Fermi National Accelerator Laboratory

FERMILAB-PUB-90/54-T

Static Effective Field Theory: 1/m Corrections

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Abstract

The static approximation, which is the zeroth order approximation in an expansion in the inverse of the mass of a heavy quark, has previously been formulated in terms of an effective field theory action. In this formulation, corrections to the approximation can be systematically included by the addition of higher dimensional operators to the action. We determine the coefficients to one loop of the dimension five operators incorporating the $1/m$ corrections to the theory.

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1. Introduction

The static approximation is the zeroth order approximation in an expansion in the inverse of the mass of a heavy quark, termed the $1/m$ expansion. The conceptually clearest and computationally most efficient way to formulate the approximation is in terms of an effective field theory action [1][2][3][4]. Computations using this action and the simple Feynman rules that result have recently been performed [4]. Once formulated in terms of an effective field theory action, it is clear [2][4] that the static effective theory is conceptually very similar to the nonrelativistic effective field theory already developed to an advanced state for QED by Caswell and Lepage [5]. The conjecture that the static effective field theory can reproduce the results of the full theory to all orders in α_S and to any fixed order in $1/m$ has been demonstrated at zeroth order in $1/m$ by Grinstein [6].

Many authors have contributed to the study of heavy quark dynamics in QCD. An interesting and pedagogical review has been written by Peskin [7]. The static potential in QCD was studied by Feinberg [8] and others [9], and applied to the definition and determination of the spin-dependent potential in heavy quark-antiquark bound states by Eichten and Feinberg [10]. The utility of the static approximation and the closely related nonrelativistic approximation for simplifying lattice calculations of matrix elements of heavy quarks was discussed by Eichten [1], and by Lepage and Thacker [2], and the perturbative corrections to the B meson decay constant measured on the lattice using the static approximation [11] have been calculated [12]. Politzer and Wise [3] used the approximation to extract logarithms of heavy quark masses previously obtained in the full theory [13]. The symmetries of heavy quarks treated at lowest order in the $1/m$ expansion have been discussed and applied to semileptonic B to D decays at the kinematic endpoint where the B and D have a common rest frame by Isgur and Wise [14]. They also generalized these symmetries to an arbitrary frame and used this to extend their results for B to D decays and relate other form factors [15]. The radiative corrections to these relations have been obtained very recently by Falk, Georgi, Grinstein and Wise [16] using a generalization of the static effective field theory to an arbitrary frame proposed by Georgi [17].

In the effective field theory formulation of the static approximation, corrections can be systematically included by adding operators of dimension greater than four to the action. There are two dimension five operators which incorporate the $1/m$ corrections to the static effective theory: the chromomagnetic moment operator and the nonrelativistic kinetic energy. The coefficients of the operators must be fixed by matching amplitudes in the effective theory to their counterparts in the full theory. These are the same operators that appear in the nonrelativistic effective field theory.

The essential difference between the static and nonrelativistic theories is that in the latter, the nonrelativistic kinetic energy is incorporated at lowest order rather than perturbatively. Thus the propagators in the static effective theory and the nonrelativistic effective theory differ for momenta of order m. Since in divergent loop integrations there are contributions from loop momenta of this order, the counterterms in the two effective theories are not related. However the dependence on light scales must be the same. Within the framework of the nonrelativistic effective theory, Lepage and Ning [2] have determined the anomalous dimension of the chromomagnetic operator, and Lepage and Thacker [18] are studying the complete one loop corrections to the counterterms. While it is somewhat more difficult to perform the loop integrals needed to calculate these counterterms in the nonrelativistic effective theory, the approximation has the advantage that it can be used in processes where more than one heavy quark can appear in an easily accessible intermediate state [2][4].

In this paper we will determine the coefficients of the dimension five operators in the static effective field theory to order α_S. In the following section we first describe the static effective field theory, and then display the dimension five operators and their tree level coefficients. In section three we perform the one loop computations needed to determine the coefficients of the dimension five operators to order α_S. We conclude with a brief discussion of applications.

