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Ratio of Charged to Neutral B Meson Production at $\Upsilon(10580)$

Nina Byers*
University of California, Los Angeles
Los Angeles, California 90024

Estia Eichten
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510

ABSTRACT

The branching ratio of $\Upsilon(10580)$ decays to B^+B^- and $B^0\bar{B}^0$ is sensitive to electromagnetic corrections and to the B^0 and B^+ masses. We report calculated values for this ratio as a function of energy in the $\Upsilon(10580)$ resonance region for various possible values of m_{B^0} and m_{B^+}.

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The determination of the $B_d\bar{B}_d$ mixing parameter x_d as well as other interesting B meson decay quantities presently requires the estimation of the $(\Upsilon(10580) \rightarrow B^+B^-)/(\Upsilon(10580) \rightarrow B^0\bar{B}^0)$ branchin g ratio.[1] Because the $\Upsilon(10580)$ resonance is so close to the kinematic threshold for production of these final states, this branching ratio may differ significantly from one. In particular it is sensitive to any small mass difference between the B^0 and B^+ meson. Atwood and Marciano[2] have called attention to the possibility that electromagnetic effects may also significantly enhance this ratio. In particular they argue that the Coulomb attraction in the charged B channel is important. The effect of the Coulomb attraction is easy to incorporate if the momentum of the outgoing B meson pair is much smaller than the inverse sizes of the initial state $(b\bar{b})$ resonance and the outgoing B meson. In this long wavelength limit, the electromagnetic enhancement factor in the charged channel is given by the modulus squared of the ratio of the Coulomb scattering wave function to the corresponding plane wave function. This enhancement factor is familiar in low energy nuclear inelastic cross sections and is given by[3]

$$|\psi_c(0)|^2 = \eta/(1-e^{-\eta}) \quad \text{where} \quad \eta = 2\pi\alpha/v_{rel}. \tag{1}$$

Here the relative velocity of the two outgoing mesons is denoted by v_{rel} and the $\ell = 0$ partial wave assumed. This factor agrees with the result of Atwood and Marciano[2].

In $\Upsilon(10580)$ decay to B^+B^- the B mesons are produced in a relative P wave and (1) should be replaced by the ratio of $\ell = 1$ partial wave functions; viz.,[3]

$$|[e^{-\eta/4}\Gamma(2-i\eta/2\pi)kre^{ikr}F(2-i\eta/2\pi,4,-2ikr)/3j_1(kr)]_{r=0}|^2 \tag{2}$$

where k is the momentum of the outgoing meson and F is the confluent hypergeometric function. Evaluation of (2) gives

$$(1+|\eta/2\pi|^2)\ \eta\ (1-e^{-\eta}) \tag{3}$$

which would be the appropriate Coulomb-enhancement factor in the long wavelength limit.

Unfortunately the inverse size of the decaying $\Upsilon(10580)$ resonance is not that much larger than the momentum of the outgoing B meson pair; hence a more careful treatment is required.[4] Furthermore, the $\Upsilon(10580)$ is a highly excited state and this enhances the sensitivity of its decay amplitudes to the shape as well as the normalization of the outgoing wave function. In particular, within the nonrelativistic framework, the $\Upsilon(10580)$ is approximately described by a 4S radial wave function with its associated rapid oscillations. Hence, its decay amplitudes include significant destructive interference between various regions of the radial wavefunction.[5]

In the Cornell model, the decay amplitude of an Υ resonance is an overlap integral of a $q\bar{q}$ pair production amplitude with the $b\bar{b}$ initial state wave function and an outgoing $B\bar{B}$ wave function.[6] In the $B^0\bar{B}^0$ channel the meson center of mass motion is described by a spherical Bessel function $j_1(kr)$. To take into account the Coulomb attraction in the B^+B^- channel, we replace $j_1(kr)$ by the $\ell = 1$ Coulomb partial wave function. In Figure 1 the Bessel function, the corresponding Coulomb function, and the remaining factor in the overlap integral are shown as curves (a), (b) and (c), respectively. The curves (a) and (b) are drawn for $B\bar{B}$ final states with m_B = 5.280 GeV and total energy W = 10.58 GeV; (c) is taken from a Cornell model calculation[6]. The curve (c) is a product of a 4S $b\bar{b}$ wave function and a light quark pair ($q\bar{q}$) production amplitude folded into $b\bar{q}$ and $q\bar{b}$ wave function. As can be seen in this figure, the factor (3) increases the amplitude but overestimates the effect because the Coulomb wave function is drawn in toward the origin increasing the cancellation that occurs in the overlap integral.

