos^2 \omega}{M^2_{H_2}} + \frac{\sin^2 \omega}{M^2_{H_1}} \right) \bar{\nu}_{\mu R}\gamma^\mu \mu_R \; \bar{e}_R \gamma_\mu \nu_{eR} \qquad [19]$$

For numerical estimates, we take the mixing angle ω to be maximal i.e. $\omega = 45^\circ$ and $M_{H_1} \sim M_{H_2} = 1$ TeV, $\frac{|M_{H_1} - M_{H_2}|}{M_H} \simeq 0.05$. So that the scalar contribution to muon decay rate not exceed the rate predicted by the $SU(2)_L \times U(1)$ theory by 1% requires $\frac{g^2_{12}}{M^2_{H_1}}$, $\frac{g^2_{12}}{M^2_{H_2}} < 10^{-3}\ G_F$. This is sufficient to suppress the contribution to the ξ' parameter due to right-handed currents well below the errors on the experimentally measured value of ξ'. The scalars H_1 and H_2 also contribute[10] to (g-2) of the electron, (g-2) of the muon, the process $\mu \to e\gamma$ and $e - \mu - c$ universality violating processes like $\mu \to \bar{\nu}_e e \nu_\tau$, $\tau \to \bar{\nu}_e\, e\, \nu_\mu$. We take

$\frac{g^2_{13}}{M^2_{H_1}}$, $\frac{g^2_{13}}{M^2_{H_2}}$, $\frac{g^2_{23}}{M^2_{H_1}}$, $\frac{g^2_{23}}{M^2_{H_2}}$ to be of order $\frac{g^2_{12}}{M^2_{H_1}}$, $\frac{g^2_{12}}{M^2_{H_2}}$ $\left(\simeq 10^{-3}\ G_F \right)$ from Yukawa

β-decay of the muon. Yukawa couplings of this order ensure that the rates of all the above mentioned rare processes are suppressed well below the present experimental bounds.[11] With scalar masses as high as one TeV all yukawa couplings still remain perturbative ($g_{\alpha\beta} \leq 0.1$).

With $g_{\alpha\beta} = 10^{-2}$, $M_{H_1} \simeq M_{H_2}$ of order 1 TeV, $|M_{H_1} - M_{H_2}|/M_{H_i} \simeq 0.05$, $\sin 2\omega = 1$, eq. (18) gives

$$M_{\nu_e} \simeq M_{\nu_\mu} \leq 10\ \text{eV},\ M_{\nu_\tau} \approx \left(\frac{M_\mu}{M_e}\right) M_{\nu_\mu} \qquad [20]$$

which are compatible with constraints from laboratory experiments and constraints from cosmology. In passing it is to be noted that what enters in the expression for neutrino masses is $\sin 2\omega \cdot \left(\frac{M_{H_1} - M_{H_2}}{M_H}\right)$. By choosing reasonably smaller values for the mixing angle ω, we can tolerate larger mass differences between the charged scalars. This eliminates further any 'fine tuned' choice for the masses H_1, H_2.

Finally we note that the discrete symmetry that prevents the neutrinos from acquiring masses at the tree level can be eliminated entirely if the model is embedded into the chirally symmetric gauge structure $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$.

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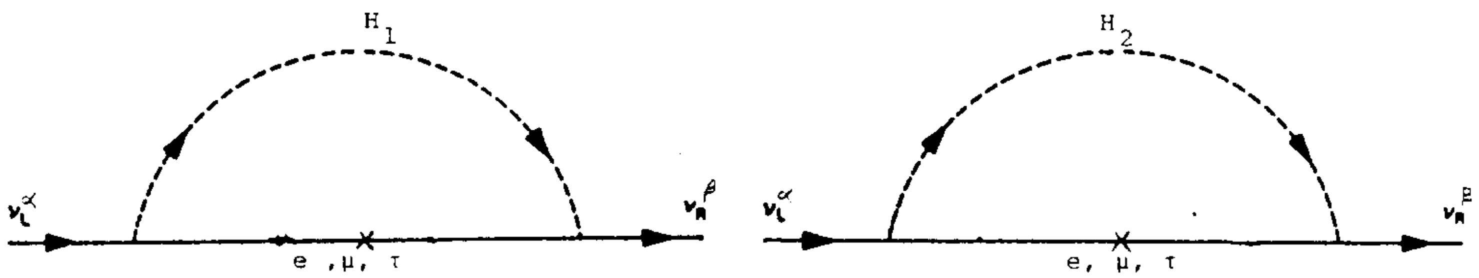

Fig.1 One-loop diagram due to the charged scalars for neutrino masses.