2. The Static Effective Field Theory

The static effective field theory is obtained by linearizing the dispersion law for a heavy quark about $(m, 0)$. At lowest order in the $1/m$ expansion, the result is that $E = m$, independent of the momentum of the heavy quark. The Lagrangian embodying this dispersion law is

$$\mathcal{L} = b^\dagger i\partial_0 b. \tag{2.1}$$

The heavy quark mass does not appear, because it is convenient to measure the momenta of external and intermediate states relative to the momentum around which the the dispersion law has been linearized. The effective theory is only applicable for processes with an external heavy quark whose four-velocity is near $(1, 0)$ because the dispersion law in the effective theory must be a good approximation to the dispersion law in the full theory for easily accessible intermediate states.* In (2.1), the field b is a two-component field which annihilates heavy quarks, and $b^\dagger$ is a two-component field that creates them. The fields do not also annihilate and create heavy antiquarks. There is no pair creation in the effective theory. Intermediate states with heavy quark-heavy antiquark pairs created will always have large energy denominators, so their elimination is a consistent part of the effective field theory treatment.

The static effective field theory Lagrangian, including gauge interactions and order $1/m$ corrections, is [5][2],

$$\mathcal{L} = Z b^\dagger i\mathcal{D}_0 b + \frac{Z Z_{kin}}{2m} b^\dagger \mathcal{D}_i \mathcal{D}_i b + i \frac{Z Z_{mag}}{2m} b^\dagger \epsilon_{ijk} \mathcal{D}_i \mathcal{D}_j \sigma_k b, \tag{2.2}$$

where $i\mathcal{D}_\mu = i\partial_\mu + gA_\mu$, is the gauge covariant derivative. We will perform our calculations using the background field gauge which has the property that the combination gA_μ is not renormalized. We have only included the terms in the Lagrangian which contain interactions of the heavy quark with the gauge fields. In particular, we have not included the mass renormalization counterterm. The two dimension five operators are the leading corrections in the 1/m expansion. Their coefficients have been chosen so that the the multiplicative renormalization factors are unity at tree level, as one can show by applying the Gordon decomposition to a tree-level amplitude with a gluon-heavy quark interaction. The wave function

* The generalization valid for heavy quarks with four-velocity near an arbitrary four-velocity U^μ can be obtained by linearizing the dispersion law about mU^μ, yielding $U^\mu p_\mu = m$. The effective Lagrangian embodying this dispersion law is $\mathcal{L} = b^\dagger U^\mu i\partial_\mu b$, which is the heavy quark piece of the Lagrangian (8) of reference [17].

renormalization, Z, is included in each factor. These definitions of Z_{kin} and Z_{mag}, which do not depend on the normalization of the heavy quark field, are the ones relevant for physical applications. In the next section we will determine Z_{kin} and Z_{mag} at one loop order.

3. Calculation

The heavy quark propagator obtained from the Lagrangian (2.1) is [4]

$$\frac{i}{p_0 + i\epsilon.} \tag{3.1}$$

In Feynman diagrams, the heavy quark propagator will be denoted with a double line. The vertices coming from the kinetic term will be denoted with a box (see figure 1), and the vertices coming from the chromomagnetic moment operator will be denoted with a triangle (see figure 2).

The coefficients of the operators in the static effective field theory are determined by matching amplitudes in the effective theory to their full theory counterparts. We find that using the background field method (see reference [19] for a review) and matching the part of the background field generating functional that is first order in the background field and has two external fermion lines is a calculationally convenient way to determine the coefficients of the dimension five operators. This is analogous to the procedure followed by 't Hooft [20] and Dashen and Gross [21] in their determinations of the counterterms in theories defined using two different regularizations. In both cases the theories being matched only differ at high energies.

The graphs in the full theory needed to determine this amplitude are depicted in figure 3. The external fermions are on shell and we calculate the amplitude at $k^2 = 0$, where k is the momentum transfer from the background field, but keep terms linear in k. The infrared divergences encountered from calculating at this point, as well as the usual ultraviolet divergences, will be regulated using dimensional regularization. Because the effective theory has the same low energy behavior as the full theory, the infrared divergences in corresponding graphs will exactly cancel the infrared divergences in the full theory graphs when the matching conditions are properly formulated. One expects that the results will thus be independent of the choice of infrared regulator. Indeed we are free to apply $\overline{MS}$ to the poles that appear as a result of infrared divergences so long as we use the same procedure on the poles that appear in the effective theory.