In Figure 2 we show calculated values for the ratio of the modulus squared decay amplitudes for charged and neutral modes as a function of energy in the resonance region. The curves (a) and (b) are for the equal mass cases $m_{B^0} = m_{B^+}$ = 5.278 GeV and $m_{B^0} = m_{B^+}$ = 5.280 GeV, respectively. The deviation of the ratio from unity is sensitive to m_B since $\eta = \pi\alpha/\sqrt{1-4m_B^2/W^2}$ and the resonance energies are close to $2m_B$. If $m_{B^0} - m_{B^+}$ is positive, there is an additional kinematic enhancement of charged B meson production relative to neutral. The curve (c) is for the case $m_{B^0} = 5.280$ GeV and $m_{B^+} = 5.278$ GeV. In this case the ratio of charged to neutral decays has a more rapid variation with W. Clearly, these effects depend on the coupling of the resonance to decay channels. However the model dependence is not as severe as might be expected because the decay amplitudes are overlap integrals of smooth functions with the $\Upsilon(4S)$ wave function and any potential model which fits the Υ spectroscopy gives essentially the same $4S$ wave function.

The results reported here are estimates rather than precise predictions. We have included only those electromagnetic corrections that depend on $\eta = 2\pi\alpha/v_{rel}$. There are additional order α corrections not enhanced by the $1/v$ threshold factor which depend on the production dynamics. Also we have made additional approximations in the production dynamics which may introduce few per cent corrections. Finally, we have neglected the B meson electromagnetic form factor.

It is clear from our analysis that the dominant elements in the determination of the charged to neutral ratio are the values of the B^+ and B^0 masses. A more precise calculation of this ratio requires more accurate knowledge of these masses than is presently available. The Coulomb correction represents the next largest contribution to the deviation of this ratio from one. If the masses of B^0 and B^+ were equal, the Coulomb correction would dominate; but even here an accurate estimation of ratio would require a more accurate mass determination. Finally at the level of a few percent this ratio is sensitive to numerous details of the production dynamics.

In conclusion, we find that the branching ratio of $\Upsilon(4S)$ decays to charged and neutral B mesons is sensitive to the Coulomb corrections at the level of five to ten percent. Taking equal masses, we estimate this ratio to be 1.07 for $m_B = 5.280$ GeV and 1.05 for $m_B = 5.278$ GeV. If $m_{B^0} - m_{B^+} = 2$ MeV, we estimate this ratio to be 1.18. Our results show that an accurate measurement of the B^0 and B^+ masses is essential for a reliable determination of the ratio of charged to neutral B meson production in the $\Upsilon(10580)$ resonance region. Futhermore, a measurement of the energy variation of this ratio over the resonance region is a sensitive probe of the $B^0 - B^+$ mass difference.

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References

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[2] D. Atwood and W. J. Marciano, Phys. Rev. D41, 1736 (1990).

[3] See, e.g., L. I. Schiff, *Quantum Mechanics, 2nd ed.* (McGraw-Hill, New York, 1955) pp. 117-119.

[4] G. P. Lepage, "Coulomb Corrections for $\Upsilon(4S) \rightarrow B\overline{B}$ ", Cornell Preprint CLNS-90/1007 (1990). This reference points out that the $\Upsilon(4S)$ form factor has structure which reduces the Coulomb enhancement.

[5] Owing to coupling of $b\bar{b}$ states with, e.g., $b\bar{q}q\bar{b}$ states, Υ resonances are configuration mixed. This is particularly the case for states above flavor threshold. Although the $\Upsilon(10580)$ is primarily a 4S $b\bar{b}$ state, it also has 3S and 5S components as well as components in the continuum.[6] Therefore $B\bar{B}$ production in this resonance region is not simply given by $\Upsilon(4S) \rightarrow B\bar{B}$ amplitudes; see ref. 6. However, the ratio $B^+B^-/B^0\bar{B}^0$ will in first approximation be given by the ratio of modulus squared decay amplitudes multiplied by ratio of phase space. The latter factor, of course, is unity if $m_{B_0} = m_{B^+}$.

[6] N. Byers and E. Eichten, "Threshold Production of B Meson Pairs in e^+e^- Annihilation", in preparation. See also E. Eichten, K. Gottfried, K. Lane, T. Kinoshita, and T. -M. Yan , Phys. Rev. D17, 3090 (1978);Phys. Rev. D21, 203 (1980).

