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REMARKS ON OPTIMIZED PREDICTIONS FOR THE DRELL-YAN PROCESS

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Abstract

We discuss the optimization of the Drell-Yan pair production cross section calculated at $O(\alpha_S)$ and compare it with the $O(\alpha_S^2)$ results of van Neerven et al. It is shown that the optimized predictions do agree analytically with the latter calculation near the phase space boundary. They also provide a good numerical approximation wherever the stability equations have a solution.

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Recently the application of optimization principles [1-3] to perturbative QCD calculations of reactions involving real or virtual photons have led to very successful phenomenological results. In particular, for the photoproduction of large p_T hadrons [4], the production of photons [5] or of heavy gauge bosons at large p_T [6], Drell-Yan pairs [7] as well as $e^+ + e^- \rightarrow jets$ [8], the optimized results have all been shown to yield better agreement to data than analyses using scales like the transverse momentum or the mass of the virtual photon. The question is then to understand the reason of this success. As a step in this direction we briefly review the optimization of the $O(\alpha_S)$ Drell-Yan cross section with respect to the arbitrary mass scales, as it has been used previously in practice, and compare it with the $O(\alpha_S^2)$ results of van Neerven et al. [9-11]. Good agreement is obtained. We then discussed the optimization with respect to the factorization scheme and it is shown that the leading terms of the $O(\alpha_S^2)$ results are correctly reproduced.

The general form of the Drell-Yan cross section up to $O(\alpha_S)$ is

$$\frac{Q^4\ d\sigma}{dQ^2} = F(M)\ F(M)\ (1 + a(\mu)(2d \ln\frac{Q}{M} + w_1)) \tag{1}$$

where we only consider the non-singlet case (with one quark flavor). We have taken moments of the cross section with respect to $\tau = Q^2/s$, but we have dropped the usual moment index n, so that we deal with products of terms rather than with convolutions. The quark distribution F depends on the arbitrary factorization scale M . The correction term w_1 has been calculated some years ago by several groups [12-14] and, like F, it depends on the factorization convention. The expansion variable is $a(\mu) = \alpha_S(\mu)/\pi$ which depends on the renormalization scale μ.

The anomalous dimension d which appears in eq. (1) enters the Altarelli-Parisi equation as

$$\frac{dF(M)}{d\ln M} = F(M)\ a(M)\ (\ d\ + a(M)\ d_1 +\ ...) \tag{2}$$

while the evolution of the coupling is

$$\frac{da(\mu)}{d\ln\mu} = -b\ a^2(\mu)\ (\ 1 + a(\mu)\ c +\ ...) \tag{3}$$

with $b = .5\ (11 - 2N_F/3)$ and $c = .5\ (153 - 19N_F)/(33 - 2N_F)$. The parameter d_1 is an unphysical variable which characterizes the factorization scheme. In eq. (1) it has been fixed by the convention $F(Q) = F_2(Q)$. We shall discuss below the optimization with respect with this parameter.

In the following we want to compare the optimized result for $Q^4\ d\sigma/dQ^2$ in eq. (1) with the results of van Neerven et al. [9-11], derived at $O(\alpha_S^2)$. After taking moments, one finds for large n, i.e. near the boundary of phase space,

$$\begin{aligned}\frac{Q^4\ d\sigma^{(2)}}{dQ^2} &= F_2(Q)F_2(Q)\big(1 + a(Q)c_F(\ln^2 n - \frac{3}{2}\ln n) \\ &\qquad + \frac{a^2(Q)}{2}(c_F^2(\ln^4 n - 3\ln^3 n) + c_F b \ln^3 n + O(\ln^2 n))\big) \\ &= F_2(Q)F_2(Q)\exp(a(Q)w_1)\big(1 + \frac{a^2(Q)}{2}(c_F b \ln^3 n + O(\ln^2 n))\big)\end{aligned} \tag{4}$$

The first line is the exact second order expression restricted here to the very large τ limit, $\tau \to 1$, whereas the last line is the improved expression where, following refs. [9-11], the abelian term has been exponentiated [15]. The moment of the coefficient function is correspondingly approximated by

$$w_1 = c_F(\ln^2 n - \frac{3}{2}\ln n) \tag{5}$$

and the anomalous dimension behaves as

$$\begin{aligned} d &= c_F(\frac{3}{2} - 2\ln n) \\ &\simeq -2c_F \ln n \end{aligned} \tag{6}$$

in the approximation we are interested in. The factorization scheme used here is to identify the quark structure function F with the deep inelastic structure function F_2 [13].