We write the amplitude for any of the graphs in figure 3 as

$$igA_{\mu a}(-k)\overline{u}'\Gamma^{\mu}T_a u. \tag{3.2}$$

The tree graph's contribution to Γ^{μ} so defined is simply γ^{μ}. The QED-like one loop graph gives a contribution to Γ^{μ} of

$$\frac{g^2}{16\pi^2}\left(C^f - \frac{1}{2}C^{adj}\right)\left[\left(3\ln\frac{\mu^2}{m^2}+4\right)\gamma^{\mu} - \frac{1}{2m}[\gamma^{\mu},\not{k}]\right], \tag{3.3}$$

while the nonabelian graph gives

$$\frac{g^2}{16\pi^2}\frac{1}{2}C^{adj}\left[\left(3\ln\frac{\mu^2}{m^2}+4\right)\gamma^{\mu} - \frac{1}{2m}[\gamma^{\mu},\not{k}]\left(\ln\frac{\mu^2}{m^2}+3\right)\right], \tag{3.4}$$

where $C^f = \Sigma_a T_a^f T_a^f$ and $C^{adj} = \Sigma_a T_a^{adj} T_a^{adj}$ are 4/3 and 3 in $SU(3)$, respectively. The background field gauge Ward identities or direct calculation show that the contribution of the heavy quark wave function renormalization counterterm determined from the one loop heavy quark self energy is,

$$\frac{-g^2}{16\pi^2}C^f\left(3\ln\frac{\mu^2}{m^2}+4\right)\gamma^{\mu}. \tag{3.5}$$

The effect of this is simply to eliminate the contributions proportional to γ^{μ} in the previous two equations. So the complete one loop result from the full theory, including the counterterm graph is:

$$\Gamma^{\mu} = \frac{g^2}{16\pi^2}\frac{-1}{2m}[\gamma^{\mu},\not{k}]\left[C^f - \frac{1}{2}C^{adj} + \frac{1}{2}C^{adj}\left(\ln\frac{\mu^2}{m^2}+3\right)\right]. \tag{3.6}$$

The corresponding graphs in the static effective field theory involving the dimension five operator kinetic energy operator are depicted in figure 4, and the graphs involving the chromomagnetic moment operator are depicted in figure 5. The graph in figure 4 denoted with a "(2)" comes from the order g^2 kinetic energy counterterm. The graph has a factor $(ZZ_{kin})^{(2)} = Z^{(2)}+Z_{kin}^{(2)}$. Similarly, the one loop counterterm graph for the chromomagnetic moment operator has a factor

$(ZZ_{mag})^{(2)} = Z^{(2)} + Z^{(2)}_{mag}$. Since the heavy quark propagator in the static effective field theory does not involve the heavy quark mass, and since we are evaluating these diagrams at $k^2 = 0$, there is no scale in any of these integrals. Using dimensional regularization to regulate the infrared as well as the ultraviolet divergences, results in the great simplification that all of the diagrams vanish. This is also true for the self energy diagrams determining Z in the effective theory. Thus we can directly obtain the one loop contributions to Z_{kin} and Z_{mag} from the full theory calculation. There is no need to apply the Gordon decomposition since the full theory result, equation (3.6), does not have a piece proportional to γ^μ. The conclusion is that $Z^{(2)}_{kin} = 0$ and

$$Z^{(2)}_{mag} = 2\frac{g^2}{16\pi^2}\left[C^f - \frac{1}{2}C^{adj} + \frac{1}{2}C^{adj}\left(\ln\frac{\mu^2}{m^2} + 3\right)\right]. \tag{3.7}$$

The coefficient of the logarithm is in agreement with the anomalous dimension calculated by Lepage and Ning [2] using the nonrelativistic effective theory.