Figure Captions

Figure 1. Contributions to the $\Upsilon(4S) \to B\bar{B}$ decay amplitude in the Cornell model[6]. The decay amplitude is the integral of the product of the oscillating amplitude (c) with either (a) or (b); (a) is $j_1(kr)$ and describes the $B^0\bar{B}^0$ final state while (b) is the corresponding Coulomb partial wave function which describes the B^+B^- including final state interactions. Here k is computed for W = 10.580 GeV and we have taken $m_{B^+} = m_{B^0} = 5.280$ GeV. The curve (c) is a product of a 4S wave function with a light quark pair production amplitude folded into B wave functions (see ref.5). The nodes in (c) arise from the three nodes characteristic of a 4S radial excitation modulated by the nodeless production amplitude expected for a pair of ground state mesons.

Figure 2. Branching ratio $(\Upsilon(4S) \to B^+B^-)/(\Upsilon(4S) \to B^0\bar{B}^0)$ as function of energy W in the resonance region showing the sensitivity of this ratio to B meson masses. Curve (a) was calculated with $m_{B^+} = m_{B^0} = 5.278$ GeV; (b) with $m_{B^+} = m_{B^0} = 5.280$ GeV; (c) with $m_{B^0} = 5.280$ GeV and $m_{B^+} = 5.278$ GeV. One sees here the steep variation with W when $m_{B^0} > m_{B^+}$ owing to kinematic suppression of $B^0\bar{B}^0$ relative to B^+B^-. These curves are approximations to the ratio of exclusive cross sections; they are the ratios of modulus squared decay amplitudes multiplied by the ratio of phase space.[5]

