



Cosmic Technicolor Nuggets

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Abstract

We study the structure of technicolor theories at finite baryon number. In particular, we investigate whether ‘technicolor matter’, containing comparable numbers of all technifermions, may be absolutely stable, as opposed to individual technibaryons. This proposal is the technicolor analogue of Witten’s conjecture regarding the stability of strange (3-flavor quark) matter. We model technicolor using a rescaled version of the MIT bag. For gauge group $SU(N)_{TC}$, the stability and cosmic durability of technimatter is enhanced with increasing N and for large flavor symmetry. For $N \gtrsim 5 - 8$, technimatter nuggets formed at the technicolor confinement transition can survive to the present epoch, and they are likely to contain virtually all of the technibaryon number of the Universe. We discuss various astrophysical and experimental constraints on the abundance of technicolor nuggets in the galactic halo.



I. Introduction

Technicolor theories, designed to provide a natural origin for spontaneous symmetry breaking in the electroweak model, bear a close resemblance to quantum chromodynamics, albeit scaled up by three orders of magnitude in energy. It is therefore natural to raise the same questions in technicolor that one asks in QCD . In this paper, we inquire into the ground state of technicolor at finite baryon number. In particular, we ask whether technifermion matter, the analogue of strange quark matter, may be the lowest energy state instead of the lightest technibaryon. Although we cannot answer this question definitively, we find it quite likely that techni-matter is stable. This result has important implications for technicolor cosmology, for it means that technibaryons present in the Universe will be condensed into large nuggets instead of roaming free. Techni-nuggets are observationally consistent candidates for the dark matter in galaxy halos, while free technibaryons, if they carry electric charge, are not.

In the next Section, we review the general framework of technicolor and estimate the mass scales involved. In Section III, we compute the energy per technibaryon number of technifermion matter and compare it with the mass of the lightest technibaryon (LTB). Unfortunately, unlike the proton, the mass of the LTB is not known, so we estimate it using a rescaled version of the MIT bag model. In Section IV, we discuss the technicolor confinement transition and the formation of nuggets during the coexistence phase in the early Universe. We show under what conditions these nuggets survive evaporation in Section V. Finally, in Section VI, we investigate the experimental and astrophysical constraints on nuggets in the galactic halo.

II. The Framework of Technicolor

The standard model of electroweak interactions requires a set of elementary scalar fields, the Higgs bosons, in order to trigger the spontaneous breakdown of gauge symmetry, explicitly break flavor symmetry, and generate mass terms for vector bosons and fermions. Although the Higgs sector provides a single explanation for these phenomena in a rather economical way, it is plagued by serious difficulties. The unprotected quadratic divergences of scalar particle masses make the model unnatural [1]. Moreover, the arbitrariness of the Higgs sector and the large number of free parameters involved suggest that we are not dealing

with a fundamental theory.

Technicolor theories* [2] hope to cure these problems by replacing the scalar sector of the standard model with a set of fermions interacting via a new, asymptotically free force. At the scale Λ_{TC} , this new interaction – technicolor – becomes strong, and it is assumed that, similar to the behavior in QCD , the technifermion chiral symmetry is dynamically broken. Now, the Goldstone bosons of the spontaneously broken chiral symmetry provide the degrees of freedom for the longitudinal components of the W^\pm and Z^0 .

In building a technicolor model, one has to specify the choice of gauge group and fermion representation. It will be convenient for us to concentrate on a definite class of technicolor theories, although most of our considerations can be generalized to different models. We take $SU(N)$, with N arbitrary, as the technicolor group and assign n massless Dirac technifermions to the N fundamental gauge representation. We assume that left- and right-handed technifermions are respectively weak doublets and singlets. Finally, m of the n technifermions are $SU(3)$ -color triplets. The hypercharge (Y) assignment is chosen to guarantee anomaly cancellation. In particular, this implies $Tr Y = 0$, where the trace is taken over the technifermions. In the following, for illustrative purposes, we will often refer to the popular “one-family” model [4], which corresponds to $n = 8$, $m = 2$. (In the literature, the “one-doublet” model, with $n = 2$ and $m = 0$, is also occasionally considered.)

In the absence of any direct experimental information on the structure of technicolor, N , n , and m are left arbitrary. However, we can infer the constraint $n < \frac{11}{2}N$ by requiring that the running of the technicolor coupling constant

$$\alpha_{TC}^{-1}(\mu) = \frac{(11N - 2n)}{6\pi} \ln \frac{\mu}{\Lambda_{TC}} \quad (2.1)$$

leads to an asymptotically free theory. In addition, the constraints $m < 10$, $n < 20$ must be imposed if one requires that the asymptotic freedom of $SU(3)$ and $SU(2)$ forces is preserved after the introduction of the technifermions.

Using the $1/N$ expansion [5], one can scale from QCD to estimate the technicolor scale Λ_{TC} [3]. We find

$$\Lambda_{TC} \simeq \sqrt{\frac{3}{N}} \sqrt{\frac{\sqrt{2} \Lambda_{QCD}}{G_F \sqrt{n} f_\pi}} \simeq \sqrt{\frac{3}{N} \frac{8}{n}} 260 \text{ GeV} \quad , \quad (2.2)$$

where we have assumed that all $n/2$ weak doublets develop dynamical condensates. At an

*For reviews, see *e.g.* ref.[3].

energy scale below Λ_{TC} , technicolor becomes confining and technifermions are bound inside technicolor singlets. Because of spontaneous breakdown of the approximate chiral symmetry ($SU(n)_L \times SU(n)_R \rightarrow SU(n)_V$), the physical spectrum contains $n^2 - 4$ pseudo-Goldstone bosons, not eaten by the W^\pm and Z^0 , that are typically much lighter than the scale Λ_{TC} . The other technimesons have masses of order the technicolor scale. A large number of technibaryons are also present in the theory. The mass of the lightest technibaryon (LTB) can be estimated by rescaling the mass of the proton (m_p) with the help of the $1/N$ expansion [5]:

$$M_{LTB} \simeq \frac{N}{3} \frac{\Lambda_{TC}}{\Lambda_{QCD}} m_p \simeq \sqrt{\frac{N}{3} \frac{8}{n}} 1.2 \text{ TeV} . \quad (2.3)$$

The LTB has spin $1/2$ (0) if N is odd (even). For the one-family model, it has been predicted [6] that the LTB has electric charge $\pm \frac{(N-1)}{2}$ ($\pm \frac{N}{2}$) and weak isospin $1/2$ (0) for N odd (even). We note that the requirement of asymptotic freedom, $n < \frac{11}{2}N$, yields a lower bound on the LTB mass, $M_{LTB} \gtrsim 0.8 \text{ TeV}$.

Here we are interested in the cosmological fate of technicolor particles. As the temperature of the Universe drops, technimesons will annihilate away, and technibaryons will rapidly decay into the LTB . If technibaryon (TB) number is conserved, then the LTB is stable, and a relic density of technicolor particles produced in the early Universe could have survived until the present.

TB number is conserved by technicolor and standard interactions. However, we know that new forces – extended technicolor (ETC) [7] – must be introduced in order to generate quark and lepton masses. ETC interactions, which explicitly violate technifermion chiral symmetry, will contribute to the masses of the pseudo-Goldstone bosons, thus explaining why these particles have not yet been observed in collider experiments. Does ETC violate TB number? Unfortunately, because of the serious difficulties involved in building a phenomenologically consistent picture of ETC , there exists no compelling model to which this question can be addressed. However, if fermions and technifermions are assigned only to fundamental representations of ETC^\dagger , TB number is conserved. In fact, in this case, TB number is carried also by the ETC gauge bosons which couple fermions to technifermions. It is straightforward to show that this quantum number is conserved in cubic and quartic gauge boson couplings. If higher dimensional ETC representations are allowed, the existence

[†]This is the generally considered case in which the ETC group $SU(N + r)$ breaks down to $SU(N)$ technicolor and each fundamental representation of ETC splits into one technifermion and r ordinary fermions. If $r > 1$, r can play the role of a generation index.

of a conserved TB number is a model dependent issue. Nevertheless, barring the possibility of exotic ETC structures, it is reasonable to assume that the LTB is stable under ETC interactions[‡]. Finally, some GUT -scale interactions could violate TB . However, since these forces are expected to be very feeble at the scale Λ_{TC} , the resulting LTB lifetime, like that of the proton, would be much longer than the age of the Universe. For our purposes, such a particle is effectively stable.

It is also worth noting that more than one technibaryon can be stable if, in addition to a conserved TB number, the technicolor model is invariant under some accidental global symmetries (like baryon and lepton number in the standard model).

As for ordinary baryons, the relic density of technicolor particles may be fixed by a nonvanishing TB cosmic asymmetry, η_{TB} . In the absence of either a complete theory or observational evidence, not much can be said about the actual value of η_{TB} . However, if the LTB is stable, η_{TB} must be smaller than the baryon asymmetry (η_B) by at least an order of magnitude, in order for the present LTB energy density to satisfy the cosmological bound $\Omega_{LTB} h^2 < 1$. It is interesting to note [9] that, if η_{TB} is indeed not much smaller than η_B , relic technicolor particles can account for the dark matter in galaxy halos, providing a natural explanation for the observed density ratio between the luminous and dark components of the Universe.

In the next sections, we will consider the possibility that technicolor particles have survived the early stage of the Universe not in the form of technibaryons, but in a different form of bound matter: technicolor nuggets.

III. Technicolor Matter

We now consider whether the ground state of technicolor might consist not of isolated technibaryons but, instead, of large technifermion nuggets. This idea is analogous to the proposal [10] that strange quark matter[§] may be the stable ground state of the hadrons. In this section, we investigate the zero temperature limit; the finite temperature case will be discussed in the next section.

We will consider the MIT bag [12][¶] as a model for technihadron structure. Because of

[‡]The cosmology of unstable $LTBs$ has been considered in ref.[8].

[§]For reviews, see ref. [11].

[¶]For reviews, see ref.[13].

the uncertainties of this model, the non-perturbative nature of the problem, and the lack of a unique technicolor theory, our considerations are necessarily approximate. However, in order to bolster confidence in the model, we also apply our simplified version of the bag model to the case of QCD, where a comparison with experimental observations is possible, and find plausible results.

We first estimate the energy per TB number of a technicolor nugget in the limit of infinite TB number (bulk techni-matter). A sufficiently large nugget can be viewed as a degenerate Fermi gas of technifermions, and surface effects can be neglected. Inside the nugget, chemical equilibrium among the different species of technifermions is maintained by color, electroweak, and ETC interactions. Ordinary quarks and leptons, although not trapped inside the nugget, participate in the reactions that establish equilibrium much in the same way that neutrinos help maintain chemical equilibrium between up- and down-type quarks in strange matter[14]. In particular, ETC gauge bosons, which couple ordinary fermions (f) to technifermions (F), mediate processes like $Ff' \leftrightarrow F'f$ as well as four-technifermion interactions. Therefore, we can assign a unique chemical potential $\mu_a = \mu$ to all species a of technifermions confined inside the nugget.

The picture above must be modified if technifermions carry extra conserved quantum numbers arising from accidental global symmetries of the model. In that case, there is an independent chemical potential corresponding to each conserved quantum number. A nugget is then specified by the total value of each of the conserved quantum numbers, in addition to its total TB number. In the following, we will not consider this possibility, and focus instead on the simplest case of a single chemical potential corresponding to TB number. We note that, in the "one-family" model, the total TB number *is* sufficient to specify the system, even if ETC conserves baryon and lepton number. For this model, because of the interactions with quarks and leptons, baryon and lepton number can flow out of the nugget, and chemical equilibrium among all species of technifermions is maintained.

