Fermi National Accelerator Laboratory

FERMILAB-PUB-90/38-T

UICPHY-TH/90-2

NUHEP-TH-90/7

QCD Corrections to CP Violation from Color Electric Dipole Moment of b Quark

[a,c]D. Chang, [b,c]W.-Y. Keung, [a,d]C. S. Li, and [a]T. C. Yuan

[a]*Department of Physics and Astronomy*
Northwestern University, Evanston, IL 60208

[b]*Physics Department, University of Illinois at Chicago, IL 60680*

[c] *Fermilab, P.O. Box 500, Batavia, IL 60510*

[d] *Department of Applied Physics, Chongqing University, Chongqing, China*

Abstract

We consider the QCD renormalization effect in the neutron electric dipole moment in models with its origin from the CP violating charged particle exchange. It is shown that, when the t quark is first integrated out, the effective Lagrangian contains a vertex of the color electric dipole moment $\mathcal{D}_b^c$ of the b quark. The CP violating three gluon operator of the Weinberg type first begins to appear only at the scale where the b quark is further eliminated from the effective theory. We calculate the QCD evolution effect on $\mathcal{D}_b^c$ for these models. Our result is applicable to both the charged Higgs models and the left-right symmetric models of the CP nonconservation.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

Recently, Weinberg[1] has pointed out that in models in which the CP violation is due to the neutral Higgs exchanges, the leading contribution to the neutron electric dipole moment is given by a CP violating gluonic operator

$$\mathcal{O}_g = \frac{1}{6} g_s^3 f^{abc} \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\rho} G^{b\rho}_{\nu} G^c_{\alpha\beta} \ , \tag{1}$$

(with the notation $\epsilon_{0123} = 1$). This Weinberg operator arises when the t quark and the neutral Higgs boson are integrated out. Then, the QCD effect can be estimated by the evolution of the operator from the t quark scale to the hadronic scale following the renormalization group equation. Subsequently, other models with the CP nonconservation due to the charged particle exchange were also studied. These include interesting models such as the left–right symmetric models[3] and the charged Higgs models[4]. The CP phase arise from the mixing of the charged gauge bosons or the charged Higgs bosons. The CP nonconservation in these models occurs even with one generation of quarks[5,6]. Therefore, we can study the effect due to the dominant contribution from the heavy quark generation (t, b). In general, the models predict[7,8], for natural choice of the CP violating parameters, sizable neutron electric dipole moment in comparison with the experimental bound[9,10]. In this paper we study the QCD renormalization corrections to this mechanism for these models.

These models share a common feature that both the b quark and the t quark appear in the two–loop diagrams when the CP violating three gluon amplitude is induced. Assuming the top quark at the electroweak scale ($M_t \sim M_W$), we encounter a problem with two distant scales: one at the t quark mass and the other at the b quark threshold. To incorporate the QCD renormalization effect, one has to remove the t quark and the b quark in two separate steps. Since according to Ref.[2], the Weinberg operator has very large anomalous dimension, the fact that it does not get induced until the low energy ($\sim m_b$) scale can make important numerical difference.

First, we integrate out the top quark. The Weinberg operator in Eq.(1) has not yet emerged in the effective theory below the t quark scale. However the b quark picks up a color electric dipole moment $\mathcal{D}^c_b$. (And, in principle, so are the s and d quarks. Nevertheless, their color electric dipole moments are suppressed by mixing angles. We shall ignore them henceforth.) A piece of CP non-conserving effective Lagrangian of $\mathcal{D}_b^c$ is produced,

$$\begin{aligned} \mathcal{L}_b &= C_b(\mu)\mathcal{O}_b(\mu) \quad , \\ \mathcal{D}_b^c(\mu) &= C_b(\mu)m_b(\mu)g_s(\mu) \quad , \\ \mathcal{O}_b(\mu) &= g_s(\mu)m_b(\mu)(\bar{b}\tfrac{i}{2}\sigma^{\alpha\beta}\gamma_5 \mathrm{T}^a b)G^a_{\alpha\beta} \ . \end{aligned} \tag{2}$$

We show explicitly the renormalization scale μ dependence in the above equation. The size of the color electric dipole moment of the b quark at the electroweak scale $\mu \simeq M_W$ is speci... by the models. It provides the boundary condition for the QCD evolution.