Before discussing the full optimization, with respect to μ, M, and d_1, of eq. (1) we first give a simplified (and therefore incomplete) treatment which nevertheless shows the main features of the optimized Drell-Yan cross section up to $O(\alpha_S)$ [3,7]. It is restricted to the leading logarithmic approximation of eqs. (2), (3) and therefore the stability condition with respect to the factorization scheme (defined by d_1) is not implemented. This approach is of practical interest since it is not possible to continuously vary the factorization prescription in the configuration space. Applying, in this approximation, the Principle of Minimal Sensitivity [1] or the Effective Charge method [2] gives the same result

$$\frac{Q^4\ d\sigma^{opt}}{dQ^2} = F(M_{opt})\ F(M_{opt}) \tag{7}$$

with

$$\ln\frac{M_{opt}}{Q} = \frac{w_1}{2\ d} \tag{8}$$

Here F is identical to F_2, the deep inelastic structure function. Using eqs. (2) and (3), in the leading order ($c = d_1 = 0$), one can express eq. (7) in terms of $F(Q)$ rather than

$F(M_{opt})$. Integrating the Altarelli-Parisi equation one finds

$$\begin{aligned} F_2(M_{opt}) &= F_2(Q)\ \exp(d\int_Q^{M_{opt}} a(M)\frac{dM}{M}) \\ &= F_2(Q)\ (\frac{a(Q)}{a(M_{opt})})^{\frac{d}{b}}. \end{aligned} \tag{9}$$

Plugging this expression into eq.(7) and using

$$a(M_{opt}) = \frac{a(Q)}{1 + ba(Q)\ln\frac{M_{opt}}{Q}} \tag{10}$$

one finds

$$\frac{Q^4\ d\sigma^{opt}}{dQ^2} = F_2(Q)\ F_2(Q)\ \exp\left(a(Q)w_1 - \frac{a^2(Q)}{2}\frac{bw_1^2}{2d} + O(a^3)\right). \tag{11}$$

After expanding to $O(a^2(Q))$ and using eqs. (5) and (6) one finds

$$\begin{aligned} \frac{Q^4\ d\sigma^{opt}}{dQ^2} = F_2(Q)\ F_2(Q)\ &(1 + a(Q)c_F(\ln^2 n - \frac{3}{2}\ln n) \\ &+ \frac{a^2(Q)}{2}(c_F^2(\ln^4 n - 3\ln^3 n) + \frac{c_F b}{4}\ln^3 n + O(\ln^2 n))) \end{aligned} \tag{12}$$

where only the dominant terms in the large n limit are kept. We see by comparing with eq. (4) above that the abelian part is correctly reproduced. The optimized result even indicates that this part should be exponentiated as conjectured in [9]. It is interesting to recall that this abelian part, which is known to dominate the correction term at $O(\alpha_S)$, is related to the "soft and collinear structure" of the diagrams. Concerning the non abelian part we note that it is, in this approximation of optimization of eq. (1), only partially reproduced, namely one finds $c_F b \ln^3 n/4$ instead $c_F b \ln^3 n$.

For illustration we make a numerical comparison of the optimized expression, eq. (7), with the results of ref. [11]. We denote by $\sigma^{(i)}$ the cross section $Q^4\ d\sigma/dQ^2$ calculated to $O(\alpha_S^i)$. In the figure we show with a solid line the ratio $\sigma^{(1)}|_{M_{opt}}/\sigma^{(1)}|_Q$ where the index refers to the choice of scales. Also shown for comparison is the ratio $\sigma^{(2)}|_Q/\sigma^{(1)}|_Q$ using the perturbative expression (dotted line) or the exponentiated one (dash-dotted line) of eq. (4). Two exemples are considered: $\pi\ p$ scattering at $\sqrt{s} = 19.1\ GeV$ and $\bar{p}\ p$ at $\sqrt{s} = 630\ GeV$. In both cases only the non-singlet contribution is kept and the exact (i.e. valid for all τ) expression for w_1 is used. It is seen that when optimization is possible a good agreement is obtained with the results of ref. [11]. We use, for our predictions, the structure functions fitted by Duke and Owens [16] for the proton and by Owens [17] for the pion which are associated to a value of $\Lambda = 200\ MeV$. This is different from the choice of leading logarithmic parametrizations made in ref. [11] for the predictions at $\sqrt{s} = 19.1\ GeV$ but, in the ratios, the dependence on the structure functions should "factor out". At $\sqrt{s} = 630\ GeV$ part of the discrepancy between the optimized results and the $O(\alpha_S^2)$ ones can be ascribed to the fact that, in ref. [11], 6 flavors instead of 4 are used in the evaluation of α_S. A question often raised concerns the value of the scale at the stability point: it is obvious from eq. (8) that for a large positive value of w_1 (w_1 is the correction term at scale Q) the optimal scale turns out to be smaller than Q since the anomalous dimension d is effectively negative for the τ range where optimization is possible. One can parametrize