If all technifermions are massless and carry the same chemical potential μ , the anomaly-free condition on the hypercharge ($TrY = 0$) guarantees that the total electric charge of the nugget is zero. Therefore, we can neglect the effect of Coulomb energy in our treatment of the bulk techni-matter. (For finite nuggets, the charge may play an important role astrophysically and experimentally; see Sec. VI.)

At zero temperature, the thermodynamic potential to first order in α_{TC} is given by [15]:

$$\Omega = - \sum_a \frac{D_a}{12\pi^2} \mu_a^4 \left(1 - \frac{3C}{2\pi} \alpha_{TC} \right). \quad (3.1)$$

The sum in eqn.(3.1) runs over all species of massless technifermions. Here, D_a is the dimension of the technicolor fermion representation ($D_a = N$), μ_a is the technifermion chemical potential of species a , and C is the quadratic Casimir in the adjoint representation, $C = \frac{N^2-1}{2N}$. The energy density is given by

$$\epsilon = \Omega + \sum_a \mu_a n_a + B, \quad (3.2)$$

where B is the vacuum energy density (the technicolor bag constant) which describes the confining properties of technicolor interactions. In eqn.(3.2), n_a is the number density of each species:

$$n_a = - \frac{\partial \Omega}{\partial \mu_a}. \quad (3.3)$$

For technifermions in the fundamental representation, the TB number density is given by:

$$n_{TB} = \frac{1}{N} \sum_a n_a. \quad (3.4)$$

The technifermion chemical potential μ can be computed by minimizing the energy per TB number:

$$\frac{\partial}{\partial \mu} \left(\frac{\epsilon}{n_{TB}} \right) = 0. \quad (3.5)$$

Substituting the value of μ found from eqn.(3.5) into eqns.(3.2), (3.4) we obtain the energy per TB number of the nugget:

$$\frac{\epsilon}{n_{TB}} = \left(\frac{12\pi^2 N^3}{n \left[1 - \frac{3(N^2-1)}{4\pi N} \alpha_{TC} \right]} B \right)^{1/4}, \quad (3.6)$$

where n is the number of technifermion flavors.

To determine whether nuggets have lower energy per TB number than technibaryons, eqn.(3.6) must be compared with the mass of the LTB . We therefore now compute the mass of LTB as a function of B and α_{TC} , using the *MIT* bag model [12].

In the *MIT* bag model, the energy of the (techni)baryon is expressed as a function of the bag radius R as

$$E(R) = \frac{4}{3}\pi B R^3 + \frac{xN}{R} + \frac{Z_0}{R} - \frac{\alpha_{TC}}{R} W . \quad (3.7)$$

The first term in eqn.(3.7) corresponds to the volume energy of the bag and mimics the confining potential of technicolor. The second term is the kinetic energy of the N constituent technifermions. For massless technifermions in the fundamental cavity mode, $x = 2.04$ [12]. The third term (Z_0) represents the zero-point energy; in *QCD* bag models, it is usually taken as a free parameter, determined by fitting the known hadron mass spectrum. The reason for this is that, when computed for a spherical bag, Z_0 is found to be divergent [16], and the necessary renormalization makes it completely arbitrary. For now we assume that Z_0 can be neglected, and will return later to this question. Finally, the last term in eqn.(3.7) gives the technichromo- electrostatic and magnetostatic energy due to one-technigluon exchange. This represents a short-distance effect and can be computed perturbatively in α_{TC} , because of the asymptotic freedom of technicolor theories. In appendix A, we evaluate this term and find, for the *LTB*:

$$W = 0.176 \left(1 + \frac{1}{N} \right) [3N - 4s(s+1)] , \quad (3.8)$$

where s is the spin of the *LTB* ($s=0$ if N is even, $s=1/2$ if N is odd).

The mass of the technibaryon is determined by minimizing $E(R)$ as a function of R , with the result:

$$M_{LTB} = \frac{4}{3} [xN + Z_0 - \alpha_{TC}W]^{3/4} (4\pi B)^{1/4} . \quad (3.9)$$

In fig.1, we plot (solid lines) the difference between M_{LTB} [eqn.(3.9)] and ϵ/n_{TB} [eqn.(3.6)] in units of M_{LTB} , for the case $Z_0 = 0$. This ratio is independent of B and also independent of N to leading order in the $1/N$ expansion; it grows with n , *i.e.*, for large flavor symmetries. In Fig.1 we have set $n = 8$ ("one-family" model) and show results for several phenomenologically plausible values of α_{TC} (inside the bag). For consistency with the $1/N$ expansion, we define $\alpha_{TC} = \alpha_N/N$ and keep α_N fixed as N varies. To get a refined estimate, in Appendix B we calculate the "center of mass" corrections [17] to M_{LTB} ; using these corrections yields the results given by the dashed lines in fig.1. In the case of *QCD* ($N = 3$), as discussed in Appendix B, this correction corresponds to taking an effective value of Z_0 which is roughly compatible with the one derived from fitting the hadronic mass spectrum.

To get an idea of the uncertainties involved in neglecting Z_0 in eqn.(3.9), we can use the

expression for the regularized zero-point energy of a spherical cavity derived by Milton [16],

$${}_M Z_0 = \frac{1}{8\pi} \left[N^2 - 1 - \frac{nN}{6} \right], \quad (3.10)$$

In this case, since ${}_M Z_0$ is positive and grows like N^2 , the nuggets are further energetically favored for large N .

Using the estimate of M_{LTB} from the $1/N$ expansion given in eqn.(2.2), in fig.2 we plot the energy difference $M_{LTB} - \epsilon/n_{TB}$ as a function of N , again for $n = 8$. As above, we show results both with (dashed curves) and without (solid curves) the ‘‘center of mass’’ corrections, assuming $Z_0 = 0$.

If we apply these estimates to QCD ($N = 3$), we find the encouraging result that, for $\alpha_c > 0.7$ ($Z_0 = 0$) or $\alpha_c > 0.8$ ($Z_0 = {}_M Z_0$), the lightest baryons are stable with respect to two-flavor quark matter ($n = 2$), but strange matter ($n = 3$) is energetically favored in the massless strange quark limit. In technicolor, the (approximate) large flavor symmetry generally present in semi-realistic models tends to favor the stability of technifermion nuggets with respect to technihadrons.

The results shown in figs. 1, 2 suggest that, for a plausible range of parameters, techninuggets rather than technibaryons constitute the true ground state of technicolor. Although these estimates contain large uncertainties, the qualitative trend is clear, and we are sufficiently encouraged to consider the fate of nuggets at finite temperature in the early universe.

IV. Nugget Formation: the Techni-confinement Transition

In this section, we discuss the transition from the technifermion-gluon plasma to confined technihadrons in the early Universe, using the bag model developed above. During and soon after this transition, stable techninuggets may have formed and possibly survive as relics today.

Investigations of lattice QCD indicate that the color confinement and/or chiral symmetry breaking transition is first-order, although the issue is not firmly settled^{||}. In addition, analytic arguments (see, *e.g.*, ref.[20]) suggest that, in absence of fermions, the $SU(N)$ transition is first order for all $N \geq 3$, and we will therefore assume the technicolor phase

^{||}For a recent discussion, see ref.[19] and references therein.

transition is first order as well. We make this assumption primarily for simplicity, and because it provides a framework for the discussion of nugget formation via the “separation of cosmic phases” scenario discussed by Witten for strange matter [10]. We can, however, easily envision scenarios in which nuggets form in a second order phase transition instead. The order of the transition also has little impact on the subsequent evolution of nuggets once they are born.

Our picture of the transition epoch is the following. At high temperatures in the early Universe ($T \gg 100$ GeV, $t \ll 10^{-11}$ sec), technicolor is unconfined. In this phase, technicolor is carried by a thermal plasma of relativistic technifermions and technigluons. As the Universe expands, the temperature drops through the coexistence temperature, T_c , below which the confined hadron phase is thermodynamically preferred. In a first order transition, the Universe initially remains in the unconfined phase, supercooling below T_c until the probability to nucleate technihadron bubbles becomes large. Once the nucleation rate is appreciable, the latent heat released by the first bubbles reheats the Universe to the coexistence temperature, whence further nucleation is suppressed. At T_c , the fermion-gluon plasma exists in pressure and possibly chemical equilibrium with the confined phase. In this coexistence epoch, the hadron bubbles grow into the unconfined phase; although the Universe is expanding, the temperature is fixed at T_c by the continuing release of latent heat by the growing bubbles. Eventually the hadron regions coalesce and percolate, and the unconfined phase regions shrink away. When the volume of remaining unconfined phase becomes small, the latent heat release is no longer sufficient to keep the Universe at T_c , and it begins to cool once again.

Whether and how technicolor nuggets arise depends upon the details of transport processes during the coexistence epoch. In this era, in chemical equilibrium the relative technibaryon number density will be higher in the unconfined phase than in the hadron phase, because TB number is carried by lighter species there (nearly massless fermions as opposed to heavy technibaryons). If the transport of TB number across the phase boundary separating confined and unconfined regions is not fast enough to maintain chemical equilibrium, the TB number density excess in the fermion-gluon plasma will grow as these regions shrink. As a result, the chemical potential μ in the unconfined phase builds up to a value of order T_c . Once this occurs, the fermion-gluon regions have become a relatively ‘cold’, low-entropy phase. From this point onward, the degeneracy pressure of the fermions becomes dynamically important and eventually stabilizes the shrinking bubbles against further demise. Qualitatively, as μ/T increases in the unconfined phase, the effective coexistence temperature drops

to zero [10]. Unfortunately, as with QCD, our understanding of technibaryon number transport across the phase boundary, and of the dynamics of bubble nucleation and growth, is insufficient to make firm predictions about the spectrum of nuggets produced. Instead we shall make approximate estimates which demonstrate the plausibility of forming large nuggets. In the next section we discuss under what conditions nuggets are expected to survive.

To study the dynamics of the confinement transition in more detail, we first need to calculate the coexistence temperature T_c . For the fermion-gluon phase, the thermodynamic potential can be written

$$\Omega_{fg} = - \sum_a \frac{D_a}{12\pi^2} \left(\frac{7\pi^4}{15} T^4 + 2\pi^2 \mu_a^2 T^2 + \mu_a^4 \right) - \frac{\pi^2}{45} (N^2 - 1) T^4, \quad (4.1)$$

where the second term is the gluon contribution. For simplicity, we have made the ideal gas approximation, ignoring $\mathcal{O}(\alpha_{TC})$ corrections; we shall similarly neglect interactions in the confined phase below. This expression then reduces to Eqn.(3.1) in the limit $T \rightarrow 0$. In this section, we will be primarily interested in the opposite limit, $\mu_a/T \ll 1$. In Eqn.(4.1), we have ignored the contribution from ordinary, technicolor-blind particles (e.g., photons and leptons), because they contribute equally to both phases. In terms of Ω , the pressure, entropy density, and energy density are given by:

$$p_{fg} = -\Omega_{fg} - B_T, \quad s_{fg} = - \left(\frac{\partial \Omega_{fg}}{\partial T} \right)_\mu, \quad \epsilon_{fg} = -p_{fg} + T s_{fg} + \sum_a \mu_a n_a, \quad (4.2)$$

where n_a is defined by Eqn.(3.3). Here, B_T is the energy density of the unconfined phase at finite temperature, and may differ from the zero temperature bag constant B . In numerical estimates below, we shall assume $B_T = B$, but we use this notation to make clear how things depend on the different variables.

