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PERIODIC SIGNATURES FOR THE DETECTION OF COSMIC AXIONS

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Abstract

In a Sikivie-type cosmic-axion detector, both the width and position of the microwave signal due to axion-photon conversion depend upon the motions of the earth. Due to the orbital and rotational motions of the earth they will be modulated with periods of 1 sidereal day and 1 sidereal year, with amplitudes of about 0.1% and 5% respectively. Because of the intrinsically-high energy resolution of Sikivie-type detectors such periodic variations should be detectable. Such modulations would not only aid in confirming the detection of cosmic axions, but, if found, would also provide important information about the distribution of axions in the halo.

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The axion is a very attractive dark-matter candidate. Moreover, at present here are several ongoing experiments[1] to detect cosmic axions in our halo by the process proposed by Sikivie:[2] Resonant conversion of axions into microwave photons in the presence of a strong magnetic field. In addition, a second generation experiment that is about 300 times more sensitive is being planned.[3] The characteristic energy of the photons produced in a Sikivie detector is $E_\gamma = m_a c^2 + m_a \bar{v}^2/2$ where m_a is the axion mass and $\bar{v}$ is the velocity dispersion of axions in the halo (we will be much more precise later). For closure density, m_a is expected to be in the range 10^{-6} eV to 10^{-4} eV (Refs. 4); the velocity dispersion of our halo is about 300 km sec$^{-1} \simeq 10^{-3}c$, so that $m_a\bar{v}^2/2 \sim 10^{-6} m_a$. The frequency of the axion microwave line is 2.4 GHz$(m_a/10^{-5}\text{eV})$, while the width corresponds to a frequency of about $(m_a/10^{-5}\text{eV})$ kHz. Because resonant-cavity experiments have very good energy resolution—Q's in excess of 10^5—both the position and the shape of the microwave line are potentially observable.

Since the earth is in motion with respect to the halo—its motion as part of the solar system, its orbital motion, and its rotational motion—the position and width of the microwave line expected from axion-photon conversion will depend upon these velocities and will be periodically modulated. (We will neglect the very small motion of the earth about the earth-moon barycenter, $v \simeq 1.3 \times 10^{-2}$ km sec^{-1} and period 27.32 sidereal days.) The periodic modulations due to the earth's orbital and rotational motions may prove crucial in confirming the detection of cosmic axions and, if detected, in determining the distribution of axions in the halo.

It is usually assumed that the distribution of matter in the halo can be described by an isothermal phase-space distribution in the rest frame of the galaxy—not so much because of any hard evidence, rather for simplicity and the lack of a more compelling model. In fact, there is essentially no evidence to preclude the possibility that the halo is rotating with respect to the galaxy. In any case, such a phase-space distribution is a very reasonable starting point and will serve to illustrate the periodic effects of interest. In the non-rotating, inertial rest frame of the galaxy (in astrophysical parlance, the Fundamental Standard of Rest) the local phase-space distribution of axions is given by

$$f d^3v = n_0 \left(\frac{\beta}{\pi}\right)^{3/2} \exp(-\beta v^2) d^3v, \tag{1}$$

where $n_0 \simeq 3 \times 10^{13}\,\text{cm}^{-3}/(m_a/10^{-5}\,\text{eV})$ is the local number density of axions (to within a factor of about 2; see Turner in Ref. 4) and $\hbar = k_B = c = 1$ throughout. In all that follows we will ignore the factor of n_0, so that $\int f d^3v = 1$; we will also be cavalier in not relabeling the distribution function f when we change its argument. The quantity β corresponds to

$m_a/2T$ where T is the "halo temperature" and determines the velocity dispersion of the halo. It is very simple to carry out the angular integrations and write the distribution in its familiar Maxwellian form:

$$f dv = 4\pi \left(\frac{\beta}{\pi}\right)^{3/2} v^2 \exp(-\beta v^2) dv. \tag{2}$$

From this expression one can compute $\langle v^2 \rangle$ and $\langle v^4 \rangle$:

$$\langle v^2 \rangle \equiv \int v^2 f d^3 v = \frac{3}{2}\beta^{-1} \qquad \langle v^4 \rangle \equiv \int v^4 f d^3 v = \frac{15}{4}\beta^{-2};$$

based upon the dynamics of our galaxy[5] $\bar{v} \equiv \langle v^2 \rangle^{1/2}$ has been determined to be about 270 km sec^{-1}, which implies that $\beta = 1.5\bar{v}^{-2} \simeq (220\,\mathrm{km\,sec^{-1}})^{-2}$. The uncertainty in the quantity β is probably 10% or so.