$$M_{opt}^2 = C\ Q^2. \tag{13}$$

In practice, at $\sqrt{s} = 19.1\ GeV$ one finds $C \simeq .1$ which is accurate to better than 10% over the whole τ range. At $\sqrt{s} = 630\ GeV$ the scales turn out to be relatively smaller since one obtains $C \simeq .05$. Although such a large difference with the so-called "natural" scale Q may be surprising it nevertheless turns out to be necessary to reproduce the $O(\alpha_S^2)$ results adequately. It is interesting to note that the same conclusion about the relatively small values of scales was obtained in an analysis of $e^+ + e^- \to n\ jets$ whether one applies the Principle of Minimal Sensitivity (Kramer and Lampe [8]) or one chooses the scales to optimize the fit of the theory to the data (Bethke [8]).

We now discuss the complete optimization of eq. (1) with respect to μ, M and d_1. The final result of this procedure can be taken from ref. [1] with the corresponding replacements for eq. (1). It is

$$\frac{Q^4\ d\sigma^{opt}}{dQ^2} = A(ca_{opt})^{-2d/b}\ (1 + ca_{opt})^{2(cd - d_1^{opt})/bc}, \tag{14}$$

where d_1^{opt} and a_{opt} are obtained from the equations which express the stability of the cross section under variation of the unphysical parameters

$$ca_{opt} + c\frac{d_1^{opt}}{d} a_{opt}^2 - \ln(1 + ca_{opt}) = 0 \tag{15}$$

$$\frac{1}{a_{opt}} + c\ln\frac{ca_{opt}}{1 + ca_{opt}} - \frac{1}{a(Q)} - c\ln\frac{ca(Q)}{1 + ca(Q)} - \frac{bt_1}{2d} - \frac{d_1^{opt}}{d} = 0 \tag{16}$$

with the factorization scheme invariant $t_1 = w_1 - 2d_1/b$. Next we express the result for $Q^4\ d\sigma^{opt}/dQ^2$ in terms of $a(Q)$ and $F(Q)$ using the stability conditions above and the expression for the quark distribution

$$F(Q) = A(ca(Q))^{-d/b}\ (1 + ca(Q))^{(cd - d_1)/bc}, \tag{17}$$

where d_1 specifies the factorization scheme for this quark distribution.

Keeping all the terms to order $a^2(Q)$, a lengthy calculation gives

$$\frac{Q^4\ d\sigma^{opt}}{dQ^2} \simeq F(Q)\ F(Q)\ \Big(1 + a(Q)w_1 + \frac{a^2(Q)}{2}(w_1^2 - \frac{bw_1^2}{2d}) + \frac{a^2(Q)}{2}(\frac{2d_1w_1}{d} - \frac{2d_1^2}{bd} - \frac{dc^2}{2b} - \frac{2d_1c}{b})\Big) \tag{18}$$

where the last term is the extra term compared to the result of eq. (12), arising from the complete optimization of eq. (1) using next to leading logarithmic expressions. In order to compare directly with eq. (4), $F(Q)$ and d_1 have to be specified for the non universal factorization scheme with $M = Q$, namely $F(Q) = F_2(Q)$ and $d_1 = -b\kappa_1$, where κ_1 is the scheme invariant of the structure function (the analogous of t_1 in the Drell-Yan cross section). Again keeping only the leading terms for large n we obtain [13,14]

$$\kappa_1 \simeq \frac{c_F}{2}\ln^2 n + O(\ln n), \tag{19}$$

and we find that the extra term in eq. (18) behaves for large n as

$$\frac{a^2(Q)}{2}(\frac{3bc_F}{4}\ln^3 n + O(\ln^2 n)). \tag{20}$$

From eq. (18) the final result is found to be

$$\frac{Q^4\ d\sigma^{opt}}{dQ^2} = \frac{Q^4\ d\sigma^{(2)}}{dQ^2} \tag{21}$$

when terms of $O(a^2(Q)\ln^2 n)$ are neglected.