In the confined techni-hadron phase, the dominant contribution to the thermodynamic potential comes from the light techni-meson degrees of freedom. Analogously to chiral-symmetric QCD, there are $n^2 - 4$ pseudo-Nambu-Goldstone modes; although they will get small masses, we assume these satisfy $m \lesssim T_c$. In addition, there will be a contribution from the technibaryon multiplets, but this is exponentially suppressed for $M_{LTB} \gg T_c$; since this condition is satisfied, as we show below, we can ignore the technibaryon contribution to Ω to good approximation. In addition, we can assume $\mu \ll T$ in the confined phase, so the

thermodynamic potential and pressure in this case are simply

$$\Omega_h = -p_h = -\frac{\pi^2}{90}(n^2 - 4)T^4 . \quad (4.3)$$

The coexistence temperature is defined implicitly by $p_{fg}(T_c) = p_h(T_c)$. Using Eqns.(4.1-4.3), we find

$$T_c = \left(\frac{180B_T}{\pi^2[4 + (4N - n)(N + 2n)]} \right)^{1/4} . \quad (4.4)$$

As an example, for $n = 8$, as N increases from 3 to 15, $T_c/B_T^{1/4}$ drops from about 0.69 to 0.33. To get a numerical estimate for T_c , as before we fix the bag constant $B_T = B$ by fitting the bag model estimate for M_{LTB} , Eqn.(3.9), to the $1/N$ estimate of Eqn.(2.2). The result is shown in Fig.3, which shows the coexistence temperature in GeV as a function of N , for different values of α_N ; as in Figs.1 and 2, we have set $Z_0 = 0$, and show results both with (dashed curves) and without (solid curves) the ‘‘center of mass’’ correction. If we instead set $Z_0 = {}_M Z_0$ using Eqn.(3.10), the curves in Fig.3 changes almost imperceptibly.

To qualitatively verify the self-consistency of not including the technibaryons in Ω_h , in Fig.4 we show the ratio M_{LTB}/T_c as a function of N . Since this ratio varies roughly between 15 and 100, the Boltzmann factor $\exp(-M_{LTB}/T_c)$ strongly suppresses the technibaryon contribution, even if the lightest technibaryon state is highly degenerate.

For nuggets to form, and in order for most of the technibaryon number of the Universe to be carried by nuggets, the TB number must become concentrated in the unconfined plasma phase during the coexistence epoch. An estimate of this concentration is found by assuming chemical equilibrium between the two phases; this corresponds to the case of rapid TB number transport across the phase boundary. From Eqns.(3.3), (3.4), and (4.1), the technibaryon number density in the unconfined phase is, to lowest order in μ/T ,

$$n_{TB}^{fg} = \frac{1}{3}T^3 \left(\frac{\mu_{TB}}{T} \right) \left(\frac{n}{N} \right) , \quad (4.5)$$

where we have used the fact that the technifermion and technibaryon chemical potentials are related by $\mu_a = \mu_{TB}/N$. In the hadron phase, the TB number density is

$$n_{TB}^h = A \left(\frac{M_{LTB}T}{2\pi} \right)^{3/2} \frac{\mu_{TB}}{T} e^{(-M_{LTB}/T)} . \quad (4.6)$$

Here, the coefficient A counts the number of ‘light’ technibaryons times their degrees of freedom. If we neglect mass splittings due to the other interactions besides technicolor and

due to technifermion current masses, the lightest technibaryon state belongs to a degenerate flavor $SU(n)$ multiplet, with

$$A = \frac{2 \left(n + \frac{N}{2} - 1\right)! \left(n + \frac{N}{2} - 2\right)!}{(n-1)!(n-2)! \left(\frac{N}{2}\right)! \left(\frac{N}{2} + 1\right)!} \quad (N \text{ even}) \quad (4.7a)$$

and

$$A = \frac{8 \left(n + \frac{N-1}{2}\right)! \left(n + \frac{N-5}{2}\right)!}{(n-1)!(n-2)! \left(\frac{N+3}{2}\right)! \left(\frac{N-1}{2}\right)!} \quad (N \text{ odd}) \quad (4.7b)$$

The equilibrium technibaryon number density ratio in the two phases is then given by

$$R(T) \equiv \frac{n_{TB}^{fg}}{n_{TB}^h} = \frac{n}{3NA} \left(\frac{2\pi T}{M_{LTB}} \right)^{3/2} e^{(M_{LTB}/T)} . \quad (4.8)$$

In Fig.5, we plot $R(T_c)$ for the case $n = 8$. Since the coexistence temperature T_c falls roughly like $N^{-3/4}$ for large N , the ratio $R(T_c)$ rises exponentially with N . This dramatic increase occurs despite the fact that the number of light technibaryon degrees of freedom A rises sharply with N . Consequently, even for moderate values of N , the technibaryon number of the Universe may be highly concentrated in the unconfined plasma phase during the coexistence epoch. Thus, if the shrinking plasma bubbles decouple and reach Fermi degeneracy before they disappear, a very high fraction of the TB number may end up in nuggets. By comparison, the baryon number contrast contemplated in the QCD transition is rather modest, $R_{QCD} \lesssim \text{few } 100$; this is consistent with Eqn.(4.8) evaluated for $N = 3$ and $n = 2$ or 3 .

In actuality, the assumption of chemical equilibrium above yields a lower limit on the TB number contrast R between the unconfined and confined phases. If TB number transport across the phase boundary is inhibited, so that chemical equilibrium is not maintained, then the technibaryon number will be trapped in the unconfined phase as the moving phase boundary sweeps it into the shrinking plasma regions. In this case, the contrast R will be increased over Eqn.(4.8) by the ratio of TB chemical potentials in the two phases, μ_{TB}^f/μ_{TB}^h . We can estimate an upper bound on the TB transport rate from the unconfined to the confined phase, using the phase space argument of Fuller, Mathews, and Alcock [21].

The total rate of technifermion recombination into both technibaryons and antibaryons at the phase boundary is

$$\Lambda = f_{TF} \Sigma_{TF} , \quad (4.9)$$

where f_{TF} is the net flux of technifermions and Σ_{TF} is the probability of combining N technifermions at the boundary into a technicolor singlet. We can write $\Sigma_{TF} = P_N \Sigma_{tf}$, where $\Sigma_{tf} \leq 1$ is the technifermion transmission probability through the phase boundary, and P_N is the probability of finding N technifermions (of the right quantum numbers to make a singlet technibaryon) in a volume equal to the size of the LTB . By Poisson statistics, the probability of finding one technifermion in such a volume is just $\bar{N}_{tf} = n_{tf} V_{LTB}$, where the average technifermion density (for each species) is $n_{tf} = \frac{3}{4} \frac{\zeta(3)}{\pi^2} T_c^3$, $\zeta(3) = 1.202\dots$, and V_{LTB} is the volume of the LTB . From the discussion of Section III, we easily find $M_{LTB} = \frac{4}{3} \pi R^3 (4B)$, so that the LTB volume is just $V_{LTB} = M_{LTB}/4B$. Thus, the recombination rate is $\Sigma_{TF} = \bar{N}_{tf}^N \Sigma_{tf}$, or using Eqn.(4.4),

$$\Sigma_{TF} = (0.86)^N \left[\frac{N}{4 + (4N - n)(N + 2n)} \right]^{3N/4} \Sigma_{tf} . \quad (4.10)$$

In Eqn.(4.10), for simplicity we have set $\alpha_{TC} = Z_0 = 0$ in expression (3.9) for the LTB mass. Using Σ_{TF} from Eqn.(4.10) and the technifermion flux $f_{TF} \simeq \frac{n_{tf} n N}{4}$ in Eqn.(4.8) yields the total recombination rate Λ .

The net TB number transport rate across the boundary (the TB number per unit time per unit area passed by the wall) is

$$\Lambda_{TB} = \Lambda \xi(T_c) , \quad (4.11)$$

where $\xi \simeq \mu_{TB}/T$ is the fractional excess of technifermions over antifermions in the unconfined phase. This is to be compared to the rate at which the wall encounters TB number as it moves into the unconfined region,

$$\frac{dn_{TB}}{dt} = n_{TB}^{fg} v_{pb} , \quad (4.12)$$

where n_{TB}^{fg} is given by Eqn.(4.5) and v_{pb} is the speed of the phase boundary. The fraction of TB number filtered through the boundary as it moves past a given volume of unconfined phase is then

$$F = \frac{\Lambda_{TB}}{\dot{n}_{TB}} \simeq 7 \cdot 10^{-2} N^2 \frac{\Sigma_{TF}}{v_{pb}} . \quad (4.13)$$

If the filter factor $F \ll 1$, then the TB transport across the boundary is suppressed, and the TB concentration in the unconfined phase is expected to be large. From Eqns.(4.10) and (4.13), we see that F depends upon the technifermion transmission probability Σ_{tf} and

the phase boundary speed v_{pb} . In addition, F drops steeply with increasing N , since the probability of finding the large number of technifermions required to make a technibaryon in a small enough volume is phase-space suppressed. Thus, at large N , TB transport is inefficient even if the transmission probability Σ_{tf} is of order unity. For example, for $n = 8$ and $v_{pb} = 0.1$, we find $F/\Sigma_{tf} \simeq 2 \times 10^{-3}, 9 \times 10^{-5}, 3 \times 10^{-6}, 1 \times 10^{-7}, 9 \times 10^{-11}$, and 6×10^{-14} for $N = 3, 4, 5, 6, 8$, and 10 . Furthermore, since nuggets are stable at low temperature, the transmission probability Σ_{tf} should be suppressed by a factor of order $T_c/(M_{LTB} - \epsilon/n_{TB})$; that is, the thermal energy of the technifermions must be comparable to the energy gap for them to pass through the boundary. From Fig.4, this leads to a further decrease in F by an order of magnitude or two.

From the preceding discussion, we conclude that, even for moderate values of N , the TB number of the Universe is essentially completely trapped inside the shrinking unconfined regions at the coexistence epoch. We therefore assume that techni-nugget formation takes place. Although we cannot predict in detail the size or mass distribution of the nuggets produced, we can get a rough estimate of the characteristic expected nugget size from the following argument**.

At the end of the initial nucleation phase, when the Universe has been reheated to T_c , the technihadron bubbles have a characteristic mean separation r . Subsequently, these bubbles expand and, soon after they percolate, the Universe will contain finite fermion-gluon bubbles with approximately the same characteristic separation. From this 'duality' argument, we find the expected size (and separation) of the proto-nugget regions to be [21]

$$r \sim \frac{\sigma^{3/2} t}{T_c^{1/2} L} . \quad (4.14)$$

Here, σ is the surface tension of the phase boundary, t is the cosmic time, and $L = 4B_T$ is the latent heat of the transition. In the absence of experimental data on the spectrum of technihadron excitations, the surface tension of the bag is largely unconstrained. On dimensional grounds, we expect $\sigma^{1/3} \sim B^{1/4}$. Defining $\gamma_\sigma \equiv \sigma^{1/3}/B^{1/4}$, we have

$$\frac{r}{t} \sim \gamma_\sigma^{9/2} \left(\frac{B^{1/4}}{T_c} \right)^{1/2} , \quad (4.15)$$

where the second factor on the right hand side, given by Eqn.(4.4), is between 1.2 and 1.7 (for $n = 8$ and $3 \leq N \leq 15$). The characteristic size r depends sensitively on the unknown

**We thank Charles Alcock for pointing out this estimate.

surface tension. If there are no small parameters in the theory, we expect $\gamma_\sigma \gtrsim 0.1$, in which case we obtain $r/t > 10^{-5}$; we will use this as a rough lower bound on the proto-nugget size. In QCD, typical recent estimates of the surface tension are in the range [14,25] $\sigma^{1/3} \simeq 50 - 70$ MeV, corresponding to $\gamma_\sigma \simeq \frac{1}{3} - \frac{1}{2}$. (An independent argument that γ_σ is not much smaller than unity comes from lattice simulations: if $\gamma_\sigma \ll 1$, the bubble nucleation probability is so large that the phase transition is effectively second order (with no supercooling), in contrast with the numerical results for QCD.)

