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Radiative Corrections to Electroweak Parameters in Technicolor Theories

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ABSTRACT

In models in which technicolor induces electroweak symmetry breaking, the relations between electroweak parameters will differ from those of the standard model. Even with the most conservative assumption that the only custodial SU(2) violating parameters are those in the standard model, there can be measurable corrections to standard model predictions. We present an effective field theory useful for calculating corrections to electroweak parameters. We then focus on technicolor models, for which we construct the low energy effective theory and calculate the correction to the W mass from light resonances, focusing on the potentially large pseudogoldstone boson contribution.

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1 Introduction

The fact of $SU(2)_W \times U(1)$ electroweak symmetry breaking is indisputable, but the dynamical model responsible for breaking the symmetry is rather poorly constrained by current data. Until experiments achieve the necessary energy and luminosity to directly study the particles involved in the symmetry breaking, we must rely on precision measurements at accessible energy scales to provide clues of the physics at higher energy. In particular, with precision measurements of the W and Z masses, radiative corrections to the standard model parameters will be strongly constrained. With a measurement of the top quark mass, nonstandard model contributions should be distinguishable from those of the standard model, putting constraints on the possible models for electroweak symmetry breaking.

Technicolor theories[1,2] are an intriguing mechanism for symmetry breaking. In these models, there is no fundamental scalar like the Higgs field which gets a vacuum expectation value; instead, the symmetry is broken by the formation of a condensate of technifermions, in analogy to the breaking of chiral symmetry in QCD. Also like QCD, there will be bound states of technifermions, the analogues of the mesons and baryons. Unfortunately, although the basic technicolor idea is quite simple, and can successfully explain $SU(2)_W \times U(1)$ symmetry breaking, it is very difficult to incorporate quark masses and mixing angles while simultaneously meeting the constraints imposed by the required suppression of flavor changing neutral currents. While there are as yet no completely realistic models, most models involve large number of technifermions, and consequently, many low energy resonances.

In technicolor models, corrections to electroweak parameters will arise from both physics at high and low energy. High energy contributions due to the existence of custodial SU(2) violating four fermion operators are model dependent, and have been considered elsewhere for certain classes of theories [3]. Moreover, it is difficult to reliably calculate such corrections in a strongly interacting theory. The second source of electroweak corrections will arise at low energy, due to the many light resonances of some technicolor theories. The largest contribution probably arises from the pseudogoldstone boson contribution, which is enhanced by the factor $\log(\Lambda_\chi^2/m^2)$, where Λ_χ a technicolor scale defined below and m is the pseudogoldstone boson mass. Unlike effects calculated from the technifermions or the remaining technicolor scale resonances, the logarithmically enhanced contribution can be reliably calculated for a particular model. Because both the spectrum of pseudogoldstone bosons and their gauge couplings are determined, the logarithmically enhanced pseudogoldstone boson contribution can be calculated, the only uncertainty being a logarithmic dependence on the unknown pseudogoldstone boson mass and technicolor scale. Strong interaction uncertainties from technifermions and heavier technicolor scale resonances are absorbed in counterterms of the low energy theory. If chiral perturbation theory is reliable, and if $\log(\Lambda_\chi^2/m^2)$ is large, their contribution will be smaller.

Previous authors have considered technipion contributions to the Z mass and ρ [4] and presented numerical results for the correction to the W mass [5] for specific models. We will derive a result using effective field theory techniques which can be readily applied to various models. We show how to derive one loop corrections to standard model parameters by determining running parameters at the Z scale in terms of which physical quantities can be calculated. We also interpret our result in this framework.

In this paper, we will calculate the corrections to the W mass from low energy resonances in technicolor models. In section 2, we review electroweak theory results, which we interpret in an effective field theory formalism. We distinguish the dominant contributions to the parameter Δr. In section 3, we present the Lagrangian describing the technicolor theory at energies low compared to the scale of fermion condensation. In section 4, we calculate technipion loop radiative corrections. In section 5, we consider possible corrections from vector meson or fermionic resonances. We conclude with a discussion of the numerical significance of our results for a particular model.

2 Effective Field Theory Interpretation of Electroweak Parameters

In the standard model at tree level the W and Z satisfy a mass relation

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\quad, \tag{2.1}$$

where G_F is the Fermi decay constant of the muon and α is the electromagnetic coupling strength, 1/137. If one substitutes the physical masses of the

W and Z, this relationship is observed to hold approximately, despite the fact that it does not automatically follow from the gauge symmetry. Indeed, it is easy to construct Higgs sectors in which it does not hold. In the standard model it can be deduced from the existence of a "custodial symmetry" [6] - an $SU(2)_C$ under which W_i, the three gauge bosons of the $SU(2)_W$, transform as a triplet. Terms in the Lagrangian which respect this symmetry, such as the Higgs potential of the one-doublet model, will not generate corrections.

At one loop the mass relation reads[7]

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{1-\Delta r} \quad , \tag{2.2}$$

where Δr includes radiative effects from various sources, each of which violates the custodial symmetry.

For the purposes of this paper, we consider only corrections of the so-called "oblique" type, *ie.* to the W, Z, or γ propagators, but not their vertices with fermions [8]. We define the correction to the two-point function of a gauge boson G to be $i\Sigma^{GG}(p^2)g^{\mu\nu}+i\tilde{\Sigma}^{GG}(p^2)p^\mu p^\nu$. The oblique correction to Δr is then [7]

$$\begin{aligned}\Delta r &= \left(\frac{\Sigma^{WW}(M_W^2)-\Sigma^{WW}(0)}{M_W^2}\right)\\ &\quad -\Pi^{\gamma\gamma}(0)+\frac{c_w^2}{s_w^2}\left(\frac{\Sigma^{ZZ}(M_Z^2)}{M_Z^2}-\frac{\Sigma^{WW}(M_W^2)}{M_W^2}\right) \quad ,\end{aligned} \tag{2.3}$$

where $\Pi^{\gamma\gamma}(k^2)=\Sigma^{\gamma\gamma}(k^2)/k^2$. Electromagnetic gauge invariance requires that the photon propagator be massless and transverse, so $\Pi^{\gamma\gamma}(0)$ is well defined. Here we have defined s_w^2 by

$$s_w^2=1-\frac{M_W^2}{M_Z^2} \quad , \tag{2.4}$$

and $c_w^2=1-s_w^2$.