Following van Neerven [9], we observe (see.eq. (4)) that with the substitution $Q^2 \to Q^2(1-\tau)$ in the running coupling constant (i.e. $Q^2 \to Q^2/n^2$) the Drell-Yan cross section becomes

$$\begin{aligned}\frac{Q^4\ d\sigma}{dQ^2} \simeq & \frac{Q^4\ d\sigma^{opt}}{dQ^2} \simeq \frac{Q^4 d\sigma^{(2)}}{dQ^2} \\ \simeq & F_2(Q)\ F_2(Q)\ \big(\exp\big(a(Q/n)c_F(\ln^2 n - 3\ln n/2)\big) + O(a^2(Q)\ln^2 n)\big),\end{aligned} \tag{22}$$

valid for $\tau \to 1$, i.e. all large corrections near the boundary of phase space can be resummed. It is encouraging to find this result either by inspecting the exact $O(\alpha_S^2)$ calculation or by the optimization of the $O(\alpha_S)$ cross section.

The above discussion suggests the usefulness of the optimization methods to other processes such as prompt photon production or photoproduction at large transverse momentum. It is known [18,19] that terms arising from the soft and collinear gluon emission can give large corrections as in the Drell-Yan process. Therefore it is reasonable to believe that the optimization methods [1-3] also improve the perturbative predictions for those reactions.

It may be interesting to confront the result, eq. (23), with the criticisms on optimization put forward in connection with the $O(\alpha_S^3)$ calculation of the total cross section for $e^+ + e^- \rightarrow hadrons$ [20,21]. The reason of the criticism is that the quantity $(R^{(3)} - R^{(2)})/R^{(2)}$ is smaller in the $\overline{MS}$ scheme with the standard choice of scale $Q = \sqrt{s}_{e^+e^-}$ than with either the choice based on the Principle of Minimal sensitivity [1] or the approach of Effective Charges [2]. Here, $R^{(n)}$ is the usual hadronic to leptonic ratio calculated to n loops [22,23]. Numerically, following the results of Maxwell and Nicholls [24] one finds for this ratio the values .092, 0.121 and .124 respectively for the schemes mentionned above (a value of $\Lambda = 100\ MeV$ and 5 flavors are assumed). So the optimized approach appears to be less stable than the standard one when one more order of perturbation theory is considered. The strength of this argument is diminished if one looks now at the relative size of the last two terms in the perturbative series: the ratio of the $O(\alpha_S^3)$ term over the $O(\alpha_S^2)$ one is 1.75, -.98 and 0. respectively and, naively, this disfavors the standard approach compared to the other two methods (for $\Lambda = 500\ MeV$, these ratios become 2.34, -.96 and 0. respectively). Note that a recent paper has explored the general sheme dependence of the ratio R and found large instabilities [25].

Admittedly, none of the arguments given above are very strong since one cannot say anything about the behavior of the still higher order terms. It is more reasonable to conclude that the perturbative series for R presents us with a very incomfortable situation where the real question is not to know which sheme is the best one but, rather, to understand the nature of the perturbative series for this particular proccess. Indeed, the problem arises from the peculiar behavior of the coefficients of the series which are (in the $\overline{MS}$ sheme for 5 flavors with standard scales) 1., 1.41 and 64.8 for the α_S/π , $(\alpha_S/\pi)^2$ and $(\alpha_S/\pi)^3$ terms respectively. Since optimization amounts to some form of exponentiation or summation it is clear that optimization based on the first two terms cannot reproduce the third term. One can say however that the uncertainty among the different predictions is small, of the order of 5%.

In principle one could try to optimize van Neerven et al. exact result up to order α_S^2 in order to estimate the yet uncalculated terms of order α_S^3. The full optimization however amounts to find the stability point with respect to five variables, namely the scales μ and M, the anomalous dimension coefficients d_1 and d_2 and the β-function coefficient c_1 at $O(\alpha_S^3)$ (cf. eqs. (2) and (3)), which is a rather formidable task. Even more, it is rather academic since, e.g. d_2 is not calculated in the $\overline{MS}$-scheme.