We expect a proto-nugget region to have a typical TB number $N_{TB}^* \sim n_{TB}(T_c)r^3$ when it begins to shrink; here, n_{TB} is the mean TB density at the coexistence epoch, given by Eqn.(4.5). The TB number will be effectively trapped inside if $F\bar{v}(\Delta t/r) \ll 1$, where F is the filter factor of Eqn.(4.13), \bar{v} is a typical thermal technifermion speed of order unity, and Δt is the duration of the phase transition. We expect $\Delta t \sim t_c$, the Hubble time at the coexistence temperature. From the estimates of F and the lower bound on r/t given above, this inequality is satisfied for $N \gtrsim 4$. Consequently, the proto-nugget retains its initial TB number,

$$N_{TB}^* \simeq 3 \times 10^{44} \frac{\Omega_{TB} h^2}{g_*^{1/2}} \left(\frac{\text{GeV}}{T_c} \right)^3 \left(\frac{\text{TeV}}{M_{LTB}} \right) \left(\frac{r}{t} \right)^3, \quad (4.16)$$

where Ω_{TB} is the fraction of critical density contributed by technibaryons. We expect the distribution of nugget TB numbers to be peaked about this value. From Fig.3, for moderate values of N we find typically $T_c \simeq 50$ GeV, so that $ct_c \simeq 3$ cm; from the lower bound on r/t above, this yields $r \gtrsim 3 \times 10^{-5}$ cm and $N_{TB}^* \gtrsim 6 \times 10^{22}$. Thus the characteristic nugget mass is $M_{nugget} \gtrsim 100$ gm. In the next section, we consider the evolution of condensed nuggets subsequent to the coexistence epoch.

V. Cosmic Evolution of Technicolor Nuggets

Armed with our heuristic understanding of techni-nugget formation, we now ask whether nuggets formed at the confinement transition could have survived to the present. Indeed, in the case of strange quark matter, it has been shown that all causally formed lumps would rapidly evaporate [22] or boil [23] away. Although stable at zero temperature, strange nuggets are not thermodynamically preferred at temperatures comparable to T_c , because they are states of low entropy (and thus high free energy). In quantitative terms, strange nuggets disappear because their binding energy per baryon number is less than (or at most comparable to) the critical temperature T_c . On the other hand, the relative binding energy

of technimatter nuggets can be quite large; comparison of Figs. 2 and 5 shows the binding energy is generally much larger than T_c , especially for large N .

Our task is to find under what conditions technicolor nuggets survive. Once a nugget is formed, its fate is determined by the competition between accretion and evaporation of technibaryons. [22,24] (It is easy to show that nugget-nugget interactions are unimportant, *i.e.*, much slower than the expansion rate, for nuggets with TB number larger than a few thousand.) For a nugget of total TB number N_{TB} , the cross-section for absorption of a technibaryon can be written,

$$\sigma_c = 4\pi f_{TB} R(N_{TB})^2 = 4\pi f_{TB} \left(\frac{\beta}{M_{LTB}} \right)^2 N_{TB}^{2/3} . \quad (5.1)$$

Here, $f_{TB} (\leq 1)$ is the technibaryon absorption efficiency, which we expect to be of order unity, and $R(N_{TB})$ is the nugget radius. The parameter β is also of order unity, and is given by

$$\beta = \left(\frac{3M_{LTB}^4(1-\Delta)}{16\pi B} \right)^{1/3} , \quad (5.2)$$

where

$$\Delta \equiv \frac{M_{LTB} - \epsilon/n_{TB}}{M_{LTB}} \quad (5.3)$$

is the nugget binding energy per TB number plotted in Fig.1. For example, for $n = 8$, $\beta(N = 3)$ ranges between 4.6 and 6.5 as the value of α_N is varied. As N increases, β shows a slow monotonic rise, reaching a value of about 31 for $N = 15$.

From detailed balance arguments, the net evolution rate for a lump of TB number N_{TB} is

$$\frac{dN_{TB}}{dt} = \frac{2}{\pi} f_{TB} \beta^2 \frac{T^2}{M_{LTB}} N_{TB}^{2/3} \left(\sqrt{2} \frac{n_{LTB}}{T^3} \left(\frac{\pi T}{M_{LTB}} \right)^{3/2} - \exp \left[-\frac{M_{LTB} \Delta}{T} \right] \right) , \quad (5.4)$$

where the first term is due to absorption and the second is from evaporation. In writing (5.4), we have assumed the lump is kept in good thermal contact (temperature equilibrium) with the environment; we have also assumed the immediate exterior of the lump is sufficiently dilute that the flow of technibaryons into or out of the nugget is not inhibited.

We now define j as the fraction of the total TB number of the Universe in the nugget phase, so the number density of light technibaryons is

$$n_{LTB}(T) = (1-j)\eta_{TB}n_\gamma . \quad (5.5)$$

Here, η_{TB} is the net technibaryon asymmetry (technibaryon to photon ratio), and $n_\gamma = 2.4T^3/\pi^2$ is the photon number density. If nugget formation is very efficient, as envisioned in Section IV, then j can be very close to 1 ($j \simeq 1 - R^{-1}$, where R is shown in Fig.5). It is convenient to introduce a dimensionless temperature (or time) variable, $x \equiv M_{LTB}/T$, in terms of which the nugget evolution equation can be written

$$\frac{dN_{TB}}{dx} = \frac{1.2}{\pi g_*^{1/2}} f_{TB} \beta^2 \left(\frac{m_{Pl}}{M_{LTB}} \right) x^{-1} N_{TB}^{2/3} [1.92\eta_{TB}(1-j)x^{-3/2} - e^{-\Delta x}] \quad (5.6)$$

Here, we have used the temperature-time relationship for a radiation-dominated Universe, and g_* is the effective number of relativistic degrees of freedom.

In the evolution of a lump, there are generally three important epochs: a) the formation epoch, $x_c = M_{LTB}/T_c$; b) the ‘turn-around’ era, $x_t = M_{LTB}/T_t$, which marks the transition from an early evaporation to a later accretion phase, and is defined by $(dN_{TB}/dx)_{x_t} = 0$; and c) the freeze-out epoch, $x_F = M_{LTB}/T_F$, when the nugget evolution rate falls below the expansion rate $H = 1/2t$, i.e., $(|\dot{N}_{TB}|/N_{TB}H)_{x_F} = 1$. Only lumps large enough to survive up to x_F or x_t are present today. The formation parameter x_c was shown in Fig.4, while the turn-around epoch is defined implicitly by

$$\ln[1.92\eta_{TB}(1-j)] = 1.5\ln x_t - \Delta x_t \quad (5.7)$$

For $\eta_{TB}(1-j) = 10^{-11}$, which corresponds approximately to $\Omega_{LTB}h^2 = 1$ if j is not extremely close to 1, we find that x_t varies between about 90 (for $\Delta = 0.3$) and 60 (for $\Delta = 0.54$). As one would expect, nuggets with larger binding energy (larger Δ) evaporate for a shorter time. Moreover, since the formation parameter x_c increases with N , for sufficiently large N we find that $x_c > x_t$. In this case, nuggets formed at the coexistence epoch undergo no evaporation at all, and either accrete immediately or are born ‘frozen out.’ For simplicity, first consider the case $\alpha_N = 0$ with no ‘center of mass’ correction; in this case, Fig.1 shows that $\Delta = 0.54$, independent of N . From above, we then have $x_t \simeq 60$, and comparison with Fig.4 shows that $x_c > x_t$ for $N > 10$. With α_N and ‘center of mass’ corrections included, the critical value N_{crit} above which evaporation is quenched will be slightly larger than this, but quite generally $N_{crit} \leq 13$. Thus, for $N \geq 13$, all lumps born in the coexistence phase should survive. In principle, such nuggets could actually grow by accretion before they freeze out. However, for these large values of N , we recall that $R(T_c)$ (Fig.5) is extremely large, of order 10^{20} or more. Thus, if TB number transport is even slightly suppressed, we expect the fraction j of TB number initially trapped inside nuggets to be very close to one. As a result,

there are very few technibaryons around to absorb. (Of course, for j very close to 1, the value of x_t increases: the evaporation phase lasts longer if there are very few technibaryons to accrete.)

For $N < N_{crit}$, and generally for j very close to 1, lumps are born evaporating or frozen-out. Approximately integrating Eqn.(5.6) during the evaporation phase (*i.e.*, neglecting the accretion term), we find that the smallest nugget which survives has net TB number

$$N_{TB}^s(x_c) \simeq 3.6 \times 10^{45} \left(\frac{M_{LTB}}{\text{TeV}} \right)^{-3} \left(\frac{f_{TB} \beta^2 e^{-\Delta x_c}}{(1 + \Delta x_c) g_*^{1/2}} \right)^3$$

As advertised at the beginning of this section, N_{TB}^s depends primarily on $\Delta x_c = (M_{LTB} - \epsilon/n_{TB})/T_c$, the ratio of the nugget binding energy per TB number to the coexistence temperature. In Fig.6, we plot this parameter as a function of N . With increasing N , Δx_c grows and therefore N_{TB}^s drops precipitously: for larger N , smaller nuggets can survive evaporation. For $N = 3$, we find N_{TB}^s is between 9×10^{32} and 3×10^{40} , depending on the value of α_N (clearly, N_{TB}^s increases with α_N , since Δx_c falls). For $N = 6$, N_{TB}^s has fallen to between 10^{20} and 10^{30} . As N approaches N_{crit} from below, N_{TB}^s falls rapidly to zero: for $N \gtrsim 10 - 12$, *all* nuggets are born frozen-out and therefore survive. These numbers should be compared to the largest nugget which can be causally formed at T_c ; this is just the TB number contained in the particle horizon at that time, which is

$$N_{TB}^H(x_c) = 1.2 \times 10^{38} \frac{\Omega_{TB} h^2 x_c^3}{g_*^{3/2}} \left(\frac{\text{TeV}}{M_{LTB}} \right)^4. \quad (5.9)$$

For $N \geq 4$, we find $N_{TB}^s < N_{TB}^H$, so the largest causally formed nuggets can survive. Of more physical interest, we can compare N_{TB}^s with N_{TB}^* , the characteristic nugget charge given by Eqn.(4.16). Using the lower bound $r/t \geq 10^{-5}$, we find $N_{TB}^s < N_{TB}^*$ for $N > 5 - 8$ (the range in N here again corresponds to varying α_N).

