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Comment on "High Resolution Simulations of Cosmic Strings I: Network Evolution" by D. Bennett and F. Bouchet

Neil Turok

Joseph Henry Laboratories

Princeton University

Princeton, NJ 08544

and

Andreas Albrecht

NASA/Fermilab Astrophysics Center

Fermilab

Batavia IL, 60510

Abstract

We comment on recent claims regarding our simulations of cosmic string evolution reported in [1]. Specifically, it was claimed that our results were dominated by a numerical artifact which rounds out kinks on a scale of the order of the correlation length on the network [2]. This claim was based on an approximate analysis of an interpolation equation

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which we solve here. We show that the typical rounding scale is actually less than one fifth of the correlation length, and comparable with our other numerical cutoffs. Our results confirm our own previous estimates of numerical uncertanties, and show that the approximations made in [2]poorly represent the real solutions to the interpolation equation.

Last year we reported on new cosmic string simulations aimed at establishing the properties of a network of cosmic strings in an expanding universe[1]. We described qualitative technical advances over previous work, and proposed a simple one parameter model to describe the network's evolution, developing an earlier idea due to Kibble. Our results were consistent with a 'scaling' model, where the scale on the string network ξ grows at the speed of light, remaining a fixed fraction of the horizon at all times. Loops of all sizes are chopped off the network, but most of the energy loss from long strings is due to loops whose length is comparable to ξ.

Recently Bennett and Bouchet published results from a new code[2], using our new variables with more sophisticated numerical evolution and crossing detection schemes. The quantitative differences between their results and ours are not large (their ξ and ours differing by a factor of two for example), and broadly within our estimated errors. However qualitatively their results are quite different - the loss of energy from the long strings occurring through loops whose size remains fixed instead of increasing with ξ. In a simultaneous paper [3] one of us (N.T.) proposes a possible explanation for this unexpected result.

Throughout their paper our work is criticised, but most of the criticisms are minor or interpretational differences. However Bennett and Bouchet make one substantive criticism of our code, intended to explain the differences between our results, which we address here. The issue involved is the rounding of kinks produced by numerical diffusion. They give an analytic estimate of this and conclude that the rounding scale this produces is 'only slightly smaller' than the scale on the long strings ξ in our simulations, adding that this probably explains why our loop production function appears to be scaling. At one point they state that our code rounds kinks on a scale of 87 numerical points on the string by the end of a run. Their analytic result is inconsistent with the behaviour we observed in tests of our code, and so it is clearly up to us to explain the discrepancy. The main cause is simple - their analytic discussion treats the rounding of kinks too crudely. Below we shall solve the case of the kink directly and find that it rounds on a significantly smaller scale. We shall concentrate on the evolution in the radiation era, when the scale factor a is proportional to $t^{\frac{1}{2}}$. The issue of whether loop production scales is crucially important in the radiation era, since it is loops produced then that produce a potentially detectable gravity wave background today.

The rounding of kinks in our code is a result of the fact that our evolution scheme involves interpolation between left and right movers on the string at each timestep. This produces numerical diffusion when the timestep is smaller than the 'ideal' timestep $dt = d\sigma$, where $d\sigma$ is the physical spacing between the points.

As Bennett and Bouchet correctly explain in Appendix A1 of [2] the equation describing this interpolation is

$$\phi_j(t + dt) = \alpha\phi_j(t) + (1 - \alpha)\phi_{j-1} \tag{1}$$

where ϕ_j is the angle at which the left mover at numerical point j lies on a great circle on the 'Kibble-Turok' sphere. We label the points so that with the 'ideal' timestep a left-mover would remain stationary. α is the ratio of the timestep to the 'ideal' one, $\alpha = dt/d\sigma$. In our runs, α decreased slowly below unity, $\alpha \approx (t_0/t)^{.25}$ at time t, where t_0 is the initial time in the simulation. We choose a timestep slightly smaller than the 'ideal' one in order to ensure numerical stability at all points along the string. It is a good approximation to treat (1) in its 'continuum limit'

$$\partial_t \phi = -\frac{(1-\alpha)}{\alpha}\partial_\sigma \phi + \frac{(1-\alpha)}{\alpha}\frac{d\sigma}{2}\partial_\sigma^2 \phi \tag{2}$$

The first term on the right hand side produces a slow drift of the left-mover rightward (not affecting its shape) with velocity $v = (1-\alpha)/\alpha$ and the second produces numerical diffusion. Substituting a solution $\phi(\sigma, t) = f(\sigma - v\tilde{t}, \tilde{t})$ and defining a new time variable $\tilde{t} = \int_{t_i}^{t} dt(1-\alpha)/\alpha = t(\frac{4}{5}(a^{\frac{1}{2}} - a_i^{\frac{5}{2}}/a^2) - 1 + a_i^2/a^2)$ (with t_i the time the kink evolution begins, $a = (t/t_0)^{\frac{1}{2}}$, $a_i = (t_i/t_0)^{\frac{1}{2}}$) one finds the diffusion equation

$$\dot{f} = \frac{d\sigma}{2} f'' \tag{3}$$

where $\dot{f} = \partial_{\tilde{t}} f(x, \tilde{t})$ and $f' = \partial_x f(x, \tilde{t})$. For initial conditions corresponding to a kink, $f(x, 0)$ is a Heaviside function. The solution for subsequent times is

$$f = \frac{N}{\sqrt{\pi}} \int_{-\infty}^{cx} e^{-y^2} dy \tag{4}$$

with $c = 1/\sqrt{2d\sigma\tilde{t}}$. N is the initial opening angle of the kink.

If we define the rounding scale w to be the scale over which ϕ changes by $N/2$, we find that at a scale factor a the ratio of w to the 'Hubble radius' $2t$ for a kink produced at a scale factor a_i is given to a good approximation by

$$\frac{w}{2t} = \sqrt{\frac{\pi d\sigma}{8t_0}} \sqrt{\frac{4}{5}(a^{-\frac{3}{2}} - a^{-4} a_i^{\frac{5}{2}}) - a^{-2} + a_i^2 a^{-4}} \tag{5}$$

In a typical run, we used initial conditions with 10 numerical points per initial correlation length ξ, so $d\sigma = .1\xi$, and near scaling, with $\xi/(2t) \approx .07$. The first square root is just .07 , so the second square root measures the ratio of the smoothing scale to the final correlation length. A conservative example is to calculate w at the end of our longest run over which a increased by 2.75, for those kinks that were present initially (i.e. $a_i = 1$). For these we find $w/\xi = .22$ at the end of the run, so the initial kinks are smoothed on a scale of approximately one fifth of a correlation length on the long string by the end of the run, corresponding to a rounding scale of about 18 numerical points. A more typical case is to take a kink produced half-way through the run ($a_i = 1.88$) , and ask how wide it has spread after five correlation times ξ. Here we take $t = t_i + 5\xi_i = 1.7t_i$, so $a = 1.3a_i = 2.44$. Then (5) gives $w/\xi = .18$, so the kinks are rounded on a scale of just over one sixth of a correlation length, about 12 numerical points.

These results are very different from those given in [2]. Our 'typical rounding scale' is seven times smaller than their estimate of 87 points, in agreement with pictures of an evolved 'kinky' loop which we published in [4], and which Bennett and Bouchet ignored. We have calculated the relevant scale exactly, whereas they merely estimated it. As a consequence we do not agree with their statement that the appearance of scaling in our numerical results was a consequence of our smoothing scale being as large as ξ.

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References

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