In the left-right symmetric model as in Ref.[3,7], we obtain

$$\mathcal{D}_b^c(\text{LR model}) = \frac{e^2 g_s}{8\sin^2\theta_W}\sum_{i=1,2}\frac{Im(v_i a_i^*)}{8\pi^2}\frac{M_t}{M_{W_i}^2}\frac{2}{(1-r_i)^2}[1+\frac{1}{4}r_i+\frac{1}{4}r_i^2+\frac{3r_i\log r_i}{2(1-r_i)}] \ , \tag{3}$$

where $r_i = M_t^2/M_{W_i}^2$, and v_i and a_i are the quark couplings to the mass eigenstates W_i of the charged gauge bosons $W_{L,R}$,

$$\mathcal{L}_C = -\frac{e}{2\sqrt{2}\sin\theta_W}\sum_{i=1,2}\bar{t}\gamma_\nu(v_i + a_i\gamma_5)bW_i^{\nu+} + \text{H.c.} \tag{4}$$

and

$$\begin{aligned} v_1 &= \cos\xi + e^{i\eta}\sin\xi, \quad & a_1 &= -\cos\xi + e^{i\eta}\sin\xi, \\ v_2 &= -\sin\xi e^{-i\eta} + \cos\xi, \quad & a_2 &= e^{-i\eta}\sin\xi + \cos\xi. \end{aligned} \tag{5}$$

Here the angle ξ and the CP violation phase η are defined by the mixing of the charged gauge bosons,

$$W_1^+ = \cos\xi W_L^+ + e^{-i\eta}\sin\xi W_R^+, \quad W_2^+ = -e^{i\eta}\sin\xi W_L^+ + \cos\xi W_R^+, \tag{6}$$

with $M_{W_2} \gg M_{W_1} \simeq M_W$. For simplicity, we have used the condition $g_L = g_R = e/(\sqrt{8}\sin\theta_W)$. For the charged Higgs exchange model as in Ref. [4,8], we get

$$\mathcal{D}_b^c(H^\pm \text{ model}) = g_s\sum_{i=1,2}\frac{Im(s_i p_i^*)}{8\pi^2}\frac{M_t}{M_{H_i}^2}\frac{1}{2(1-h_i)^2}[h_i - 3 - \frac{2\log h_i}{1-h_i}] \ , \tag{7}$$

where $h_i = M_t^2/M_{H_i}^2$, and s_i and p_i are the quark couplings to the mass eigenstates H_i of the charged Higgs bosons,

$$\mathcal{L}_H = -\sum_{i=1,2}\bar{t}(s_i + p_i\gamma_5)bH_i^+ + \text{H.c.} \tag{8}$$

These Yukawa couplings are related to the quark masses in the minimal model with CP nonconservation. There are three Higgs doublets, ϕ_1, ϕ_2 and ϕ_3, with the first two responsible for the masses of the t-like quarks and the b-like quarks respectively, while the last doublet is mainly responsible for the electroweak breaking. The mass eigenstates H_1^+ and H_2^+ together with the unphysical charged Goldstone boson H_3^+ are linear combinations of ϕ_1^+, ϕ_2^+ and ϕ_3^+,

$$\phi_i^+ = \sum_{j=1}^{3} U_{ij} H_j^+ \quad (i = 1,2,3) \quad . \tag{9}$$

In general, the mixing amplitudes U_{ij} cannot all be real. The complexity gives rise to the CP nonconservation through the Yukawa couplings,

$$s_i = \frac{M_t}{\langle\phi_1^0\rangle} U_{1i} - \frac{m_b}{\langle\phi_2^0\rangle} U_{2i}, \quad -p_i = \frac{M_t}{\langle\phi_1^0\rangle} U_{1i} + \frac{m_b}{\langle\phi_2^0\rangle} U_{2i} \quad . \tag{10}$$

Therefore, unlike the left–right symmetric model, the m_b factor explicitly occurs in the CP violating coupling in Eq.(7) because

$$Im(s_i p_i^*) = -2 m_b M_t \; Im\left[\frac{U_{1i} U_{2i}^*}{\langle\phi_1^0\rangle \langle\phi_2^0\rangle}\right] \quad . \tag{11}$$

The results in Eqs.(3,7) agree with similar previous calculations on electric dipole moments[11].