As N increases, then, we have the following rough sequence (assuming $n = 8$): i) for $N < 4$, all lumps smaller than the horizon evaporate away; ii) for $N \gtrsim 5 - 8$, small lumps (if formed) still disappear, but nuggets of the expected characteristic size are born frozen-out; iii) for $N \gtrsim 10 - 12$, all lumps are born frozen-out. We thus expect the majority of lumps to survive for $N \gtrsim 5 - 8$. Furthermore, if the initial nugget size distribution is peaked about the characteristic size [Eqn.(4.16)], these surviving lumps will carry essentially all of the TB number of the Universe.

So far, we have assumed surface evaporation is the dominant decay mode for techninuggets at finite temperature. In addition, however, a techni-lump may boil away by catastrophic nucleation of technihadron bubbles throughout its interior [23]. The probability of bubble nucleation is very sensitive to the surface tension of the bubble wall, σ . Generally, if σ is greater than a critical value σ_{crit} , then the free energy associated with the bubbles is sufficiently large that boiling is unimportant. From Section VI of ref.[24], we find $\sigma_{crit}(x_c)/M_{LTB} \ll 10^{-2}$ for all $N > 3$. Therefore, if $\gamma_\sigma = \sigma^{1/3}/B^{1/4} \gtrsim 10^{-1}$, we always have $\sigma \gg \sigma_{crit}$, and boiling may be safely ignored.

In the preceding discussion, the technibaryon number of the forming and evolving nuggets was a function of the cosmic TB number asymmetry η_{TB} . For $\eta_{TB} \simeq 10^{-11}$, the mass density of technibaryons (either free or in lumps) is comparable to the critical density for a spatially flat Universe. What happens if there is no TB asymmetry, or if it is much smaller than 10^{-11} ? The discussion of TB number transport given in Section IV would go through essentially unchanged, because the chemical potential drops out of the filter factor F [Eqn.(4.13)]. Thus, we expect to form proto-nugget regions of the same characteristic size r , given by Eqn.(4.14), and they will trap the enclosed TB number. In this case, however, the net TB number in a contracting proto-nugget arises from random Poisson fluctuations instead of a cosmic asymmetry:

$$\tilde{N}_{TB}^* \simeq N_{tf}^{1/2}(r) \simeq (n_{tf} r^3)^{1/2} \quad , \quad (5.10)$$

where $n_{tf} \sim T_c^3$ is the technifermion density. Since the proto-nugget number density is of order $n_L \simeq 1/r^3$, the energy density in lumps is

$$\tilde{\rho}_L \simeq \tilde{N}_{TB}^* n_L M_{LTB} \sim T_c^{3/2} M_{LTB} r^{-3/2} \quad . \quad (5.11)$$

Using our previous estimates of $T_c \simeq 50$ GeV, $M_{LTB} \simeq 1$ TeV, and $r \gtrsim 10^{-5} t_c \simeq 10^{-4}$ cm, we find that these lumps contribute at most $\tilde{\Omega}_L \simeq 10^{-6}$ to the energy density of the Universe today. Thus, nuggets formed from TB number fluctuations, as opposed to a cosmic asymmetry, are not cosmologically interesting. (Nuggets formed from fluctuations could close the Universe if the proto-nugget size distribution is relatively flat down to a scale of order $10^{-4} r \simeq 10^{-9} t_c$. However, they would have a rather small TB number, $N_{TB} \sim 10^{12}$; they would only survive if $N \gtrsim 8 - 9$, and would possibly contravene observational constraints (Sec. VI) if they carry electric charge.)

VI. Astrophysical and Experimental Constraints

What are the observable consequences of technicolor nuggets? The phenomenology of techni-lumps depends primarily on two factors: their electric charge and their size (or mass) distribution. In general, the technifermion density in a nugget will not be the same for all species. Consequently, nuggets smaller than the Compton wavelength of the electron or proton may be charged; by analogy with strange matter [14], we expect the charge to scale with the TB number of the nugget as $Q \simeq N_{TB}^{1/3}$ for large N_{TB} ^{††}. Unlike nuclei, here the Coulomb energy becomes less important as N_{TB} increases, $E_C/N_{TB} \sim Q^2/RN_{TB} \sim N_{TB}^{-2/3}$, and large nuggets are stable against fission. Although structurally irrelevant, the electric charge strongly boosts the interaction rate of a nugget with ordinary matter.

As with baryons, the lightest technibaryon is likely to be charged [6]. Charged particles with masses of order a few TeV are strongly excluded on experimental and astrophysical grounds as the dominant component of the galactic halo [6,26]. Obviously, similar arguments rule out charged nuggets with small TB number (although, as shown in the previous sections, very small nuggets are unlikely either to have formed or to have survived the early Universe). If the dark matter is to be made of technicolor particles, it must be hidden in relatively large lumps. Such a scenario is perfectly natural from the considerations of the preceding sections: for $N \gtrsim 5 - 8$, we found that, to 1 part in $R(T_c)^{-1}$ (see Fig.5), the entire TB number of the Universe would be concentrated in lumps of characteristic size N_{TB}^* [Eqn.(4.16)]. We therefore assume that no isolated technibaryons are present today and place lower bounds on the mass (or TB number) of nuggets, assuming they form the dark matter in the galactic halo.

We first consider techni-nuggets carrying positive electric charge (in the absence of electrons to neutralize them). As far as their interactions with ordinary matter are concerned, there are two regimes to distinguish: nuggets smaller than or larger than the Bohr radius, $r_b = 1/m_e\alpha \simeq 5 \times 10^{-9}$ cm, corresponding to lump TB number smaller or larger than $N_{TB}^b \simeq 10^{22}$. Hereafter, we distinguish these two classes by the technical terms ‘small’ and ‘large’ nuggets. A large nugget, with $N_{TB} \gg N_{TB}^b$, is neutralized by an electron cloud spread throughout its interior. Its interaction cross-section with matter is roughly geometric, *i.e.*,

^{††}By a simple energetics argument, the chemical potentials of the different technifermion species always adjust so that the nugget charge grows no faster than $Q \sim N_{TB}^{2/3}$. If this limit is saturated, the Coulomb energy may reduce the binding energy of bulk technimatter somewhat.

just its effective cross-sectional area $\sigma_c \simeq \pi R^2(N_{TB})$, given by Eqn.(5.1) with $f_{TB} = 1$. From the discussion following Eqn.(4.16), we note that $N_{TB}^* \gtrsim N_{TB}^b$, so that nuggets of the characteristic size produced at the confinement transition are 'large' and approximately neutral.

A large nugget with speed v loses energy at a rate [27]

$$\frac{dE}{dx} = -\sigma_c \rho v^2 \quad , \quad (6.1)$$

where ρ is the density of the scattering medium. The characteristic stopping distance of a lump of mass $M_L = N_{TB}(\epsilon/n_{TB}) \sim N_{TB}M_{LTB}$ is then $\rho L_s = M_L/\sigma_c \simeq M_{LTB}^3 N_{TB}^{1/3} \simeq 10^{12} N_{TB}^{1/3}$ gm/cm² for a lump with $N_{TB} \gtrsim 10^{22}$ ($M_L \gtrsim 20$ gm). Since the column density of the Earth is only 10^{10} gm/cm², such large lumps pass freely through our planet. For neutron stars, the column density is of order $2\rho_{nuc}R_{ns} \simeq 5 \times 10^{20}$ gm/cm², so nuggets with TB number $N_{TB} \lesssim 10^{26}$ (or mass $M_L \lesssim 2 \times 10^5$ gm) will be stopped inside them. Since $N_{TB}^* \simeq 5 \times 10^{22} - 6 \times 10^{37}$ (for $10^{-5} < (r/t_e) < 1$), it is possible that nuggets of the characteristic size are captured by neutron stars. However, as we show below, for nuggets this massive, the total nugget mass condensed with the protostellar cloud that eventually forms a neutron star is negligible. In addition, the total mass captured from the galaxy halo over the lifetime of an old neutron star is only of order 10^{12} gm, independent of N_{TB} . As a result, neutron stars and other astrophysical bodies are not in danger of disruption by large nuggets. As will be clear from the discussion below, large nuggets also easily evade current limits from terrestrial and spaceborne detectors, so we turn now to consider small lumps.

Small nuggets, with TB number $N_{TB} \ll N_{TB}^b$ and size $R \ll r_b$, are surrounded by an electron cloud much like ordinary atomic nuclei. In this case, the extent of the electron shells (of order r_b) determines the interaction cross-section, which is therefore much larger than the geometric cross-section of the nugget itself. Nuggets with charge $Q \lesssim 100$ (corresponding roughly to $N_{TB} \lesssim 10^3 - 10^6$) resemble superheavy isotopes of known nuclei. Those with supercritical charge, $Q \gtrsim 137$, break down the vacuum, creating electron-positron pairs which screen the Coulomb field. In this case, the effective charge is of order $Q_{eff} \simeq 137$, the radius of the innermost electron shell is $r_0 \simeq 1/m_e Q_{eff} \alpha \simeq 1/m_e \simeq 4 \times 10^{-11}$ cm, and the outermost orbits have radii of order the Bohr radius, r_b . For small nuggets, the energy loss rate due to Coulomb scattering off atomic electrons and nuclei is [28]

$$\frac{1}{\rho} \frac{dE}{dx} = \frac{1}{Am_p m_e} \left(\frac{\pi^2}{e} \frac{ZQ}{(Z^{2/3} + Q^{2/3})^{1/2}} + \frac{8\pi v}{\alpha} \frac{ZQ}{(Z^{2/3} + Q^{2/3})^{3/2}} \right) \quad , \quad (6.2)$$

where Z , A are the nuclear charge and atomic number of the scattering atom. For the Earth's atmosphere, the dominant component is N^{14} ; scaling from the results of [27], the range of a small nugget in the atmosphere is roughly $\rho L_s \simeq 10^{-3} N_{TB} Q_{eff}^{-2/3}$ gm/cm². Note that this is within roughly an order of magnitude of the naive estimate obtained using the area of the electron cloud, $\sigma_b \simeq 4\pi r_b^2$. The atmosphere's column density of 10^3 gm/cm² stops lumps with $N_{TB} \lesssim 10^6$, while nuggets with $N_{TB} \lesssim 10^{14}$ are stopped in the Earth.

We can now, *mutatis mutandis*, run through the host of constraints on charged dark matter particles previously discussed in refs.[6,26,27,29]. For charged halo particles, the strongest constraints come from gamma ray and cosmic ray detectors, from galactic halo infall, and from the survival of neutron stars. For small nuggets, the characteristic interaction cross-section with ordinary matter [Eqn.(6.2)] is approximately atomic, $\sigma \sim \sigma_b \simeq 10^{-15}$ cm². From the solid state cosmic ray detector aboard Pioneer 11 and plastic track detectors, a halo particle with this cross-section must have a mass larger than $M_L \gtrsim 10^9 - 10^{10}$ GeV, or $N_{TB} \gtrsim 10^6 - 10^7$. Lumps smaller than this are excluded from having a mass density comparable to the halo.

Charged lumps in the halo lose energy by collisions as they traverse the disk of the galaxy and eventually fall into it. Requiring that lumps remain in the halo for a Hubble time yields a lower bound on the cross-section, $\sigma_{int} \lesssim 5 \times 10^{-21} (M_L / \text{TeV})$ cm². For small nugget cross-section $\sigma \simeq 10^{-15}$ cm², this implies that halo nuggets must have $N_{TB} \gtrsim 2 \times 10^5$. A nearly identical bound arises from requiring that lumps do not collisionally ionize neutral molecular clouds [30].