The expectation that the results are independent of the infrared regularization scheme has been checked for the QED-like diagrams by using the scheme where the gluon propagators are given a small mass, and the results are in agreement for this part of the contributions to $Z^{(2)}_{kin}$ and $Z^{(2)}_{mag}$. In addition, one can use the differentiated Ward identity to relate the total contribution to $Z^{(2)}_{kin}$ to the order $1/m$ one loop correction to the heavy quark self energy. The latter diagram is QED-like, and so regulating its infrared divergences with a small mass, and combining this with the differentiated Ward identity, we are able to confirm the dimensional regularization determination of $Z^{(2)}_{kin}$.

4. Conclusions

We have calculated the full order α_S contributions to the coefficients of the dimension five operators in the static effective field theory. These operators give the leading corrections to the properties of heavy-light mesons. For example, the expectation value of the chromomagnetic operator between B or B^* states determines their mass splitting. At present these matrix elements can only be evaluated using lattice gauge theory. This lattice calculation requires a choice of discretization of the chromomagnetic operator. The perturbative corrections to the matrix element measured on the lattice can be determined by matching a matrix element of the chromomagnetic operator in the lattice regularized effective theory to the matrix element of the chromomagnetic operator in the dimensionally regularized effective theory, whose coefficient we have just determined. This will provide a quantitative prediction of this splitting and a test of this approach for the determination of properties of heavy-light systems [1]. The approach is already being used to determine other phenomenologically important parameters such as the B meson decay constant [11].

We expect that $1/m$ corrections are of practical importance in D and B meson systems and that inclusion of these terms should make possible improved phenomenological relationships, in the spirit of those derived by Isgur and Wise [14]. We also expect that the determination of the dimension six operators in the static effective field theory which appear in order $1/m^2$ will be tractable using the techniques employed here.

Acknowledgements

We have had useful conversations with many members of the Fermilab theory group. We would especially like to thank Bill Bardeen, R. Keith Ellis, Nigel Glover, Walter Giele, and Lisa Randall.

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Figure Captions

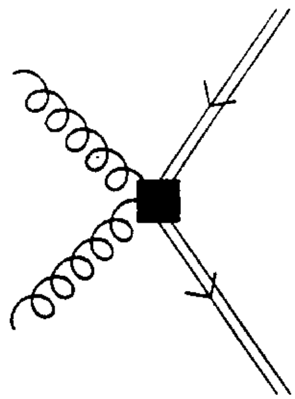
Fig. 1: Vertices originating from the dimension five kinetic energy operator.

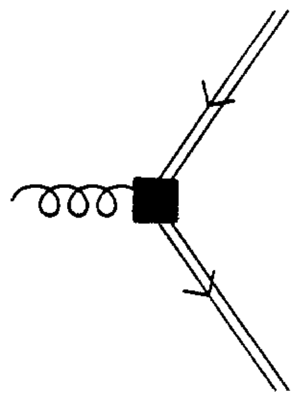
Fig. 2: Vertices originating from the chromomagnetic moment operator.

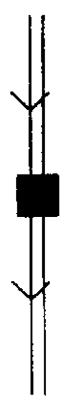
Fig. 3: Full theory graphs.

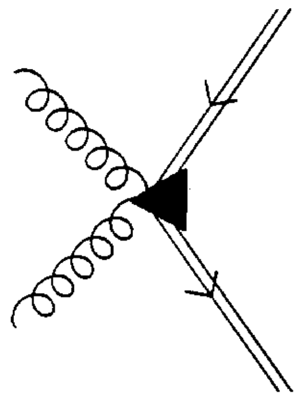
Fig. 4: Effective theory graphs involving the kinetic energy operator.

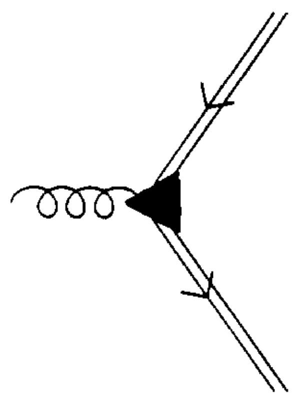
Fig. 5: Effective theory graphs involving the chromomagnetic moment operator.