Now, we come back to the QCD evolution effect. The running strong coupling $g_s(\mu)$, the b quark running mass $m_b(\mu)$ and the Wilson coefficient $C_b(\mu)$, obey the following RG equations,

$$\mu \frac{d}{d\mu} g_s(\mu) = -\frac{g_s^2(\mu)}{8\pi^2} \beta_n g_s(\mu) \ , \tag{12}$$

$$\mu \frac{d}{d\mu} m_b(\mu) = -\frac{g_s^2(\mu)}{8\pi^2} \gamma_m m_b(\mu) \ , \tag{13}$$

$$\mu \frac{d}{d\mu} C_b(\mu) = -\frac{g_s^2(\mu)}{8\pi^2} \gamma_b C_b(\mu) \quad . \tag{14}$$

It is known that $\beta_n = \frac{1}{6}(33 - 2n)$ for n active flavors of quarks, and $\gamma_m = 4$. To find the renormalization effect on this color electric dipole moment term, we observed that its anomalous dimension γ_b is given by the similar calculation[12] on the color magnetic dipole transition operator ($\Delta S = 1$) in kaon decays. The γ_5 in Eq.(2) is irrelevant in the calculation due to the fact that QCD is a vector–like theory. We therefore have

$$\gamma_b = -14/3 \quad . \tag{15}$$

The evolution from the weak scale to the b quark mass scale $\bar{m}_b$ gives

$$C_b(\bar{m}_b) = C_b(M_W)(\frac{g_s(\bar{m}_b)}{g_s(M_W)})^{\gamma_b/\beta_b} \quad . \tag{16}$$

Here the physical b quark mass is defined by $m_b(\bar{m}_b) = \bar{m}_b$. Since $\gamma_b < 0$, the Wilson coefficient at the b quark energy scale is suppressed by the renormalization.

On the second step of eliminating the b quark in the remaining effective theory, the b quark color electric dipole moment disappears and turns into a Weinberg operator (1) below b threshold. Explicit calculation defines the relevant effective interaction,

$$\mathcal{L}_g = C_g(\mu)\mathcal{O}_g(\mu) \quad , \tag{17}$$

$$C_g(\bar{m}_b^-) = \frac{1}{32\pi^2} C_b(\bar{m}_b^+) \quad . \tag{18}$$

The superscripts '$\pm$' label two calculations just before and after the removal of the b quark field.

If the Weinberg operator already exists together with the b quark color electric dipole moment $\mathcal{D}_b^c$ at the electroweak scale such as in models in which both the neutral Higgs and the charged particle exchanges are important, we have to include the operator mixing effect. The renormalization group equations are

$$\mu \frac{d}{d\mu} \begin{pmatrix} g_s^3 C_g \\ g_s C_b \end{pmatrix} = -\frac{g_s^2(\mu)}{8\pi^2} \begin{pmatrix} \gamma_g + 3\beta_n & 0 \\ \gamma_{gb} & \gamma_b + \beta_n \end{pmatrix} \begin{pmatrix} g_s^3 C_g \\ g_s C_b \end{pmatrix} \quad . \tag{19}$$

Dai and Dykstra[2] obtained a value of $\gamma_g = 18$ and thus concluded a large renormalization enhancement. Lately, Braaten *et al.* claimed[13] that $\gamma_g = -18$, which would yield suppression instead. Braaten *et al.* also gave $\gamma_{gb} = 3$. For the evolution between M_W and m_b, we have

$$C_g(\mu) = C_g(M_W)\mathcal{K}^{\gamma_g/\beta_b} \quad , \quad \mathcal{K} = g_s(\mu)/g_s(M_W) \quad ,$$

$$C_b(\mu) = C_b(M_W)\mathcal{K}^{\gamma_b/\beta_5} + C_g(M_W)\gamma_{gb}\frac{g_s^2(\mu)\mathcal{K}^{\gamma_g/\beta_5} - g_s^2(M_W)\mathcal{K}^{\gamma_b/\beta_5}}{\gamma_g - \gamma_b + 2\beta_5} \quad . \tag{20}$$

When the b quark is removed, $C_g(\bar{m}_b^-)$ is given by the right handed side of Eq.(18) plus the already existing $C_g(\bar{m}_b^+)$, if any.