(Because the galaxy has an escape velocity, thought to be about 650 km sec^{-1}, the axion distribution should be truncated at this velocity. For our purposes this fact is not significant. In addition, Griest[6] has pointed out that owing to the gravitational effects of the sun there is a *minimum* axion velocity, the escape velocity from the solar system, locally about 42 km sec^{-1}, and a small additional modulation—due to focusing by the sun—that is orthogonal to the one discussed here. We will also ignore the small gravitational effects of the sun.)

The earth is not at rest with respect to the galaxy; its velocity can be written as

$$\mathbf{v}_E = \mathbf{v}_S + \mathbf{v}_O + \mathbf{v}_R,$$

where $\mathbf{v}_S$ is the velocity of the sun with respect to the galaxy, $\mathbf{v}_O$ is the orbital velocity of the earth around the sun, and $\mathbf{v}_R$ is the rotational velocity of the position on earth where the axion detector is. To orient one, the velocity of the sun has a magnitude of about 230 km sec^{-1} and is in a direction that is about $\alpha = 60°$ above (north of) the ecliptic (plane of the earth's orbit); when projected down onto the ecliptic, it intersects the earth's orbit at a point that is about 19 days before the autumnal equinox, i.e., near the earth's position on 4 September. (The uncertainties in these numbers correspond to a few days, a few degrees and about 20 km sec^{-1}; see Refs. 5) The angle between $\mathbf{v}_S$ and the equatorial plane is about $\psi = 47°$, and at local midnight, 19 days before the summer solstice (2 June) the projection of $\mathbf{v}_S$ onto the equatorial plane and $\mathbf{v}_R$ are parallel. The magnitude of the earth's mean orbital velocity is $v_O \simeq 29.8\,\mathrm{km\,sec^{-1}}$, while that of its rotational velocity at the equator is $v_R = 0.465\,\mathrm{km\,sec^{-1}}$. Of course at latitude l, the rotational speed is smaller by a factor of $\cos l$.

What is of interest to the experimenter on earth are the velocities of halo axions with respect to the experiment. Let $\mathbf{v}$ be the velocity of an axion in the galactic rest frame, then its velocity with respect to the experiment is

$$\mathbf{v}_a = \mathbf{v} - \mathbf{v}_E.$$

In the earth frame, the phase-space distribution of axions is given by

$$f d^3 v_a = \left(\frac{\beta}{\pi}\right)^{3/2} \exp(-\beta v_a^2) d^3 v_a, \tag{3a}$$

$$f dv_a = 2\left(\frac{\beta}{\pi}\right)^{1/2} \frac{v_a}{v_E} \exp(-\beta v_a^2 - \beta v_E^2) \sinh(2\beta v_E v_a) dv_a, \tag{3b}$$

where of course $v_a^2 = v^2 - 2\mathbf{v}_E \cdot \mathbf{v} + v_E^2$, and Eq. (3b) follows from Eq. (3a) by carrying out the angular integrations. The square of the earth's velocity is

$$\mathbf{v}_E^2 = (\mathbf{v}_S + \mathbf{v}_O + \mathbf{v}_R)^2, \tag{4a}$$

$$\mathbf{v}_E^2 \simeq v_S^2 \left(1 + 2\frac{v_O}{v_S}\hat{v}_O \cdot \hat{v}_S + 2\frac{v_R}{v_S}\hat{v}_R \cdot \hat{v}_S\right), \tag{4b}$$

where in Eq. (4b) we have dropped all terms that are quadratic in v_O and v_R since $v_O \simeq 0.13 v_S$ and $v_R \lesssim 0.002 v_S$. (Formally, the v_O^2 term is larger than the term involving $v_S v_R$; however, we are interested only in the periodic terms, and $\mathbf{v}_S^2$ is constant. If one wishes to, it is straightforward to retain all the terms in $\mathbf{v}_E^2$.)