There are several sources of custodial SU(2) violating effects which could enter the above formula. In the standard model, the largest effect arises from the large mass splitting of the third generation quarks. Thus one finds the well-known correction to the W mass from a heavy top quark[9]

$$(\Delta r)_{\rm top}\approx-\frac{\alpha}{4\pi}\frac{3c_w^2}{4s_w^4}\frac{m_t^2}{M_W^2} \quad . \tag{2.5}$$

Custodial SU(2) is also broken by the hypercharge gauge coupling. The custodial symmetry transforms the three left $\mathrm{SU(2)_W}$ gauge bosons W_i among themselves, and thus the mixing of the W_3 with the $U(1)$ boson B breaks the custodial symmetry. This custodial violating effect appears explicitly in two loop radiative corrections in which a photon or Z is exchanged. A more subtle point is that it is present also in the one-loop radiative corrections. Although it has no effect on the low energy ρ parameter, an ultra-heavy degenerate quark doublet will cause a small shift in M_W[10]:

$$(\Delta r)_{\mathrm{Q}} = \frac{\alpha}{4\pi}\frac{1}{s_w^2} + O(M_W^2/M_Q^2) \quad . \tag{2.6}$$

The heavy quark doublet does not decouple as one ordinarily expects [11], rather it serves as a mechanism for feeding the U(1) breaking of the custodial symmetry to the W mass. This point is made explicit by dividing the previous two equations:

$$\frac{(\Delta r)_{\mathrm{top}}}{(\Delta r)_{\mathrm{Q}}} = -\frac{3f_{\mathrm{top}}^2}{g'^2} \quad , \tag{2.7}$$

where f_{top} is the Yukawa coupling of the top quark, which is the term which violates the custodial symmetry when the top is heavy.

Notice that neither the mass splitting nor the U(1) contribution decouples when the quark is heavy. In both cases, the heavy degrees of freedom contribute finite effects in the low energy theory. This is because the mass term of a heavy quark in the standard model implies a large Yukawa coupling, and thus the quark does not truly decouple as its mass goes to infinity.

It is of interest to interpret the formula for Δr in an effective field theory approach. One can then more readily distinguish the different sources of custodial SU(2) violation, and moreover, one can more easily calculate any large logarithmic corrections. We will derive the formula for Δr from an effective field theory. The formalism we develop can also be applied to other quantities which will be precisely measured. We will then show why technicolor model contributions can be logarithmically enhanced, while contributions for the fundamental Higgs theory only have important logarithmic contributions from the scaling of α_{em} and in $\log(M_{\mathrm{Higgs}}/M_Z)$. These results are similar to those in Refs. [12,4]. The effective field theory is particularly useful for calculating such logarithmically enhanced contributions. Of course, when high level of precision is reached, one would want to include the full expression with additional terms which we neglect (see ref. [13] for example). We first present the derivation of the effective field theory lagrangian at the scale M_Z along the lines outlined by Jenkins and Manohar [14]. We then derive the formula for Δr.

Assume we start with a Lagrangian appropriate for physics at a scale Λ, above the W and Z masses. The lagrangian includes the fields W, Z and γ:

$$\begin{aligned} \mathcal{L} \;=\; & -(1+\Delta Z_W(\mu)+\delta Z_W)\partial^\mu W^\nu \partial_\mu W_\nu \\ & -\frac{1}{2}(1+\Delta Z_Z(\mu)+\delta Z_Z)\partial^\mu Z^\nu \partial_\mu Z_\nu \\ & -\frac{1}{2}(1+\Delta Z_\gamma(\mu)+\delta Z_\gamma)\partial^\mu A^\nu \partial_\mu A_\nu \\ & -(\Delta Z_{\gamma Z}(\mu)+\delta Z_{\gamma Z})\partial^\mu A_\nu \partial_\mu Z_\nu \\ & +(M_W^2+\Delta M_W^2(\mu)+\delta M_W^2)W^2 \\ & +\frac{1}{2}(M_Z^2+\Delta M_Z^2(\mu)+\delta M_Z^2)Z^2+\cdots \quad , \end{aligned} \tag{2.8}$$

where the ellipsis represents the three- and four-point couplings of the gauge bosons, the gauge fixing terms, terms involving the matter fields, and non-renormalizable terms of higher order in p^2, which we will ignore. The factors multiplying the kinetic energy terms are wave function renormalizations of the gauge boson fields. The infinite counterterms, δZ, are chosen so that they absorb the $2/\epsilon - \gamma + \log(4\pi)$ terms of the loop integrals in dimensional regularization. (That is, we work in the $\overline{MS}$ prescription.) The coefficients ΔZ and ΔM^2 are one loop radiative corrections proportional to the coupling constants g^2 and g'^2. $\Delta Z(\Lambda)$ and $\Delta M(\Lambda)$ are finite and are in principle determined by matching to the high energy theory. If Λ is larger than the mass of all particles transforming under the electroweak interactions, we can choose our parameters such that they vanish (i.e. if $\Delta r(\Lambda) = 0$). At any lower scale, they are determined by matching at heavy particle thresholds and the renormalization group equations.

This approach is somewhat unconventional; normally one chooses to keep the fields properly normalized, instead shifting the effects of the factors ΔZ above to the couplings. Our approach proves to be convenient when considering only "oblique" corrections to the gauge boson propagators.