The remaining evolution is again obtained by dressing up the Weinberg operator following the standard procedure from $\bar{m}_b$ down to the hadronic scale via the renormalization group machinery. If we neglect the small effect[13] of the operator mixing and assume $C_g(M_W) = 0$, we have

$$C_g(\mu) = C_g(\bar{m}_b^-)\left[\frac{g_s(\bar{m}_c)}{g_s(\bar{m}_b)}\right]^{\gamma_g/\beta_4}\left[\frac{g_s(\mu)}{g_s(\bar{m}_c)}\right]^{\gamma_g/\beta_3} \quad . \tag{21}$$

Finally, we present some numerical study. In the charged Higgs exchange model, we obtain the QCD evolution factor from Eqs. (16,21)

$$\zeta_{CH} = \left[\frac{g_s(\bar{m}_b)}{g_s(M_W)}\right]^{\gamma_b/\beta_5}\left[\frac{g_s(\bar{m}_c)}{g_s(\bar{m}_b)}\right]^{\gamma_g/\beta_4}\left[\frac{g_s(\mu)}{g_s(\bar{m}_c)}\right]^{\gamma_g/\beta_3} \quad . \tag{22}$$

There is a difference in the QCD correction in the left–right symmetric model, where the factor m_b in Eqs.(2,11) is not actually present. The factor becomes

$$\zeta_{LR} = \left[\frac{g_s(\bar{m}_b)}{g_s(M_W)}\right]^{(\gamma_b+\gamma_m)/\beta_5}\left[\frac{g_s(\bar{m}_c)}{g_s(\bar{m}_b)}\right]^{\gamma_g/\beta_4}\left[\frac{g_s(\mu)}{g_s(\bar{m}_c)}\right]^{\gamma_g/\beta_3} \quad . \tag{23}$$

If we turn off the renormalization effect, our formula reproduces the same result as that in the previous calculations[7,8]. Thus, our present approach can be regarded as a betterment of the previous calculations by including all the leading logarithmic corrections of the form $g_s^{2n}\log^n(M_t/m_b)$. In Fig. 1, we show these factors versus the renormalization scale μ around the hadronic scale for both values[2,13] of γ_g. The corrections are weaker than that in the model with the neutral Higgs exchange, where the Weingberg operator is already present at the weak scale. The renormalization point μ was chosen in Ref.[1] as low as about 250 MeV, where the perturbative calculations become questionable. If one chooses μ around the chiral symmetry breaking scale $2\pi f_\pi$, the effect of the renormalization is reduced by almost two orders of magnitudes. Such dependence on μ is absorbed by the matrirx element in Eq.(1), which needs a systematic investigation.

In conclusion, we have shown that the neutron electric dipole moment comes indirectly from the color electric dipole moment of the b quark in models of CP nonconservation due to the exchange of charged bosons.

We thank useful discussion with Eric Braaten. This work is supported by the U.S. Department of Energy.