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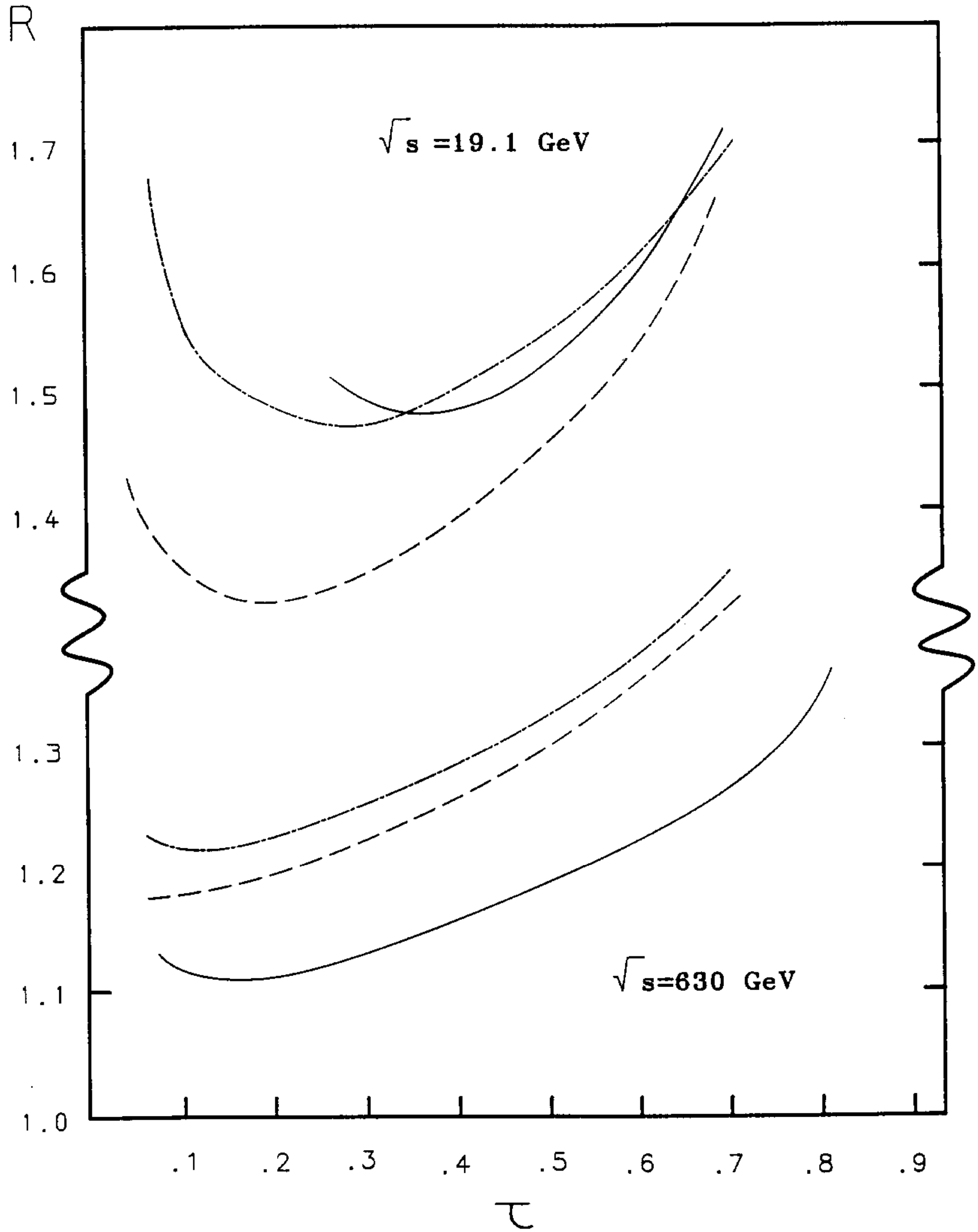
Figure caption

Higher order predictions for Drell-Yan pair production normalized to the $O(\alpha_S)$ cross section evaluated with the scales $\mu = M = Q$. Only the non singlet part of the cross section is considered. The solid lines are the optimized results, eq. (7), for $\pi\ p$ scattering at $\sqrt{s} = 19.1\ GeV$ and $\bar{p}\ p$ at $\sqrt{s} = 630\ GeV$. The dotted lines are the $O(\alpha_S^2)$ perturbative predictions and the dash-dotted lines are the "exponentiated" $O(\alpha_S^2)$ predictions taken from ref. [11].



R
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
$\sqrt{s}$ =19.1 GeV
$\sqrt{s}$=630 GeV
.1
.2
.3
.4
.5
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.7
.8
.9
τ