An additional bound on N_{TB} arises from the survival of neutron stars [26]. A protostellar cloud of mass M_c , the progenitor of a neutron star, captures a total mass M_x in nuggets given by [26]

$$\eta = \frac{M_x}{M_c} \simeq 4 \times 10^{-3} \left(\frac{100 \text{TeV}}{M_L} \right)^{7/4}, \quad (6.3)$$

where M_L is the nugget mass. For a cloud of mass $M_c = 10M_\odot$, the captured mass in nuggets is

$$M_x \simeq 100 \left(\frac{\text{TeV}}{M_{LTB}} \right)^{7/4} N_{TB}^{-7/4} M_\odot. \quad (6.4)$$

If the nuggets become self-gravitating in the core of the star, and if the captured mass M_x is larger than the Chandrasekhar mass M_{Ch} for techni-matter, the neutron star will be swallowed up by the black hole which forms at its center. The equation of state of techni-matter is identical in form to that of quark matter, $p = (\rho - 4B)/3$, so the Chandrasekhar

mass can be found by scaling from the results for strange stars [31],

$$M_{Ch} = 2 \left(\frac{B}{(145 \text{ MeV})^4} \right)^{-1/2} M_{\odot} . \quad (6.5)$$

For a bag constant in the range $B^{1/4} = 80 - 160$ GeV, we find

$$M_{Ch} \simeq (2 - 7) \times 10^{-6} M_{\odot} . \quad (6.6)$$

Comparing Eqns.(6.4) and (6.7), we find $M_x < M_{Ch}$ for $N_{TB} > 10^4$; neutron stars are safe for nuggets larger than this bound.

Negatively charged nuggets will bind protons and, after nucleosynthesis, light nuclei. For nuggets with negative charge, we can repeat the arguments above, replacing the electron mass everywhere with the proton mass, m_p . The transition from small to large nuggets now occurs at $N_{TB} \simeq 10^{12}$. Small nuggets have a characteristic strong interaction cross-section, $\sigma \sim 10^{-23}$ cm². In this cross-section range, the strongest experimental bounds come from balloon-borne and underground searches for particle dark matter [29]. Roughly, one finds the constraint $N_{TB} \gtrsim 10^9$ for nuggets in the halo. The astrophysical limits are less severe.

Summarizing the experimental and astrophysical constraints, we find that positive (negative) charged nuggets with TB number larger than 10^7 (10^9) are observationally consistent candidates for the dark matter in galactic halos. Even taking into account the uncertainties involved, this lower bound is many orders of magnitude below the expected nugget size N_{TB}^* .

VII. Conclusion

We have investigated the ground state of technicolor at finite baryon number using a rescaled version of the MIT bag model. Given the large number of flavors present in popular models, we have found that the lowest energy state is likely to be techni-matter, the analogue of strange quark matter, rather than individual technibaryons. We have also studied the formation of techni-nuggets in the technicolor confinement transition as well as their subsequent evolution. For technicolor group $SU(N)_{TC}$ with $N \gtrsim 5 - 8$, large nuggets survive the early Universe and are expected to contain virtually all the TB number. Nuggets produced in the coexistence epoch are natural candidates for the dark matter in galaxy halos. Unless the nugget size distribution has a tail down to extremely small TB number, halo nuggets are consistent with all known experimental and astrophysical constraints.

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Appendix A

In this appendix, we derive expressions for the technichromo- electrostatic and magneto-static energy inside the bag arising from one-technigluon exchange. Following ref. [12], we write the exchange energy contribution as:

$$\Delta E = 2\pi\alpha_{TC} \sum_a \sum_{i,j} \int_{bag} d^3x \left[\vec{E}_i^a(x) \cdot \vec{E}_j^a(x) - \vec{B}_i^a(x) \cdot \vec{B}_j^a(x) \right], \quad (A.1)$$

where \vec{E}_i^a and \vec{B}_i^a ($a = 1, \dots, N^2 - 1$) are the technichromo- electric and magnetic fields generated by the i th constituent technifermion. \vec{E}_i^a and \vec{B}_i^a are computed by solving the Maxwell equations in the presence of the technifermion currents, using appropriate boundary conditions on the surface of the bag.

For massless technifermions, the total electric field $\vec{E}^a \equiv \sum_i \vec{E}_i^a$ is proportional to the technicolor generator T^a [12] and therefore vanishes for technibaryons, since they are technicolor singlets. Thus, the net electrostatic energy in eqn.(A.1) is zero. (We will neglect small contributions due to nondegeneracy of technifermion masses.)

In the computation of the magnetostatic energy, we follow the usual prescription of dropping the self-energy terms ($i = j$), which are partly absorbed in the renormalization of the fermion propagator^{††}. In this approximation, the magnetostatic energy becomes:

$$\Delta E = -2\alpha_{TC} \frac{\mu}{R} \sum_a \sum_{i \neq j} T_i^a \vec{\sigma}_i \cdot T_j^a \vec{\sigma}_j, \quad (A.2)$$

where μ is an integral over cavity wavefunctions and, for massless fermions in the fundamental mode, $\mu = 0.176$ [12]. T_i^a and $\vec{\sigma}_i/2$ are the technicolor and spin generators of the i th technifermion.

We now want to evaluate the sum in eqn.(A.2). Since the technibaryon is a technicolor singlet, the technicolor wavefunction is antisymmetric and thus

$$P_{ij}^{TC} P_{ij}^S = -P_{ij}^S, \quad (A.3)$$

^{††}For a discussion of the magnetostatic self-energy, see ref. [18].

for each pair of technifermions i, j . In eqn. (A.3), P_{ij}^{TC} , P_{ij}^S are respectively the technicolor and spin permutation operators:

$$P_{ij}^{TC} = \frac{1}{N} + 2 \sum_a T_i^a T_j^a, \quad (A.4)$$

$$P_{ij}^S = \frac{1}{2} + \frac{1}{2} \vec{\sigma}_i \cdot \vec{\sigma}_j. \quad (A.5)$$

With the help of eqns.(A.4) and (A.5), eqn.(A.3) becomes:

$$- \sum_a T_i^a \vec{\sigma}_i \cdot T_j^a \vec{\sigma}_j = \frac{1}{2} \left(1 + \frac{1}{N} \right) (1 + \vec{\sigma}_i \cdot \vec{\sigma}_j) + \sum_a T_i^a T_j^a. \quad (A.6)$$

Summing eqn.(A.6) over N constituent technifermions under the condition of technicolor neutrality ($\sum_i T_i^a = 0$) and substituting the result into eqn.(A.2), we get the one-gluon exchange contribution to the energy of a technibaryon with spin s , eqn.(3.8).

In the presence of only technicolor interactions, the theory has an exact $SU(n)$ flavor symmetry and a multiplet of states is degenerate with the LTB . Color, electroweak and ETC interactions break the flavor symmetry and lift the degeneracy. In our estimates, we have neglected the mass splittings inside flavor multiplets.

Appendix B

In this appendix we compute the ‘‘center of mass’’ corrections [17] to the technihadron mass formula in the bag model.

The expression for the energy of the bag, eqn.(3.7), is related to the physical mass of the bound state by

$$E = \langle (M^2 + P^2)^{\frac{1}{2}} \rangle \simeq (M^2 + \langle P^2 \rangle)^{\frac{1}{2}}, \quad (B.1)$$

where the average is taken over the appropriate bag wave packet. Therefore, if momentum fluctuations $\langle P^2 \rangle^{1/2}$ are nonvanishing, they must be subtracted from eqn.(3.7) in order to derive the correct value of the physical mass. The contribution to momentum fluctuations from N constituent technifermions is estimated to be [17]:

$$\langle P^2 \rangle \simeq \frac{x^2}{R^2} N, \quad (B.2)$$

where, for massless technifermions in the fundamental cavity mode, $x = 2.04$. The physical

mass is now obtained by minimizing

$$M(R) = (E(R)^2 - \langle P^2 \rangle)^{\frac{1}{2}} \quad (B.3)$$

as a function of R . Using the expression for E given in eqn.(3.7), we obtain:

$$M = \frac{2}{3} \left(2a^2 - 3b + a\sqrt{4a^2 - 3b} \right)^{1/2} \cdot \left(\frac{4\pi B}{\sqrt{4a^2 - 3b} - a} \right)^{1/4}, \quad (B.4)$$

where

$$a \equiv xN + Z_0 - \alpha_{TC}W, \quad b \equiv x^2N. \quad (B.5)$$

Note that, for $b \rightarrow 0$, eqn.(B.4) reduces to eqn.(3.9).

To first order in $\langle P^2 \rangle^{1/2}$, the corrections given by eqn.(B.3) correspond to introducing an effective contribution to Z_0 of the form:

$$\delta Z_0 \simeq -\frac{3}{8}x \left(1 + \frac{Z_0}{xN} - \frac{\alpha_{TC}W}{xN} \right)^{-1}. \quad (B.6)$$

Fits of the hadronic mass spectrum, where B , α_c and Z_0 are treated as free parameters, are optimized for $\alpha_c = 0.55$, $Z_0 = -1.84$ [12]. For QCD ($N = 3$) with $\alpha_c = 0.55$, eqn.(B.6) gives $\delta Z_0 = -0.88$ for $Z_0 = 0$ and $\delta Z_0 = -0.84$ for $Z_0 = {}_M Z_0$. Therefore, in QCD bags, the ‘‘center of mass’’ correction provides an estimate for the phenomenological value of Z_0 . Note that, for large N , the ‘‘center of mass’’ contribution to eqn.(B.4) becomes negligible.

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Figure Captions

Fig.1 - The fractional techni-matter binding energy per unit TB number, $(M_{LTB} - \epsilon/n_{TB})/M_{LTB}$ as a function of N , for $n = 8$ and $\alpha_N \equiv N\alpha_{TC} = 0, 1, 2$. We have set the zero-point energy $Z_0 = 0$. For the dashed curves, the "center of mass" corrections (Appendix B) are taken into account.

Fig.2 - The techni-matter binding energy per unit TB number, $(M_{LTB} - \epsilon/n_{TB})/M_{LTB}$, in TeV. The conventions are the same as in Fig.1.

Fig.3 - The techni-confinement coexistence temperature T_c as a function of N , with conventions as in Fig.1. Progressing from lower to upper curves, $\alpha_N = 0, 1, 2$.

Fig.4 - The ratio M_{LTB}/T_c as a function of N , with conventions as in Fig.1. Progressing from upper to lower curves, $\alpha_N = 0, 1, 2$.

Fig.5 - The technibaryon number density ratio $R(T_c)$ in chemical equilibrium between unconfined and confined phases, as a function of N . Going from the upper to the lower curves, the coupling constant α_N increases as above.