In the standard model, the gauge invariance implies there are only two independent infinite counterterms: one common δZ_{W^i} for the three W^i fields, and one δZ_B for the hypercharge. The four gauge field wave function counterterms therefore satisfy:

$$\begin{aligned} \delta Z_W &= \delta Z_{W^i} \\ \delta Z_Z &= \bar{c}^2 \delta Z_{W^i} + \bar{s}^2 \delta Z_B \\ \delta Z_{\gamma Z} &= \overline{sc}(\delta Z_{W^i} - \delta Z_B) \\ \delta Z_\gamma &= \bar{s}^2 Z_{W^i} + \bar{c}^2 Z_B \end{aligned} \tag{2.9}$$

In any model without a custodial symmetry violating mass counterterm, there is one further relationship:

$$\delta M_W^2 = \bar{c}^2 \delta M_Z^2 \quad , \tag{2.10}$$

and also $M_W^2 = \bar{c}^2 M_Z^2$. These relationships are not required for the derivation of Δr, but we will use them below.

Here we have defined the parameters $\bar{c}^2 = g^2/(g^2+g'^2)$, $\bar{s}^2 = 1-\bar{c}^2$. These quantities are not running couplings, but parameters of the theory which will be determined by the physical renormalization conditions below.

The physical Z mass is now given by

$$M_{Z\mathbf{phys}}^2 = \frac{M_Z^2 + \Delta M_Z^2(M_W)}{1 + \Delta Z_Z(M_W)} \quad , \tag{2.11}$$

and similarly for M_W^2.

At the W scale (which we identify with the Z scale since $\log(M_W/M_Z)$ is small), we integrate out the heavy gauge bosons, calculating the one-loop corrections to the electroweak parameters of the low energy theory. Large logarithms have been incorporated in the parameters ΔM and ΔZ; there will also be finite matching conditions in general as well. Below the W mass, we match to an effective theory involving just the photon and the four fermion operator responsible for muon decay. We want $\alpha_{EM}(\mu)$ in this theory be the running coupling of the properly normalized photon. We therefore choose

$$4\pi\alpha_{EM}(M_W) = \frac{g^2\bar{s}^2}{1 + \Delta Z_\gamma(M_W)} \quad . \tag{2.12}$$

The Fermi constant renormalized at the scale $\mu = M_W$ is the square of the coupling g divided by 8 times the W propagator at zero momentum. That is,

$$\frac{G_F(M_W)}{\sqrt{2}} = \frac{g^2}{8(M_W^2 + \Delta M_W^2(M_W))} = \frac{g^2}{8M_{W\mathrm{phys}}^2(1 + \Delta Z_W(M_W))} \quad . \tag{2.13}$$

In the low-energy theory, the four-fermion operator does not run with μ, so $G_F(M_W) = G_F(0) = (1.16637 \pm 0.00002) \cdot 10^{-5}$ GeV. This expression may be solved for $\bar{s}$:

$$\begin{aligned} \bar{s}^2 &= \tfrac{1}{2}\Bigg\{ \quad 1 + \frac{\Delta M_W^2(M_W)}{M_{Z\mathrm{phys}}^2} - \Bigg(1 + \frac{2\Delta M_W^2(M_W)}{M_{Z\mathrm{phys}}^2} - \\ & \qquad \frac{2\sqrt{2}\pi\alpha_{EM}(M_W)(1 + \Delta Z_\gamma(M_W))}{G_F(M_{Z\mathrm{phys}}^2(1 + \Delta Z_Z(M_W)) - \Delta M_Z^2(M_W))}\Bigg)^{1/2}\Bigg\} \quad , \end{aligned} \tag{2.14}$$

where here and below, we drop terms of order g^4 or higher. Taken together, equations 2.11, 2.12, and 2.14 allow us to fix the parameters in the lagrangian in terms of known quantities, once ΔZ and ΔM^2 have been calculated.

One can now compare the expression for G_F above to that of Sirlin to derive the formula for Δr. The definition of Δr is

$$\frac{G_F}{\sqrt{2}} = \frac{4\pi\alpha_{EM}(m_e)}{8M_{W\mathrm{phys}}^2 s_w^2(1 - \Delta r)} \quad . \tag{2.15}$$

One concludes that

$$\frac{1}{1 - \Delta r} = \frac{\alpha(M_W)}{\alpha(m_e)} \frac{s_W^2}{\bar{s}^2} \frac{1 + \Delta Z_\gamma(M_W)}{1 + \Delta Z_W(M_W)} \quad . \tag{2.16}$$

The two definitions of the weak mixing angle are related by

$$c_w^2 = \bar{c}^2(1 + \frac{\Delta M_W^2}{M_W^2} - \frac{\Delta M_Z^2}{M_Z^2} - Z_W + Z_Z) \quad . \tag{2.17}$$

So, in sum,

$$\begin{aligned} 1 + \Delta r &= \frac{\alpha(M_W)}{\alpha(m_e)}\Bigg\{ 1 - \Delta Z_W(M_W) + \Delta Z_\gamma(M_W) \\ &+ \frac{c_w^2}{s_w^2}\Bigg(\frac{\Delta M_Z^2(M_W)}{M_{Z\mathrm{phys}}^2} - \frac{\Delta M_W^2(M_W)}{M_{W\mathrm{phys}}^2} \\ & + \Delta Z_W(M_W) - \Delta Z_Z(M_W)\Bigg)\Bigg\} \end{aligned} \tag{2.18}$$

Expanding the terms in 2.3, we will arrive at this expression.

Of course, as a byproduct of deriving the theory at the scale M_W, we have directly solved for the three parameters of the theory, g, g' and M_Z, in terms of α, G_F, and $M_{Z\mathbf{phys}}$. We can now compute the remaining ones, including the one-loop correction terms. One could thereby use our results of the following sections to derive deviations in the standard model for the W mass directly and for other quantities of interest such as the forward backward asymmetry for leptons or b's simply by computing the results at the scale M_W and comparing to the standard prediction. One would thereby account for both direct effects and those due to the fact that the parameters extracted from the physical quantities are different from their standard model counterparts. The net deviation would be constrained to be less than the difference between experimental results and the standard model prediction.