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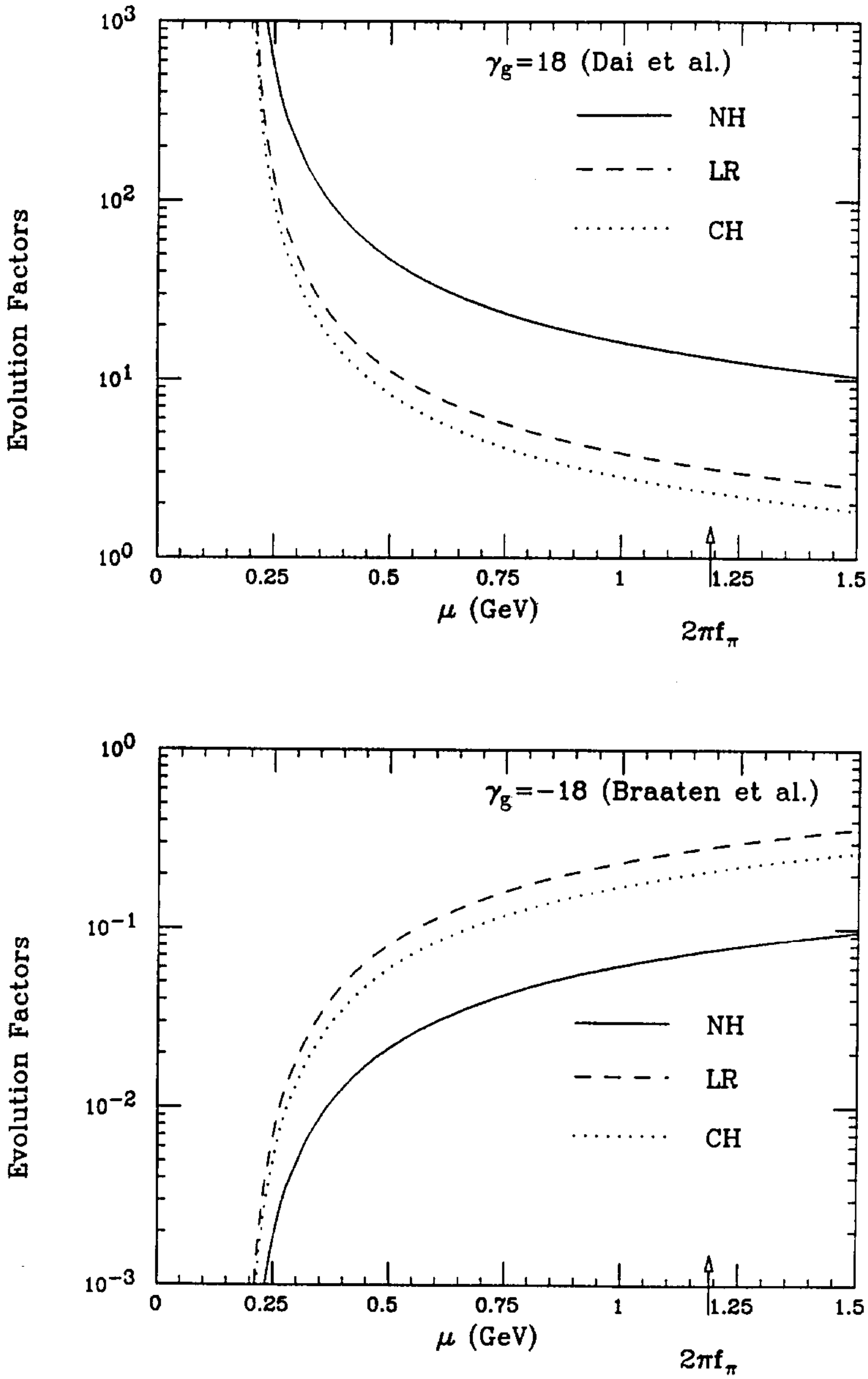
FIGURE CAPTIONS

1. The evolution factors in Eqs. (22,23) of the charge Higgs model (CH) and the left–right model (LR) in comparison with that in the neutral Higgs model (NH) of Ref.[1]. Both possible values of γ_g are illustrated.



γ_g=18 (Dai et al.)
NH
LR
CH
Evolution Factors
10^3
10^2
10^1
10^0
0
0.25
0.5
0.75
1
1.25
1.5
μ (GeV)
$2\pi f_\pi$
γ_g=−18 (Braaten et al.)
NH
LR
CH
Evolution Factors
10^0
10^{-1}
10^{-2}
10^{-3}
0
0.25
0.5
0.75
1
1.25
1.5
μ (GeV)
$2\pi f_\pi$

Fig. 1