Fig.6 - The factor $\Delta x_c = (M_{LTB} - \epsilon/n_{TB})/T_c$ vs. N , with the same conventions as before. The coupling α_N increases moving downward through the figure.

$$(M_{\text{LTB}} - \epsilon/n_{\text{TB}})/M_{\text{LTB}}$$

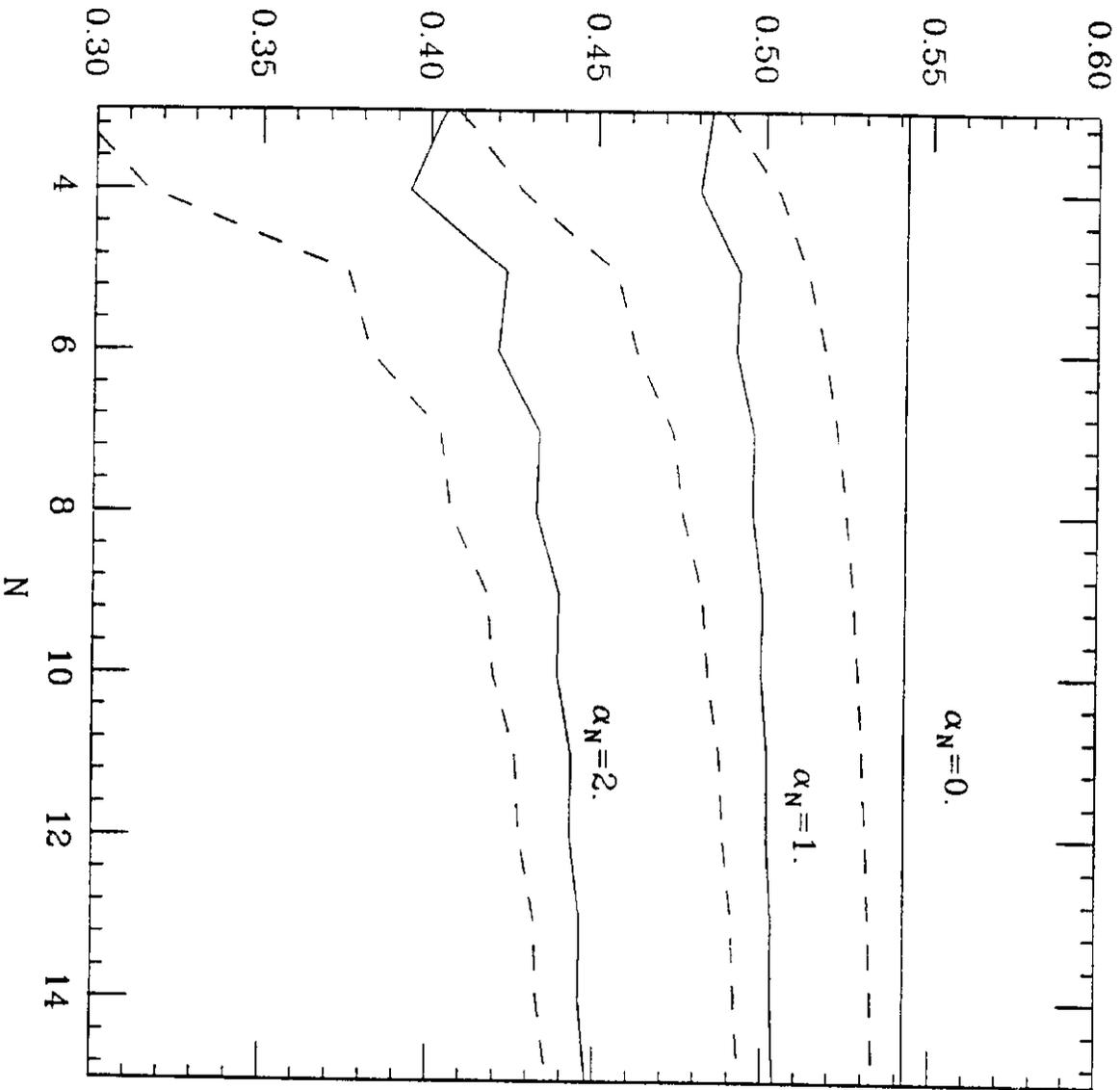


Fig. 1

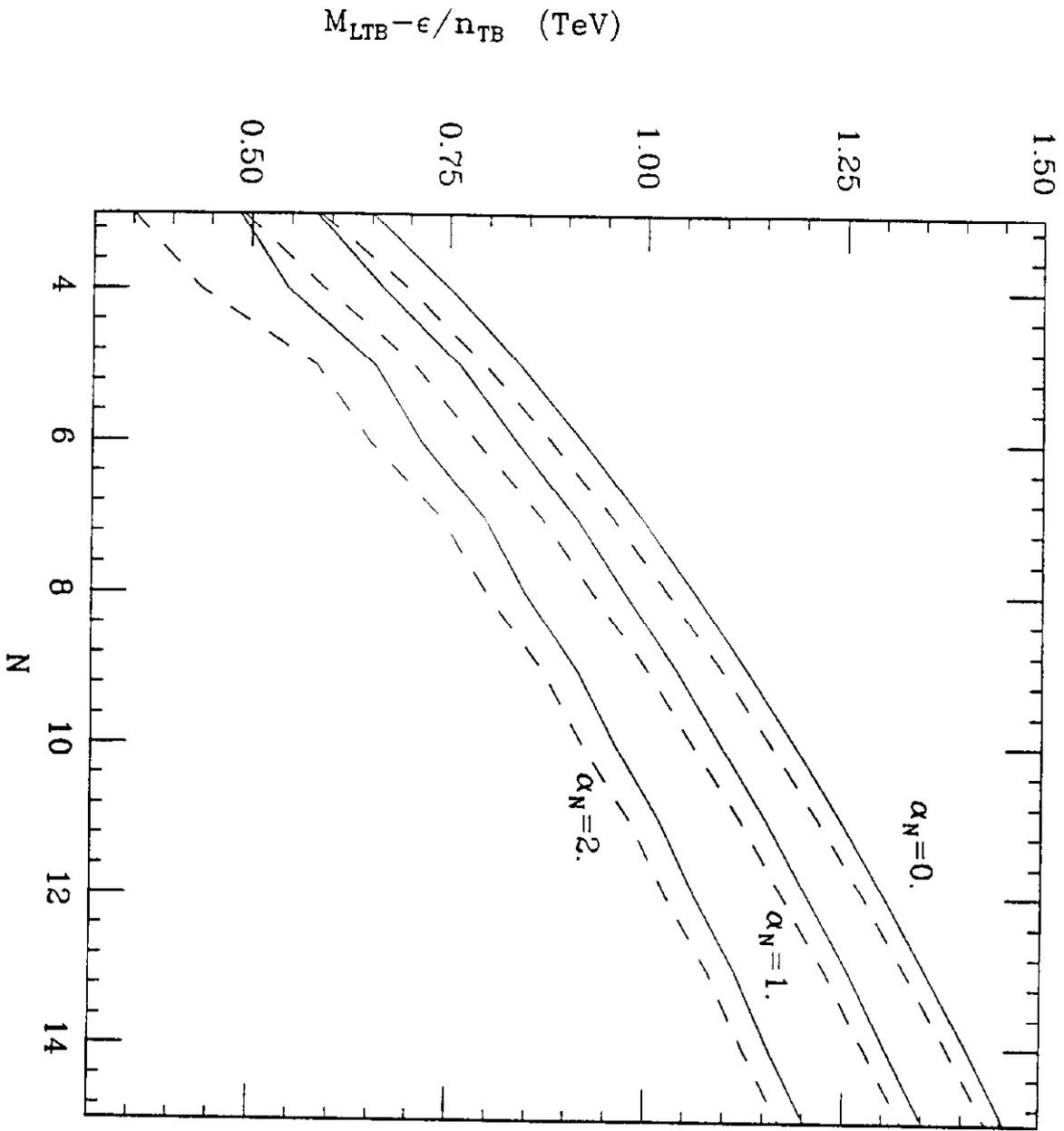


Fig. 2