As an illustration of the simplicity of this formulation, one can derive the difference, Δ, between the value of $\sin^2\theta_W$ which would be extracted from a measurement of the forward–backward asymmetry and that obtained from the gauge boson masses [13]. Accounting for both the $\gamma - Z$ mixing contribution and the difference in our value of $\overline{s}$ from the Sirlin convention (see eq. 2.17) one finds

$$\Delta = Z_{\gamma Z}(M_Z) - \frac{c_w}{s_w}\left(\frac{\Delta M_Z^2(M_W)}{M_{Z\mathbf{phys}}^2} - \frac{\Delta M_W^2(M_W)}{M_{W\mathbf{phys}}^2} + \Delta Z_W(M_W) - \Delta Z_Z(M_W)\right) \quad . \tag{2.19}$$

For the remainder of this paper, we will restrict our discussion to Δr. Before presenting our results, we discuss how one can simply extract the largest contributions. We will see that for the standard model, the most important logarithmically enhanced contribution will arise from the scaling of α_{em}, but that for technicolor models, one gets large logarithms of the form $\log(\Lambda/m)$.

We will retain terms in Σ only up to order p^2 and work to leading order in α_{em}. We calculate the corrections to ΔZ and ΔM^2 by scaling between physical thresholds and matching at the heavy particle mass scales. However, we will neglect finite matching corrections, retaining only the logarithmically enhanced scaling contribution. In dimensional regularization, the anomalous dimension can be extracted from the pole from the loop of particles. This contribution will scale $\Delta Z(\mu)$ and $\Delta M(\mu)$ to the scale of the particle mass or μ, whichever is larger. However, because we are only working to leading order in α, the log we thereby obtain is equivalent to the log which would appear were we simply to evaluate the loop at the particle mass, applying the $\overline{MS}$ prescription. Therefore we can write

$$\begin{aligned}\Delta Z_G(\mu) - \Delta Z_G(\Lambda) &= \sum_m \Sigma'^{GG}(0;\Lambda) - \Sigma'^{GG}(0;\overline{\mu}) \\ \Delta M^2(\mu) - \Delta M^2(\Lambda) &= \sum_m -\Sigma^{GG}(0;\Lambda) + \Sigma^{GG}(0;\overline{\mu}) \quad , \end{aligned} \tag{2.20}$$

where $\Sigma^{GG}(p^2;\mu)$ is the propagator correction defined above, regulated at the scale μ. The sum in the above expression represents the loops involving particles of mass m, and $\overline{\mu} = \mu$ if $\mu > m$ and $\mu = m$ otherwise.

For simplicity, first consider the case where the corrections to the self energy arise only from degenerate custodial SU(2) multiplets. In this case, we are interested only in the logarithmically enhanced U(1) corrections. From equations 2.9, we derive the relation

$$\delta Z_W - \delta Z_\gamma - \overline{c}^2/\overline{s}^2(\delta Z_W - \delta Z_Z) = 0 \quad . \tag{2.21}$$

Because of the relation between the logarithmic μ dependence and the infinite counterterms, we can therefore conclude that all $\log(\mu)$ dependence cancels from the sum of the ΔZ terms appearing in the expression for $1 + \Delta r$ when we work to leading order in α_{em} and neglect $\log(M_Z/M_W)$.

Therefore, if only heavy degenerate SU(2) representations run in the loops determining the gauge boson self energies, there will be no large logarithms in the expression for Δr. However, there could be light particles in the theory (that is, with mass smaller than that of the W). In this case, there can be large logarithms, but only coming from the factor $(\alpha(M_W)/\alpha(m_e))$. Such light particles contribute to the running of α between these two scales. This is the only logarithmically enhanced effect surviving in Δr.

If we now allow the particles of an SU(2) multiplet to be nondegenerate, there can be additional custodial SU(2) violation in the term $(\Delta M_Z^2/M_Z^2 - \Delta M_W^2/M_W^2)$. These effects must also be included. There will also be additional logarithmic dependence on the ratio of heavy and light masses. However, such terms will in general be smaller than the mass splitting effects.

This agrees with previous estimates of large standard model effects, (see refs. [7,14], for example) where it is found that the largest U(1) effect is simply from scaling the electromagnetic coupling e between zero momentum and the W mass. This greatly simplifies the calculation of Δr in the standard model, if one is only interested in the largest contributions.

We now compare this result to the one we obtain in technicolor theories. In this case, we will show in the next section that there is an independent wave function renormalization constant so that the above relations between the renormalization of the gauge boson fields no longer apply. This counterterm induces an extra term in the lagrangian of the form

$$Z_X \left(\partial^\mu W^\nu \partial_\mu W_\nu + \frac{1}{2\bar{c}^2} \partial^\mu Z^\nu \partial_\mu Z_\nu \right) \quad . \tag{2.22}$$

The relation for Z_Z from Eq. 2.9 therefore becomes

$$\delta Z_Z = \bar{c}^2 \delta Z_{W^3} + \bar{s}^2 \delta Z_B + \frac{1}{\bar{c}^2} \delta Z_X \tag{2.23}$$

while that for Z_W becomes

$$\delta Z_W = \delta Z_{W^3} + \delta Z_X \tag{2.24}$$

where δZ_X absorbs the remaining infinity in Z_W and Z_Z encountered in technicolor theories.

Now the relation 2.21 is destroyed, so there can indeed be a logarithm remaining in the expression for Δr, even when only particles heavier than the W run in the loop. In technicolor theories, we expect this logarithm to be cut off at a scale Λ_χ of order of the technicolor condensate scale. Corrections to Δr from light pseudogoldstone bosons in technicolor theories will therefore be enhanced by the factor $\log(\Lambda_\chi^2/m^2)$.