It is of some use to rewrite the distribution function in terms of the axion kinetic energy, $E = m_a v_a^2/2$:

$$f dE = 2\left(\frac{\beta}{\pi}\right)^{1/2} \frac{dE}{m_a v_E} \exp(-\beta v_E^2 - 2\beta E/m_a) \sinh[2\beta(2E/m_a)^{1/2} v_E]; \tag{5a}$$

$$f du = \left(\frac{3}{2\pi}\right)^{1/2} \frac{du}{r} \exp[-1.5(r^2 + u)] \sinh(3r\sqrt{u}); \tag{5b}$$

where $r \equiv v_S/\bar{v} \simeq 0.85$ and the kinetic energy of the axion $E = (m_a \bar{v}^2/2)u$, which corresponds to a frequency of $980\,\mathrm{Hz}(m_a/10^{-5}\mathrm{eV})u$. The distribution function for $v_E = 0$, i.e., that in the rest frame of the galaxy, as well as those for 2 June (v_E is maximum) and for 2 December (v_E is minimum) are shown in Fig. 1. The large effect of the motion of the solar system on the distribution function is apparent, as is the smaller effect of the earth's orbital motion (no attempt was made to show the even smaller effect of the rotational motion of the earth).

From the distribution function we may compute any and all quantities of interest. Moreover, we can use the distribution function in any of its forms, cf. Eqs. (1), (2), (3a), (3b), and (5). It will be most convenient to use the distribution function in the rest frame of the galaxy, Eq. (2). Remembering that $v_a^2 = v^2 - 2\mathbf{v}\cdot\mathbf{v_E} + v_E^2$ it is simple to compute $\langle v_a^2\rangle$ and $\langle v_a^4\rangle$ (and higher moments if wanted):

$$\langle v_a^2\rangle \equiv \int v_a^2 f d^3v = v_E^2 + \frac{3}{2}\beta^{-1} = v_E^2 + \bar{v}^2; \tag{6a}$$

$$\langle v_a^4\rangle \equiv \int v_a^4 f d^3v = \frac{15}{4}\beta^{-2}\left[1 + \frac{4}{3}\beta v_E^2 + \frac{4}{15}\beta^2 v_E^4\right] = \frac{15}{9}\bar{v}^4\left[1 + 2\frac{v_E^2}{\bar{v}^2} + \frac{9}{15}\frac{v_E^4}{\bar{v}^4}\right]; \tag{6b}$$

$$\langle(v_a^2 - \langle v_a^2\rangle)^2\rangle = \frac{3}{2}\beta^{-2}\left[1 + \frac{4}{3}\beta v_E^2\right] = \frac{2}{3}\bar{v}^4\left[1 + 2\frac{v_E^2}{\bar{v}^2}\right]; \tag{6c}$$

Substituting in numbers, and for the moment neglecting the small contributions of the orbital and rotational velocities to v_E, we find that

$$\bar{v}_a \equiv \langle v_a^2\rangle^{1/2} \simeq 355\,\mathrm{km\,sec^{-1}},$$

$$v_{dis}^2 \equiv \langle(v_a^2 - \langle v_a^2\rangle)^2\rangle^{1/2} \simeq (305\,\mathrm{km\,sec^{-1}})^2,$$

$$\frac{m_a\bar{v}_a^2/2}{m_a c^2} \simeq 7\times 10^{-7}, \qquad \frac{m_a v_{dis}^2/2}{m_a c^2} \simeq \frac{1}{1.9\times 10^6}.$$

Note that because of the motion of the solar system relative to the galaxy the *rms* velocity of axions as seen on earth is about 355 km sec^{-1}, compared to about 270 km sec^{-1} as seen in the galactic rest frame. Since the energy of the axion-microwave line is about $m_a c^2$ and its width is $m_a v_{dis}^2/2$, the final expression corresponds to $1/Q_a$—the axion line is very narrow indeed.

Now consider the average axion kinetic energy ($\bar{E}_a = m_a\bar{v}_a^2/2$) and the dispersion in the axion kinetic energy ($\Delta E_a = m_a v_{dis}^2/2$), taking into account the orbital and rotational motions of the earth:

$$\bar{E}_a = \frac{m_a\bar{v}^2}{2}(1+r^2)\left[1 + \frac{2r^2}{1+r^2}\left(\frac{v_O}{v_S}\cos\alpha\cos\omega_O t + \frac{v_R}{v_S}\cos l\cos\psi\cos\omega_R t\right)\right]; \tag{7a}$$

$$\Delta E_a = \frac{m_a\bar{v}^2}{2}\left[\frac{2(1+2r^2)}{3}\right]^{1/2}$$
$$\times\left(1 + \frac{4r^2}{1+2r^2}\left[\frac{v_O}{v_S}\cos\alpha\cos\omega_O t + \frac{v_R}{v_S}\cos l\cos\psi\cos\omega_R t\right]\right)^{1/2}; \tag{7b}$$