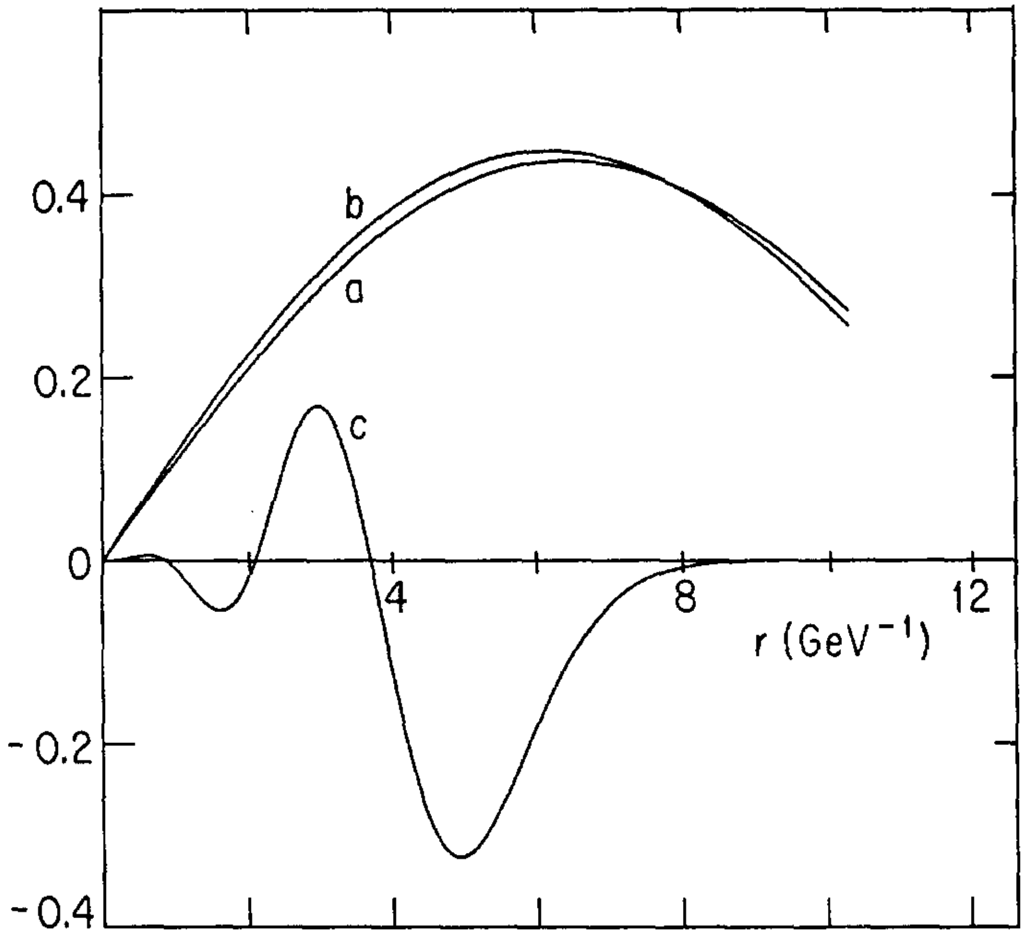

Figure 1

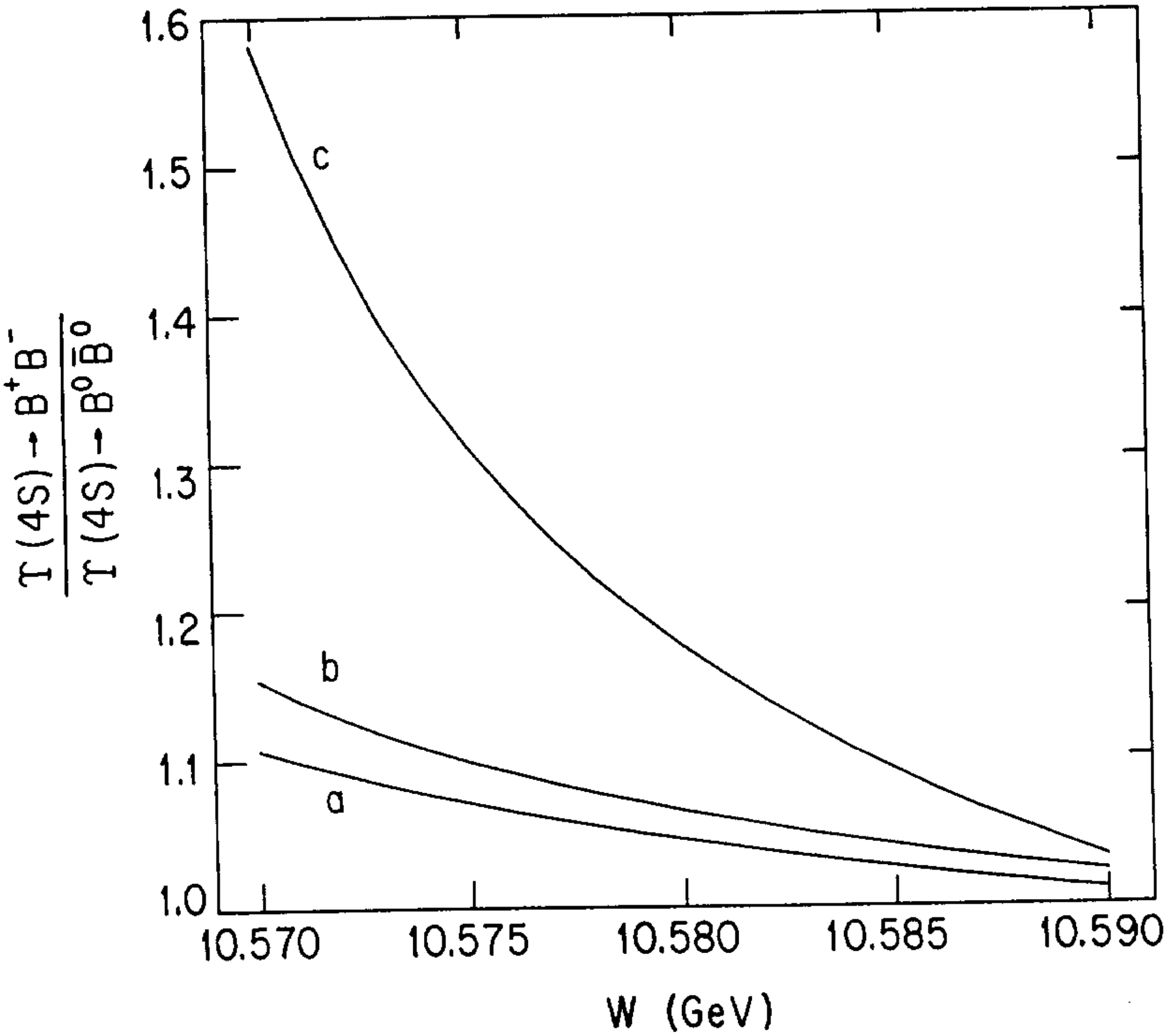

Figure 2