In fact, in the standard model such a logarithm also occurs. There will be similar contribution to Δr when there is a heavy field which contributes to the β function of an operator which reduces to 2.22 when the Higgs has a nonzero VEV. From dimensional considerations, it is easy to see that fermion loops for example do not give a divergent contribution to the β function of such an operator. However, the unphysical Goldstone boson states responsible for the masses of the heavy gauge bosons[12] do contribute. These states act like the pseudogoldstone bosons of technicolor, and contribute to the scaling of nonrenormalizable operators which reduce to 2.22 when the Higgs takes its vacuum expectation value. In the standard model, the logarithm is cut off by the Higgs mass. For a single standard model Higgs field however, even with the $\log(m_H/m_Z)$ enhancement factor, the contribution to Δr is fairly small.

Of course, the presence of this additional counterterm means that Δr is in fact a new parameter of the technicolor theory with an arbitrary value at the cutoff scale. However, one expects the logarithmically enhanced contribution to dominate over that determined with a chiral coefficient determined by naive dimensional analysis [15].

In our calculations, we will simply assume the pseudogoldstone bosons are heavier than the W and Z gauge bosons. Also, since we work to lowest order in α_{em}, we can extract the relevant logarithms simply by evaluating Σ at the relevant momentum scales (without explicitly scaling). We retain the logarithms and discard the additional finite corrections and poles. This is the simple procedure which we apply in the next section.

3 Low Energy Effective Theory

In this section, we consider the leading contributions to Δr, or equivalently, $\Sigma(k^2)$ in technicolor models. To do so, we first construct the low energy chiral Lagrangian[16] for a general technicolor model.

To simplify the analysis, we will not consider the full set of technicolor theories, but only those theories whose global symmetry group is of the form $G_L \times G_R \times U(1)_V$. The two copies of the group G are the transformations on the left- and right-handed techniquark fields, and the $U(1)$ is an overall phase for all the techniquarks. The formation of the condensate breaks the global symmetry down to the vectorial part, $G_V \times U(1)_V$. We assume that the condensate $\langle \bar{Q}_\alpha Q_\beta \rangle$ is proportional to the identity.

We now need to specify the embedding of the gauged $SU(2)_W$. We will focus our analysis on theories which preserve custodial SU(2), aside from the symmetry breaking present in the standard model. A simple class of such theories can be constructed by only allowing left handed technifermions to transform under the gauged SU(2) symmetry. Then generator of the custodial $SU(2)_C$ symmetry is then the direct sum of the generator of $SU(2)_W$ and the corresponding global symmetry generator embedded in G_R, under which right handed fermions transform like their left handed counterparts.

We restrict our attention to this class of theories.

We denote the generators of $\mathrm{SU}(2)_W$ by τ^i. They satisfy

$$[\tau^i, \tau^j] = i\epsilon^{ijk}\tau^k \quad . \tag{3.1}$$

The action of the hypercharge on the left-handed techniquarks is generated by some other element y of the algebra of $G_L \times U(1)_V$ (though y may be zero). We know that the condensate does not break electric charge; therefore, we can deduce that the action of the hypercharge on the right-handed techniquarks is $y + \tau_L^3$.

To construct the chiral Lagrangian, the technipions π_T are exponentiated in a field Σ

$$\Sigma = \exp\left(\frac{2i\pi_T \cdot T}{h}\right) \quad , \tag{3.2}$$

where T^i are the generators of the group G, and h is the order parameter parameter associated with the condensate. This field transforms linearly under $G_L \times G_R$,

$$\Sigma \to L\Sigma R^\dagger \quad , \tag{3.3}$$

and it is a singlet under $U(1)_V$. Because the electroweak generators are embedded in G_L and G_R, the gauge covariant derivative of Σ is given by

$$\begin{aligned} D^\mu\Sigma = \partial^\mu\Sigma \quad &- \quad i(gW^\mu \cdot \tau + g'B^\mu y)\Sigma \\ &+ \quad i\Sigma(g'B^\mu(y + \tau_L^3)) \quad . \end{aligned} \tag{3.4}$$

The chiral Lagrangian is an expansion in powers of derivatives. In the absence of ETC interactions, the term with the fewest derivatives is the kinetic energy

$$\mathcal{L}_{\mathrm{KE}} = \frac{h^2}{4} \operatorname{tr} (D^\mu\Sigma)^\dagger(D_\mu\Sigma) \quad . \tag{3.5}$$

The coefficient of this term is dictated by the normalization of the technipion kinetic energy, which is obtained by expanding out the exponential and using the normalization condition of the generators

$$\operatorname{tr} T^iT^j = \frac{1}{2}\delta^{ij} \quad . \tag{3.6}$$

There are no other terms with two derivatives.

In addition to the technipion propagators, $\mathcal{L}_{\rm KE}$ also includes gauge couplings. If we set $\pi_T = 0$, then $\Sigma = I$, and the W and Z will get a mass:

$$M_W^2 = g^2 \frac{h^2}{4} \text{ tr } ((\tau^1)^2) \tag{3.7}$$

$$M_Z^2 = (g^2 + g'^2) \frac{h^2}{4} \text{ tr } ((\tau^3)^2) \quad , \tag{3.8}$$

which fixes h in terms of known quantities.

From this equation, we can determine how the scale, h, scales with the number of flavors which condense through the technicolor gauge symmetry. For example, if all left-handed fermions were in doublets, while their right-handed counterparts were singlets, we would find $h = v/\sqrt{N_f}$, where $v \approx 250\,\text{GeV}$ is the $\text{SU}(2)_\text{W}$ breaking scale, and N_f is the number of doublets.

From equation 3.5 we may also determine the "swallowed" technipions, ϕ^i, which become the longitudinal components of the gauge bosons:

$$\phi^i \propto \pi_T^m \text{ tr } (\tau^i T^m) \quad . \tag{3.9}$$

This kinetic energy term determined the gauge couplings of the Goldstone bosons which will be required in order to calculate radiative corrections. Had the theory contained a field with the properties of the standard model Higgs, the couplings would be determined by this kinetic energy times $((H+v)/v)^2$, where H is the Higgs field and v is the VEV. We assume there is no such field in our theory.