where $\omega_O \equiv 2\pi/\text{sidereal year} = 1.991\times10^{-7}$ rad sec^{-1} is the earth's orbital angular velocity, $\omega_R \equiv 2\pi/$ sidereal day $= 7.292\times10^{-5}$ rad sec^{-1} is the earth's rotational angular velocity, and time t is measured from local midnight on 2 June. The prefactors in expressions (7a,b), expressed as frequencies, are: $m_a\bar{v}^2(1+r^2)/2 \simeq 1.70\,\mathrm{kHz}(m_a/10^{-5}\mathrm{eV})$ and $m_a\bar{v}^2[2(1+2r^2)/3]^{1/2}/2 \simeq 1.25\,\mathrm{kHz}(m_a/10^{-5}\mathrm{eV})$.

The (relative) amplitude of the sinusoidal variation of $\bar{E}_a$ is: $2r^2(v_O/v_S)\cos\alpha/(1+r^2) \simeq 5.5\%$ (orbital) and $2r^2(v_R/v_S)\cos l\cos\psi/(1+r^2) \simeq 0.093\%$ (rotational for a latitude of $37.5°$). The periods are 1 sidereal year (3.1558×10^7 sec) and 1 sidereal day (86,164.091 sec) respectively. The (relative) amplitude of the sinusoidal variation of ΔE_a is: $2r^2(v_O/v_S)\cos\alpha/(1+2r^2) \simeq 3.9\%$ (orbital) and $2r^2(v_R/v_S)\cos\psi\cos l/(1+2r^2) \simeq 0.066\%$ (rotational for a latitude of $37.5°$). The maxima of the orbital variations occur about 19 days before the summer solstice (2 June) and the minimum about 19 days before the winter solstice (2 December).

(Because the earth's orbit is not circular, orbital eccentricity $e = 0.016750$, there are higher harmonics in the moduations of $\bar{E}_a$ and ΔE_a, proportional to $e^{N-1}\cos N\omega_O t$. To order e, the orbital velocity of the earth is $\mathbf{v}_O = a\omega_O[-(\sin\omega_O t + e\sin 2\omega_O t)\hat{\mathbf{x}} + (\cos\omega_O t + e\cos 2\omega_O t)\hat{\mathbf{y}}]$, where $a = 1\mathrm{AU} = 1.496 \times 10^{13}$ cm is the semi-major axis of the earth's orbit, time is measured from perihelion (approximately 3 January), and at perihelion the earth is located on the positive $\mathbf{x}$-axis.[7] The sinusoidal variation of $\bar{E}_a$ at frequency $2\omega_O$ is $2er^2(v_O/v_S)\cos\alpha\cos 2\omega_O t'/(1+r^2)$ and that of ΔE_a is $2er^2(v_O/v_S)\cos\alpha\cos 2\omega_O t'/(1+2r^2)$; each smaller than that at frequency ω_O by a factor of $e \simeq 0.017$. Here time t' is measured since 20 March.)

Note that the amplitudes of the periodic variations of $\bar{E}_a$ and ΔE_a depend upon the astronomical parameters v_O/v_S, v_R/v_S, α, ψ, l, and $r = v_S/\bar{v}$. Moreover, the functional dependences of the modulation factors for $\bar{E}_a$ and ΔE_a are different. Thus, if these effects are measured, they will prove extremely useful in probing the distribution of axions in the halo. For example, the periodic orbital variations of $\bar{E}_a$ and ΔE_a differ by a factor of $(1+2r^2)/(1+r^2)$, a fact which could be used to infer $\bar{v}$ and therefore β. In addition, the phases of the modulations can be used to infer the direction of the sun's velocity, as projected onto the ecliptic.

The importance of the yearly modulation due to the earth's orbital motion in bolometric dark matter detection experiments has been previously emphasized.[8] There, the modulation is typically expected to be of order 10% and is crucial to identifying the signal—in that case energy-deposition rate—as being due to dark matter interactions in the detector. Whether such detectors can achieve the required sensitivity and energy resolution to observe periodic varitations remains to be seen. While the yearly modulations in an axion detector are slightly smaller, owing to the excellent intrinsic energy resolution of

Sikivie-type detectors these variations should be easily detectable, and can be used both to confirm that the signal is associated with cosmic axions and to probe the halo distribution of axions.[9] It may even be possible to detect the daily variations due to the earth's rotation.