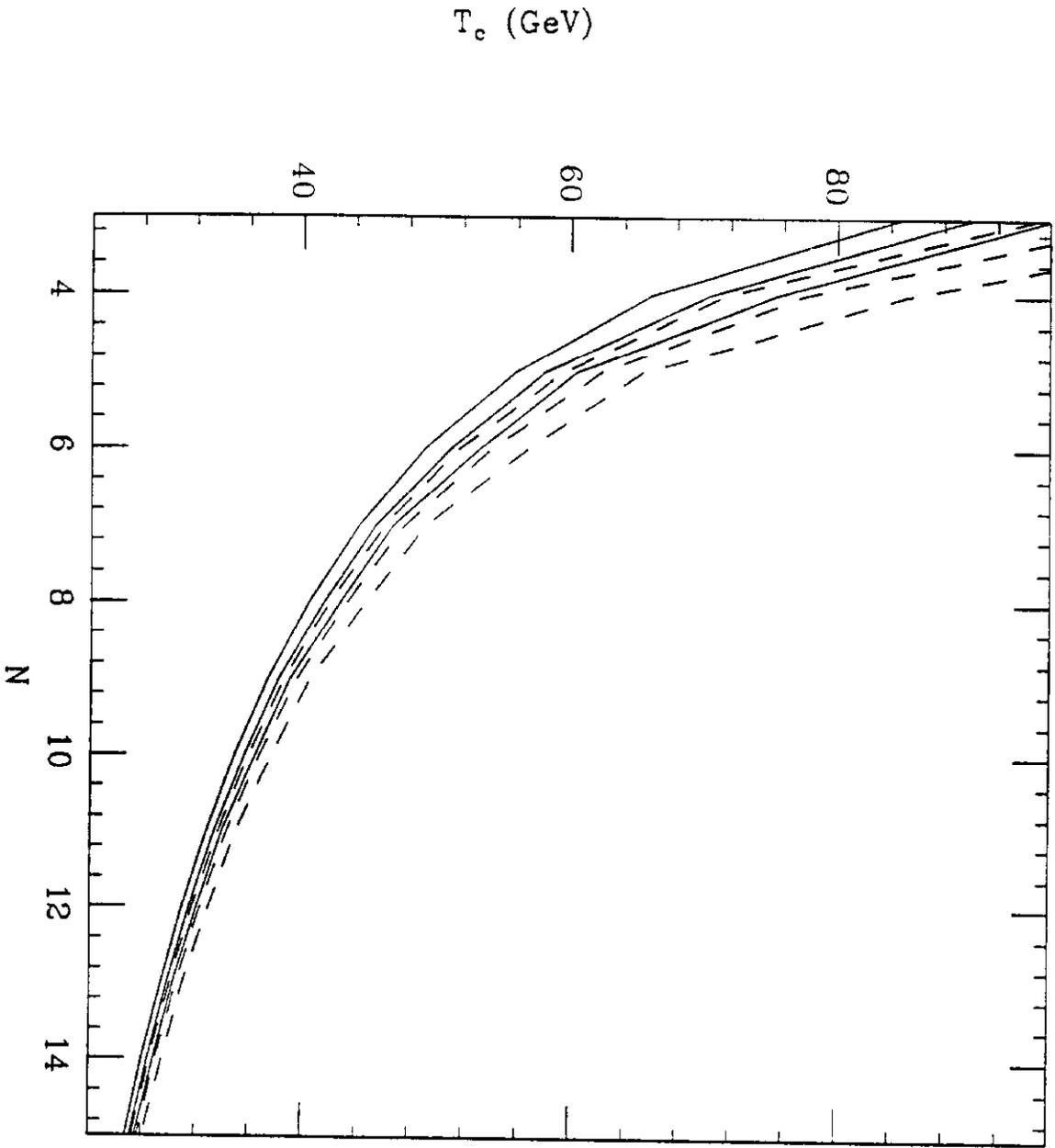


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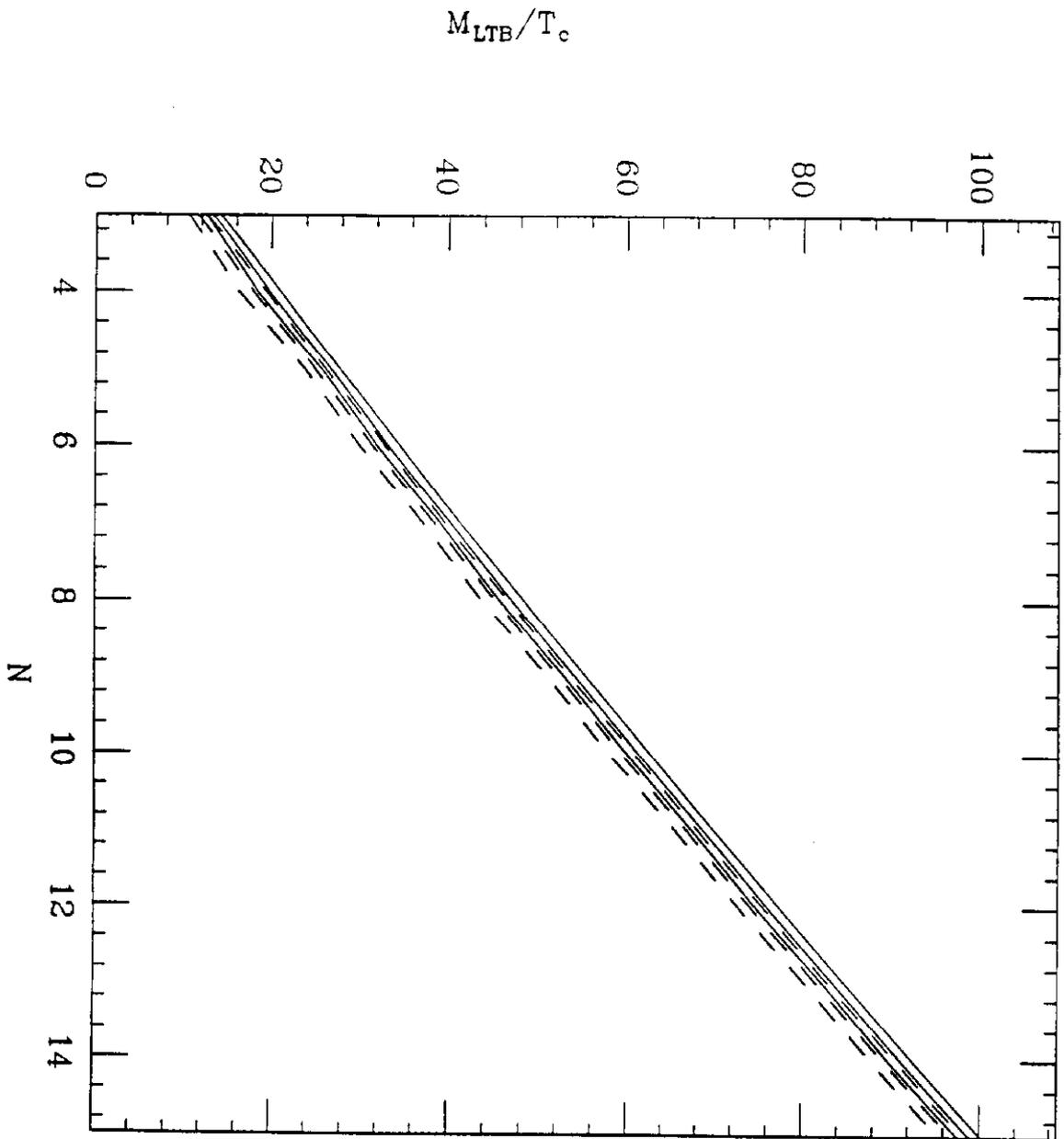


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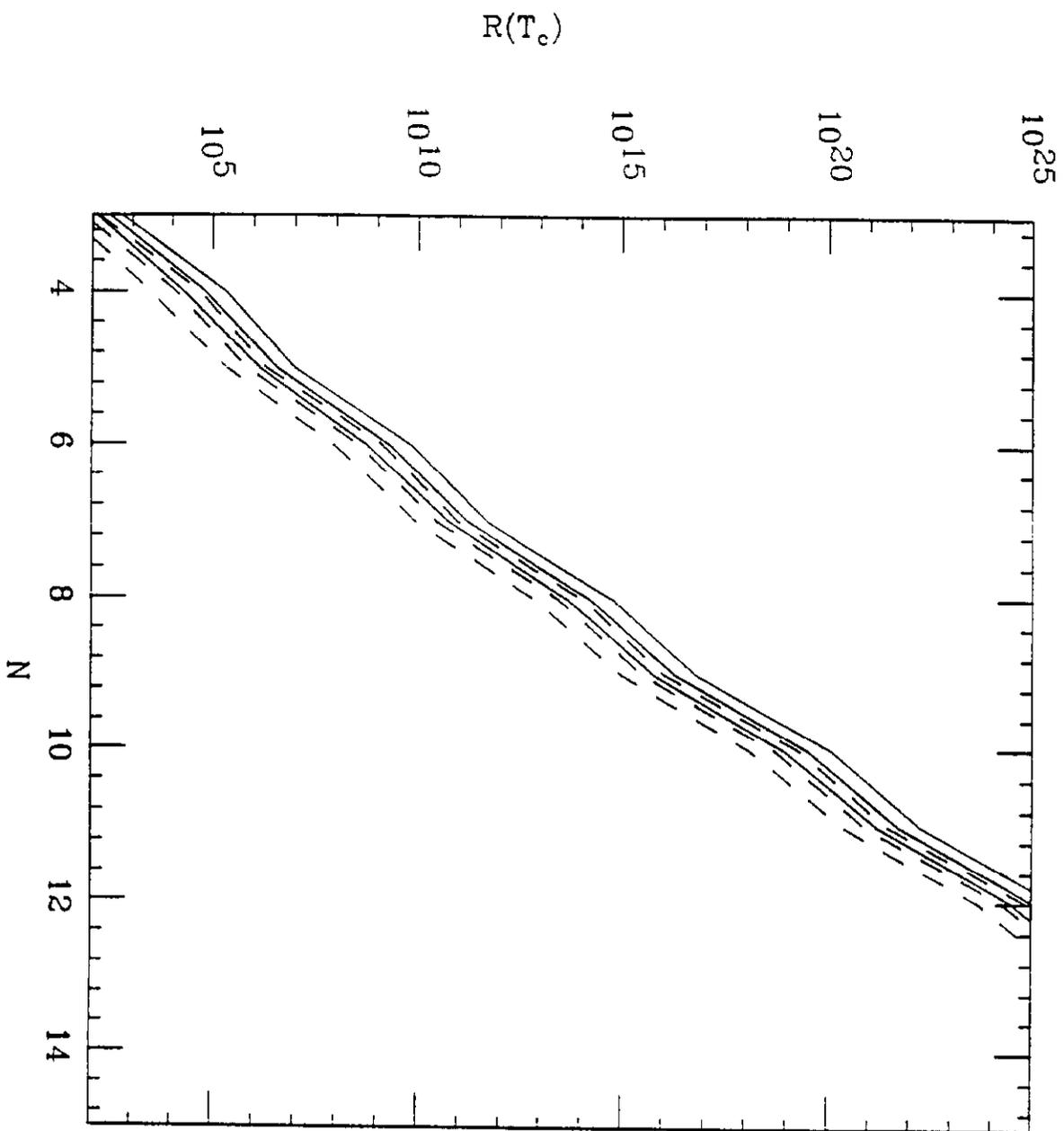


Fig. 5

$$(M_{\text{LTB}} - \epsilon / n_{\text{TB}}) / T_c$$

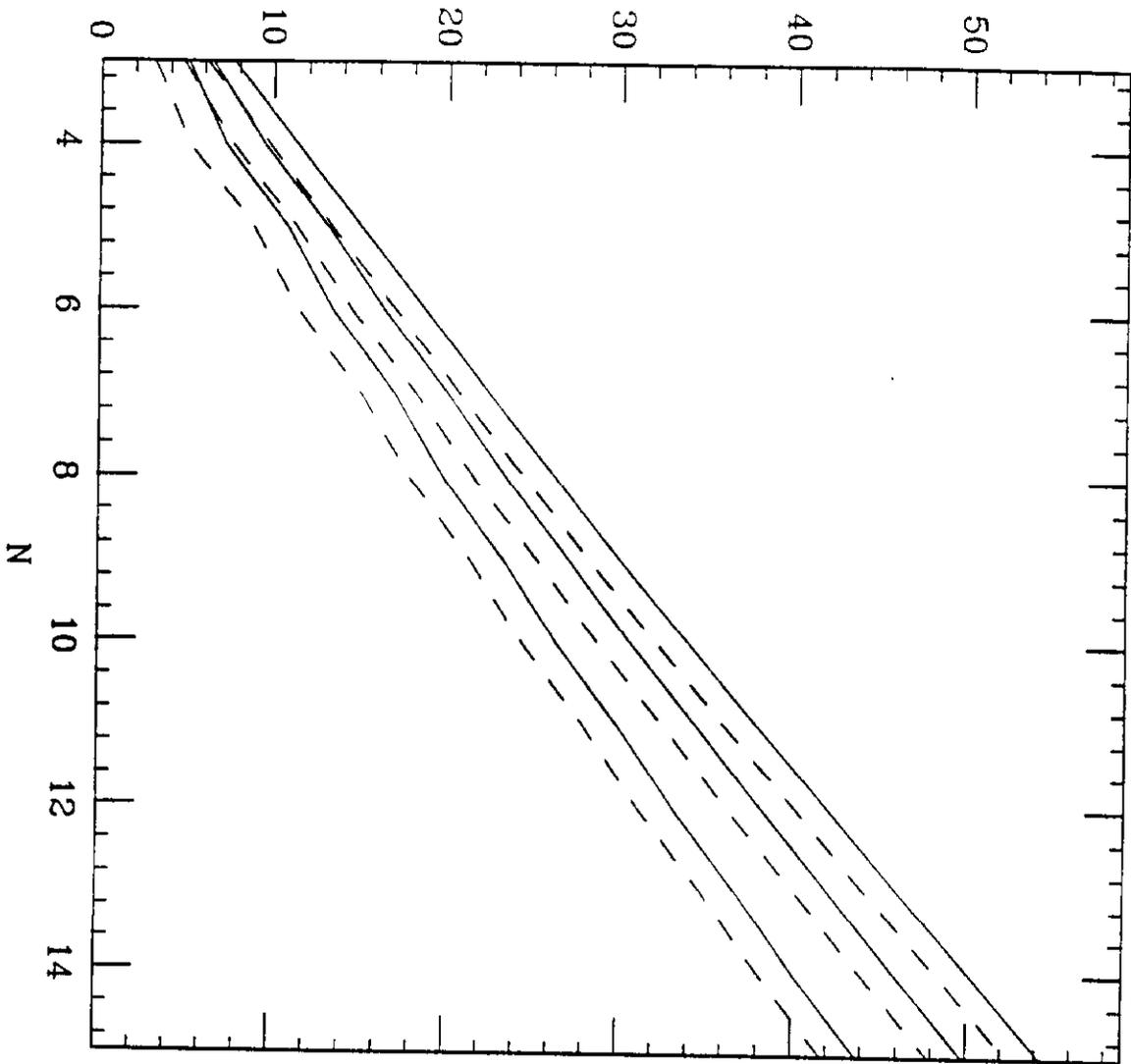


Fig. 6