In the foregoing we have assumed that there is only one type of technifermion which condenses to break the chiral symmetry. In two-scale models[17], for example, we would have to allow for more than one parameter, h.

In general, the technipions are not necessarily exact Goldstone bosons. In fact, from recent LEP data [18], we know the charged Goldstone bosons will be heavier than 35 GeV. Photon radiative corrections are expected to give a smaller mass [2], so we will assume the existence of additional mass terms in our theory, and treat radiative corrections as higher order custodial SU(2) violating effects. We will however assume that the leading order mass term preserves $\text{SU}(2)_\text{C}$. Therefore, the technipions in a given $\text{SU}(2)_\text{C}$ multiplet will be degenerate at leading order with a mass, m.

Having constructed the leading order terms in derivatives and symmetry breaking, we now consider higher order terms. From Eq. 2.3, one sees that both corrections to the kinetic energy and mass terms for the W and Z gauge bosons will give a contribution to Δr. Because the U(1) coupling violates the custodial symmetry, the nonrenormalizable theory below the electroweak scale requires counterterms which will also contribute to Δr. One example of such a term is

$$\mathcal{L}_{\text{h.o.}} = \frac{\beta}{4}\frac{1}{16\pi^2}\ \text{tr}\left(([D^\mu, D^\nu]\Sigma)^\dagger([D^\mu, D^\nu]\Sigma)\right) \quad , \tag{3.10}$$

where β is an undetermined coefficient which we expect to be of order unity. This is one of many possible four derivative terms.

4 Technipion Radiative Corrections

Having constructed the leading order chiral lagrangian, we are now prepared to calculate the corrections to Δr in the low energy theory. We expect the dominant contribution to come from technipion loops, since their contribution is enhanced by a potentially large logarithm of the form $\log(\Lambda_\chi^2/m^2)$, where m is the mass of the technipion multiplet and Λ_χ is the symmetry breaking scale, which, from general considerations[15], we expect to be about $4\pi h$. Contributions from more massive states not appearing in the low energy theory can be absorbed in counterterms of the form in equation 3.10.

In fact these counterterms are required since the contribution from pseudogoldstone boson loops alone would be divergent.

We calculate corrections of the so-called "oblique" type, *ie.* to the W, Z, or γ propagators, but not their vertices with fermions. The technipion loops correcting the gauge boson propagators are shown in figure 1.

The particles which run in the diagrams of figure 1 are the technipion mass eigenstates described above. We consider the contribution from a single degenerate representation of $\text{SU}(2)_\text{W}$. If we denote these states by π^s, they satisfy

$$\begin{aligned}\left[\tau^3, \pi^s\right] &= s\pi^s \\ \left[\tau^+, \pi^s\right] &= \sqrt{\ell(\ell+1) - s(s+1)}\pi^{(s+1)} \quad ,\end{aligned} \tag{4.1}$$

where ℓ is the isospin of the multiplet.

We may now use the gauge couplings of the term in equation 3.5 and the relationships of equation 4.1 to compute the diagrams of figure 1. The integrals are regulated using a dimensional regulator with a scale parameter μ in $4-\epsilon$ dimensions. First define

$$
\begin{aligned}
C^{\gamma\gamma} &= \frac{e^2}{16\pi^2}\left[\sum_{-\ell}^{\ell}(s+y_m)^2\right] \\
C_1^{ZZ} &= \frac{e^2}{16\pi^2}\left[\sum_{-\ell}^{\ell} -s(s+y_m)+\frac{s_w^2}{c_w^2}y_m(s+y_m)\right] \\
C_2^{ZZ} &= \frac{e^2}{16\pi^2}\left[\sum_{-\ell}^{\ell}\left(-\frac{s_w}{c_w}y_m+\frac{1}{2}\left(\frac{c_w}{s_w}-\frac{s_w}{c_w}\right)s\right)^2\right] \\
C^{WW} &= \frac{e^2}{16\pi^2}\frac{1}{8s_w^2}\left[\sum_{-\ell}^{\ell}\ell(\ell+1)-s(s+1)\right] \\
d(x) &= \frac{8}{3}-\frac{8}{9x}-\frac{2}{3x}(4x-1)^{3/2}\tan^{-1}((4x-1)^{-1/2}) \\
D(m^2) &= \frac{2}{\epsilon}-\gamma_E-\log\left(\frac{m^2}{4\pi\mu^2}\right) \quad , \qquad (4.2)
\end{aligned}
$$

where e is the electronic charge, y_m is the common hypercharge of the multiplet, and γ_E is Euler's constant. We then find

$$
\begin{aligned}
\Sigma^{\gamma\gamma}(p^2) &= C^{\gamma\gamma}\left(-\left(\frac{1}{3}\right)p^2D(m^2)+m^2d\left(\frac{m^2}{p^2}\right)\right)F \\
\Sigma^{WW}(p^2) &= C^{WW}\left(2m^2(D(m^2)+1)-\left(\frac{1}{3}\right)p^2D(m^2)+m^2d\left(\frac{m^2}{p^2}\right)\right)F \\
\Sigma^{ZZ}(p^2) &= C_2^{ZZ}\left(2m^2(D(m^2)+1)-\left(\frac{1}{3}\right)p^2D(m^2)+m^2d\left(\frac{m^2}{p^2}\right)\right)F \\
&\quad +C_1^{ZZ}(-2m^2(D(m^2)+1))F \quad . \qquad (4.3)
\end{aligned}
$$

In writing the answer in this form, we have used the tree level relations $g=e/s_w$ and $g'=e/c_w$. Here $F=1/2$ when the multiplet of technipions is self conjugate, and $F=1$ otherwise. For example, the ordinary pion triplet (π^+,π^0,π^-) is self conjugate, but the kaon doublet (K^+,K^0) is not. The function $d(x)$ is never very important; for all values of m bigger than about

35 GeV its contribution to Δr is less than about 10%. In what follows, we will ignore $d(x)$.

Now

$$\left[\sum_{-\ell}^{\ell} s^2\right] = \frac{1}{2}\left[\sum_{-\ell}^{\ell} \ell(\ell+1) - s(s+1)\right] = \frac{\ell(\ell+1)(2\ell+1)}{3} \quad . \tag{4.4}$$

Substituting into the standard relation for Δr, we find

$$\Delta r = \frac{\alpha}{4\pi}\frac{\ell(\ell+1)(2\ell+1)}{18 s_w^2} D(m^2) F \quad , \tag{4.5}$$

where $\alpha = e^2/4\pi$.