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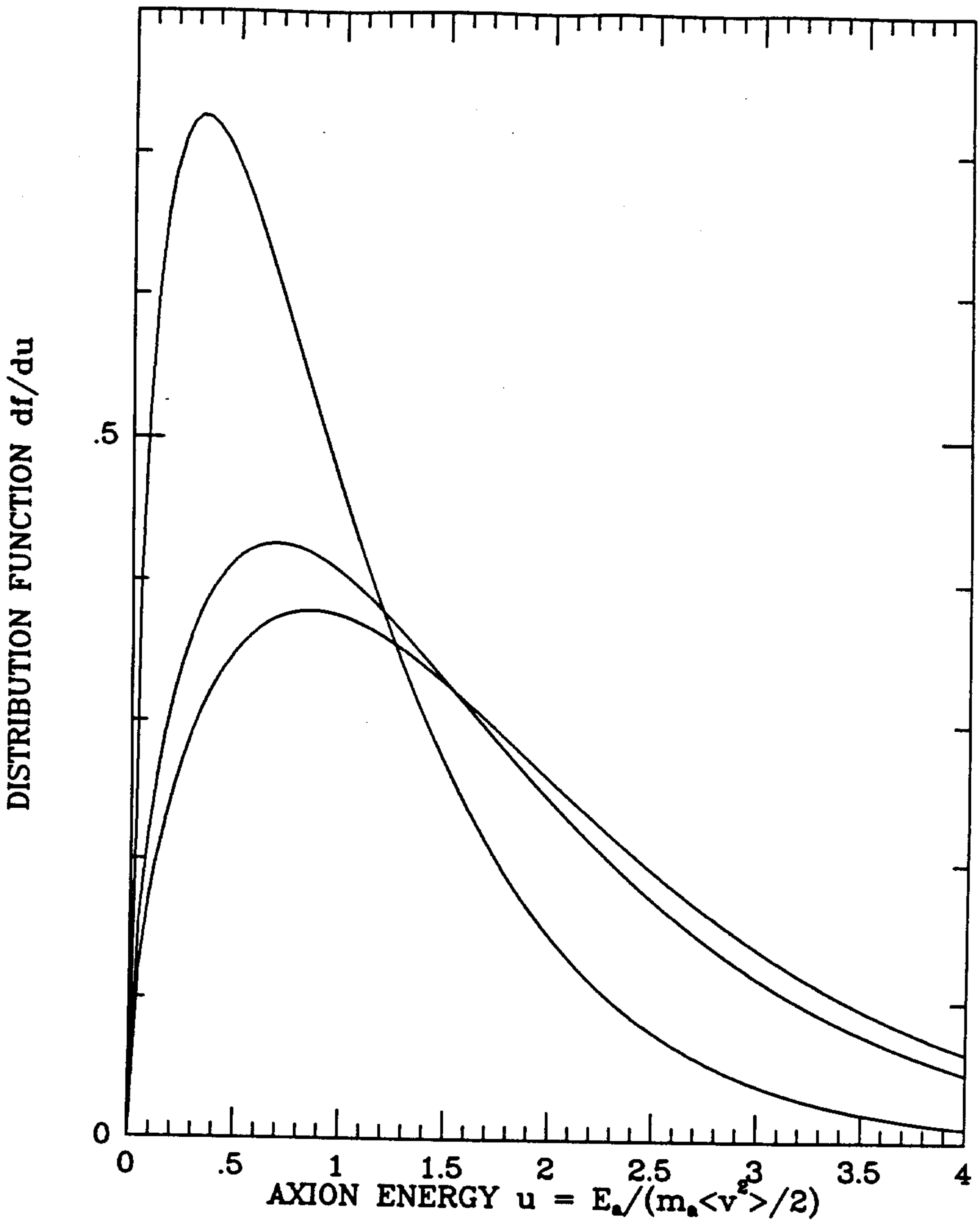
FIGURE CAPTION

Fig. 1. Local phase-space distribution function df/du for cosmic axions. The axion kinetic energy $E_a = u(m_a\bar{v}^2/2)$, which corresponds to a frequency of $980u\,(m_a/10^{-5}\text{eV})\,\text{Hz}$. The distribution function in the rest frame of the galaxy (tallest and narrowest), in the laboratory frame in June (shortest and broadest), and in the laboratory frame in December are shown.



DISTRIBUTION FUNCTION df/du
.5
0
0
.5
1
1.5
2
2.5
3
3.5
4
AXION ENERGY $u = E_a/(m_a\langle v^2\rangle/2)$