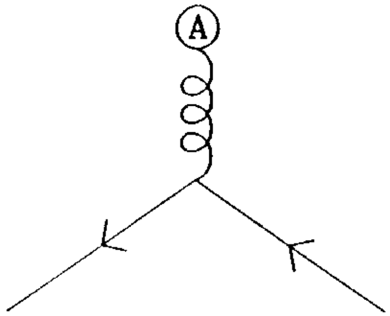








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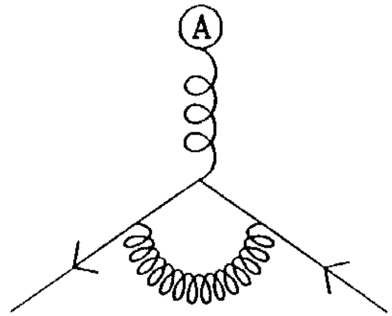
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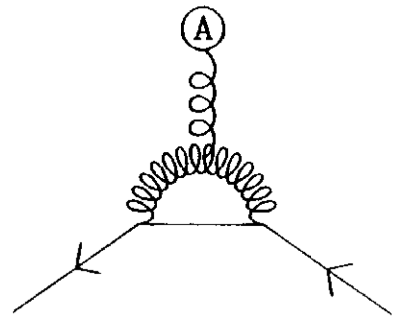