As explained in the previous section, the infinite result arises when we neglect the contribution from the additional four derivative term of the form 3.10. Such terms also contribute to Δr through their contribution to $\Sigma'^{GG}(0)$. They contain a part with no technipions and only two gauge bosons:

$$\frac{\beta}{8\pi^2}(g^2 \partial^\mu W^{\pm\nu} \partial_\mu W_\nu^{\mp} + (g^2 + g'^2)\frac{1}{2}\partial^\mu Z^\nu \partial_\mu Z_\nu) \quad , \tag{4.6}$$

and this term therefore contributes to $\Sigma'^{WW}(0)$ and $\Sigma'^{ZZ}(0)$. The contribution to Δr is

$$-\frac{\alpha\beta}{2\pi s_w^2}\frac{\ell(\ell+1)(2\ell+1)}{6} \tag{4.7}$$

for each multiplet. When we choose β appropriately, Δr is finite.

In fact, one can check that this single additional counterterm is adequate to render the theory finite. This is because of the remaining relation between the divergences in $\Sigma^{\gamma\gamma}$, $\Sigma^{\gamma Z}$, Σ^{ZZ}, and Σ^{WW}.

To the extent to which the logarithmically enhanced terms dominate, we can derive the radiative corrections to Δr which are

$$\Delta r \approx \frac{\alpha}{4\pi}\sum_I \frac{\ell_I(\ell_I+1)(2\ell_I+1)}{18 s_w^2} F \log\left(\frac{\Lambda_\chi^2}{m_I^2}\right) \tag{4.8}$$

where we are including the physical Goldstone boson representations I of custodial SU(2) (unphysical Goldstone boson contributions were considered for example in reference [12]). The $1/s_w^2$ dependence is expected for the custodial SU(2) violating effects from U(1) (and not mass splitting).

In a model with only doublets of fermions, this contribution grows as $N_f^2 - 1$. For $\Lambda_\chi \approx 4\pi v$, $m \approx M_Z$, the logarithm would lead to a factor of 7 enhancement. However, as we saw above, $h = v/\sqrt{N_f}$, so with a larger number of fermions, the logarithm is reduced.

5 Vector Technimesons

Having considered the low energy contributions to the parameter Δr, we now show that the remaining contributions from heavier resonances will probably not be large, so their contribution can probably be adequately represented by the four derivative counterterms discussed in the previous section with a coefficient determined by naive dimensional analysis.

The technimesons which are expected to be next lowest in mass above the technipions are the vectors, the analogues of the $\rho(770)$ and $\omega(783)$. These particles too will contribute corrections to the low energy electroweak parameters. In addition to radiative corrections to Δr, there is a tree level contribution. However, this contribution is probably not large, as can be deduced from the following estimate, based on vector meson dominance and naive dimensional analysis[15].

The diagram of figure 2 is shows a "vector dominance" correction to the propagator of the gauge bosons. This type of process is well described by pointlike vertices for the intermediate particle because the momentum through the diagram is only of order M_W. Each of the gauge boson–vector meson vertices introduce a factor of p^2/f, where p^2 is the gauge boson mass squared and f, the analog of f_ρ, is approximately 4π. Computing the diagram introduces a propagator suppression of order m_ρ^2, so the net contribution is approximated by

$$\Delta r \approx \frac{e^2 M_W^2}{m_\rho^2 f_\rho^2} \quad . \tag{5.1}$$

If we substitute $m_\rho \approx 4\pi v$, we see this contribution is suppressed by $(\alpha/4\pi)^2$, which is smaller than one loop radiative contributions.

One loop corrections will probably not be very large either. For technirho mass large compared to the cutoff, we expect the contribution to Δr to decouple; for masses near Λ_χ, the logarithm $\log((\Lambda_\chi^2 + M^2)/M^2)$ is not large.

6 Fermionic Techniparticles

Because so little is known about how to construct technicolor models, it is possible that there are viable models in which there are relatively light resonances other than the technipions and technirhos. In this section we consider the effects of a loop of spin-1/2 resonances. The discussion is similar to that of the second section. The formulation of the chiral lagrangian is the same as that for baryons, given for example in reference [15].

In order to construct a chiral Lagrangian for these particles, we must first define their transformation properties. To do this, we define the field ξ, such that

$$\xi\xi = \Sigma \quad . \tag{6.1}$$

The field ξ transforms as

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger \quad , \tag{6.2}$$

where U, L, and R are elements of G. This equation implicitly defines U as a function of L, R, and the technipion fields. We now take the fermions to transform as

$$f \rightarrow Uf \quad , \tag{6.3}$$

but an equivalent theory may be constructed using any other transformation law[16].

We may construct a covariant derivative of f:

$$D^\mu f = \partial^\mu f - iV^\mu f \quad , \tag{6.4}$$

where

$$V^\mu = \frac{i}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger - i\xi^\dagger(gW^\mu\cdot\tau + g'B^\mu y)\xi - i\xi g'B^\mu(\tau^3 + y)\xi^\dagger) \quad . \tag{6.5}$$

This covariant derivative obeys

$$D^\mu(Uf) = U(D^\mu f) \quad . \tag{6.6}$$

The chiral lagrangian for these fermions begins with terms of zero or one derivative

$$\mathcal{L}_f = \bar{f}(i \not{D} + c_A \not{A}\gamma_5 + m_f)f \quad . \tag{6.7}$$

Here A^μ is a field made out of technipions and gauge bosons

$$A^\mu = \frac{i}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger - i\xi^\dagger (gW^\mu \cdot \tau + g' B^\mu y)\xi + i\xi g' B^\mu (\tau^3 + y)\xi^\dagger) \quad . \tag{6.8}$$

The field A^μ transforms linearly under the action of the group: $A^\mu \to U A^\mu U^\dagger$.

The vectorial couplings of the fermions are fixed by the normalization of the kinetic energy, but the unknown axial coupling is determined by the strong dynamics and is an additional parameter of the theory. Of course, since the photon field does not appear in A^μ, its coupling is purely vectorial and therefore determined.

The m_f term generates the mass of the fermion, and in most models it will be at or above the scale Λ_χ. On the other hand, one might imagine that the dynamics of the model creates fermions of small enough mass that the chiral lagrangian can be used to compute the technifermion loops. As before, the ETC operators will split the masses of the fermions, yielding degenerate custodial symmetry multiplets.

Calculating the fermion loop in dimensional regularization we find

$$\begin{aligned}\Sigma^{GG}(p^2) &= 8\frac{g_A^2}{16\pi^2} m^2 D(m_f^2) \\ &\quad - \frac{4}{3(16\pi^2)} p^2 \left[(g_V^2 + g_A^2) D(m_f^2) - g_A^2\right] + O\left(\frac{p^4}{m_f^4}\right) \quad ,\end{aligned} \tag{6.9}$$

where g_V and g_A are the vector and axial couplings respectively of gauge boson G. This yields the correction to Δr from the fermions:

$$\Delta r \approx \frac{\alpha}{4\pi} \frac{2\ell(\ell+1)(2\ell+1)}{9 s_w^2} (c_A^2 + (1 - c_A^2) D(m_f^2)) \quad . \tag{6.10}$$

Here the correction is infinite again, at least in the case where $c_A \neq 1$, so once again the counterterm above will have to be adjusted to cancel this divergence. In the standard model, the couplings of a heavy degenerate doublet of fermions are like those generated with $c_A = 1$. As stated above, such fermions contribute a finite correction to Δr.

7 Conclusion

At present, the measured values of the W and Z masses are not accurate enough for the corrections above to constrain technicolor models with a reasonable number of technifermions. It is possible, however, that in the not too distant future the W mass may be measured to 45 MeV, the Z mass to 20 MeV[19], and the top to better than 10 GeV[20]. If, for example, we assume that the values are $M_Z = 91.000 \pm .020$ GeV, and $M_W = 80.000 \pm .045$ GeV, then the measured value of Δr is $(4.4 \pm 0.40) \times 10^{-2}$.

The largest uncertainty in the standard model prediction of Δr will come from the lack of precision in the top quark mass measurement. If the mass of the top is measured to be 150 ± 10 GeV, then $\Delta r_{top} \approx -2.2 \pm 0.3 \times 10^{-2}$. The standard model prediction of course depends on the value of the Higgs boson mass, but since technicolor models generally lack light resonances with the properties of the Higgs boson, we would expect that these models should agree with the standard model prediction with M_H above 1 TeV. (A light Higgs boson generates rather substantial corrections to Δr.) If the W mass agrees with this scenario, then there will be little room available for complex technicolor models. We conclude the uncertainty in the measured minus the predicted value of Δr will be $\approx 5 \times 10^{-3}$. Models which generate corrections appreciably larger than this will be ruled out.

Consider, for example, the one-family model [21], in which the technifermions have the charges under SU(3) $\times$ SU(2)$_W$ $\times$ U(1) of an ordinary family of quarks and leptons. In this model, the global flavor symmetry group G is SU(8), and, since there are four doublets, $h = v/2$. For this model equation 4.8 gives a correction of $\approx 3.6 \times 10^{-2}$, corresponding to a shift in M_W of about 560 MeV. This value is clearly sufficiently large to be tested in future experiments. Moreover, it has the opposite sign as the top quark (or other splitting) contribution, so it can be distinguished. The opposite sign applies in models with degenerate fermions as well. Moreover, the effects described in equation 4.8 grow roughly quadratically with the number of generations. On the other hand, this value is somewhat uncertain because of the lack of knowledge of the technipion masses, and of the ambiguity in the scale Λ_χ. If technipion masses are significantly enhanced relative to the values we've chosen, then other contributions could be comparible to the one we've calculated.

An interesting case is also presented by the two-scale models[17]. In these models, there are two types of techniquarks, which transform under different representations of the technicolor group (TC). For example, one may imagine a model in which there a techniquarks q which transform as a fundamental of $SU(N)_{TC}$, and techniquarks Q which transform as adjoints. In this model, there will not only be the corrections from the technipions, but there may also be light fermionic resonances of the form $q\bar{q}Q$. These may generate additional corrections of the type discussed in section 5.

In this paper we have restricted ourselves to the correction of the mass of the W. There are other experiments, such as the forward-backward asymmetry of e^+e^- scattering at the Z pole, which could also be sensitive to the presence of technicolor. These effects could depend on a different combination of standard model and non standard model one-loop contributions, which could in principle enable one to distinguish between the two.

In realistic technicolor models, the shift in the W mass we have considered in this paper from degenerate custodial SU(2) violations is not necessarily the leading effect. There can be model dependent corrections due to additional sources of custodial SU(2) violation, which will generally occur with the opposite sign. So we can only view our result as indicative of the fact that without fine tuned cancellations, one would expect technicolor models yield measurable corrections to electroweak parameters, in particular, Δr. These effects should be useful in restricting and constraining technicolor models.

If deviations from the standard model prediction of Δr are observed, they might be the first indication of an underlying technicolor theory or some other physics associated with electroweak symmetry breaking. In general, by calculating the parameters of the effective field theory presented in Section 2, one should be able to readily compute corrections to electroweak parameters from nonstandard models.

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9 Figure Captions

Figure 1a, Figure 1b - Technipion loops contributing to Σ^{GG}.
Figure 2 - A "vector dominance" contribution to Σ^{GG}.

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