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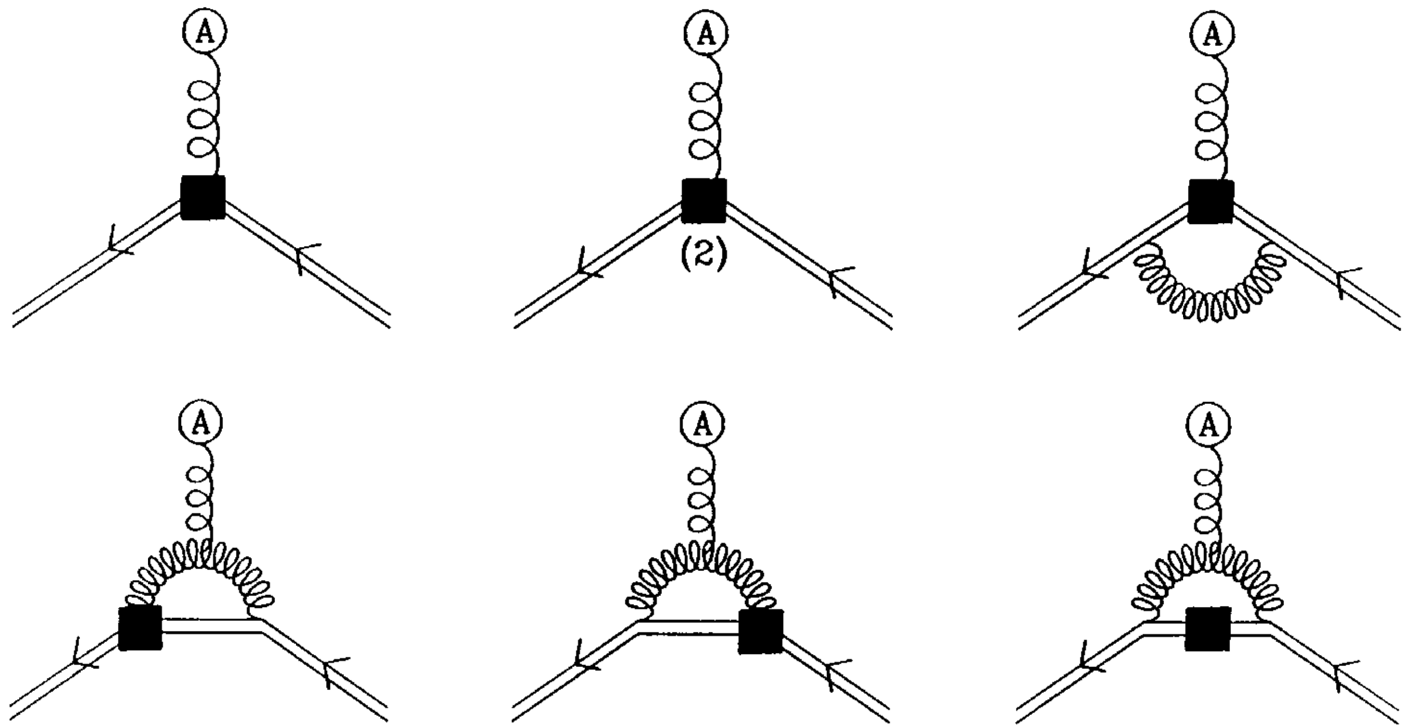
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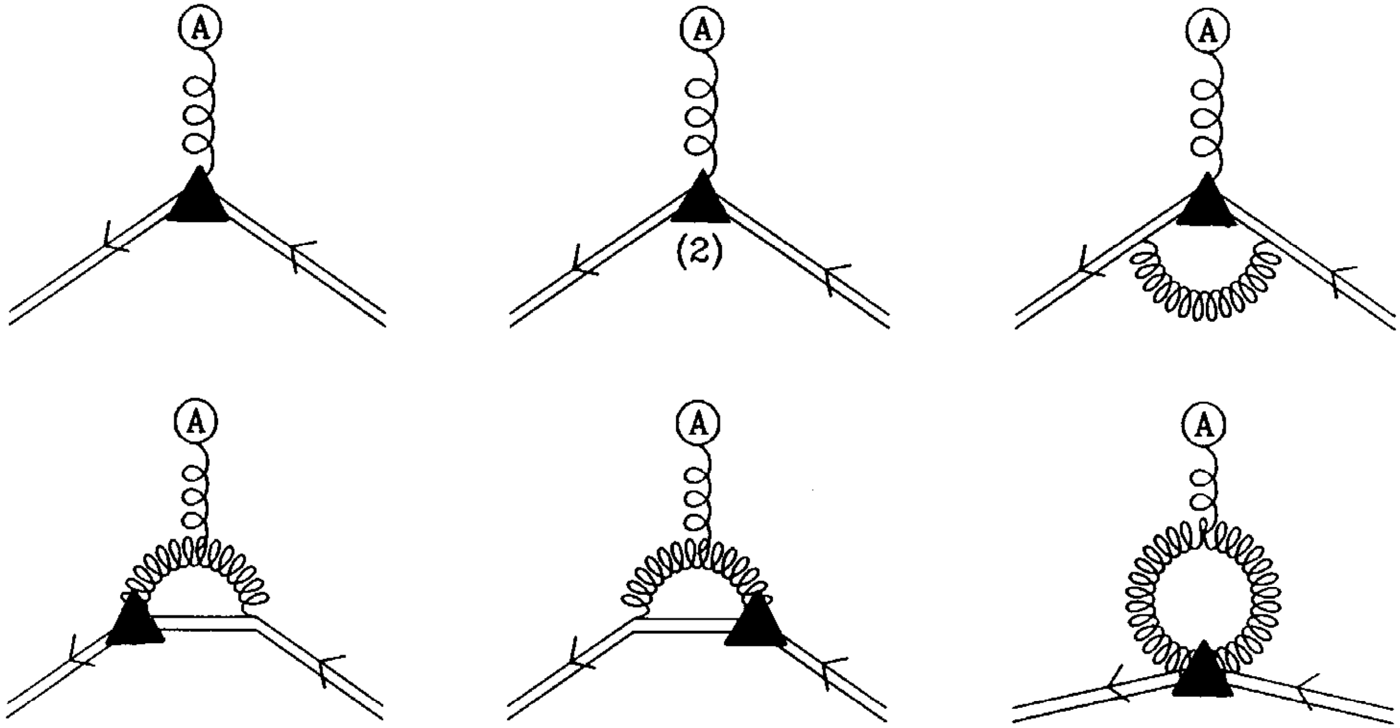
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