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PERIODICITY OF REDSHIFT DISTRIBUTION IN A T-3 UNIVERSE

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Summary. The periodicity in redshift distribution in a T-3 universe has been investigated. We show that, in a small T-3 universe, if the mean number density of the considered objects is large enough, their redshift distribution must bear an observably periodic component. In particular, such periodicity can significantly be confirmed by means of the power spectrum of the redshift distribution. All observed features related to the periodicity in quasar's redshift distribution, including the argument of the periodicity, the wavelength and the mean number density of quasars, can be fitted in with the periodicity given by a T-3 universe with adoptable parameters. Therefore, the redshift distribution of large redshift objects may have the potential ability of determining the topology of the universe.

Key words: topology of the universe - quasar distribution

1. Introduction

The periodicity in the redshift distribution of quasars is a slippery topic in quasar physics and cosmology. More than twenty years ago Burbidge (1968) first pointed out the probable existence of periodicity in the redshift distribution of quasars. In subsequent investigations, some results (Cowan, 1969; Burbidge and O'Dell, 1972) confirmed Burbidge's guess, while some others (Plegemann et al, 1969; Wills and Ricklefs, 1976) were negative. Using a much larger number of quasars, we (Fang et al., 1982) confirmed the periodicity in the redshift distribution of quasar emission lines with respect to the variable $w=\ln(1+z)$. No such periodicity has been found, however, in the redshift distribution of quasars given by the Texas radio survey (Wills and Wills, 1987). Recently, the periodicity has once again been analyzed and confirmed for samples with a larger number of quasars (Arp et al., 1989).

There has also been considerable debate over how to explain the periodicity in redshift distribution. At first Burbidge wanted to explain the periodicity as a result of the non-cosmological origin of quasar's redshifts. The gravitational lensing of quasars, however, directly shows that the redshifts of the lensed quasars are cosmological. The observational selection effects, such as the preference for identifications of certain emission lines (Lyα, C IV, Mg II) in quasar spectra, may also be a reason for the periodicity. It has recently been confirmed, however, that a periodic component also signifcantly exists in the redshift distribution of Lyα absorption lines in quasar spectrum (Chu and Zhu, 1989). In 1983 we proposed that the periodicity might be due to the multiply-connected topology of the universe (Fang and Sato, 1983; Fang and Mo, 1987). For instance, in a small-scale 3-dimensional torus (T-3) universe the redshift distribution of the multiple images of a quasar will be periodic. Nevertheless, a later study (Ellis and Schreiber, 1986; Ellis, 1987) concluded that no remarkable features can be found in the redshift distribution of a small T-3 universe.

The aim of this paper is to re-investigate the periodicity of redshift distribution in a T-3 universe. We will show that, if the scale of a T-3 universe is small, then so long as the number density of the considered objects is large enough, their redshift distribution must bear an observably periodic component. In particular, in this case the power spectrum of the redshift distribution of the objects must bear an observable resonance peak, which is remarkably different from that of a simply-connected flat universe. That is, the periodicity in the redshift distribution is one of the observable features which are sensitively dependent on the spatial topology of the universe. All features of observed periodicity in quasar's redshift distribution can naturally be fitted in with the periodicity of a small T-3 universe. This seems to strongly imply that the periodicity of redshift distribution of large redshift objects, such as quasars and Lyα clouds, would be able to play an important role in the observational cosmology of detecting the topology of the universe.

2. Redshift distributions in a T-3 universe

Let us consider a T-3 universe, which is constructed from a flat universe by identifying point (x_1, x_2, x_3) with points $(x_1+la_1, x_2+ma_2, x_3+na_3)$ with all integers l, m and n. In such a universe an observer will find that the space is divided into a periodic lattice with side lengths a_1, a_2, a_3 in the directions of x_1, x_2, x_3, respectively. Each cell of the lattice space can be denoted by integers (l, m, n). The basic cell is given by l=m=n=0.

With no loss of generality, we can choose the observer to be located at the origin of the basic cell, i.e. $x_1=x_2=x_3=0$. If an object is located in (x_1, x_2, x_3) of the basic cell, then the redshifts of the object and its images are given by

$$z = \left\{ 1 - \frac{R(t_0)}{2(c/H_0)} [(x_1+la_1)^2+(x_2+ma_2)^2+(x_3+na_3)^2]^{1/2} \right\}^{-2} -1, \qquad (1)$$

where $R(t)$ is the scale factor of the flat universe, H_0 is the Hubble constant and t_0 the present time.

The size of the universe is described by $L_i = R(t_0)a_i$, $i = 1, 2, 3$. The lower limit of L_i can be obtained from the fact that there are no opposite-side twins in some surveys of galaxies or quasars. Up to now, it can only be confirmed $L_i > 400 - 600h^{-1}$ Mpc (Fang and Liu, 1988). Therefore, one cannot now rule out that the universe is so small that the present horizn size $L_h \sim (c/H_0)$ is already at least five times as large as L_i. That is, we have

$$P = \frac{L_h}{L_i} \qquad 5 \tag{3}$$

Eq. (3) is equal to say that the whole observable range may already cover on N_c cells of the lattice space, where N_c is given by

$$N_c = \frac{4\pi}{3} L_h^3 \;/\; L_1 L_2 L_3 \sim \frac{4\pi}{3} P^3 \sim 500 \tag{4}$$

Therefore, every object has, on average, N_c images.

In fact, the coordinates of images are not precisely given by $(x_1 + la_1,\ x_2 + ma_2,\ x_3 + na_3)$. Since the object and its images belong to different era, peculiar motion will lead to a deviation from rigorous periodicity of (a_1, a_2, a_3). Another reason for the deviation from rigorous periodicity is the clusting of objects. As we know, in some clumps of matter in the universe formations of quasars and galaxies are more active. Therefore, for short-lived objects like quasars, the brightest object for a given clump in different time was located at different places in the clump. In this case, it would be most likely that the observed images in different cells belong to the same clump, but not the same object. Hence, the coordinates of such images may also deviate from (a_1 , a_2 , a_3) periodicity by a scale of the size of the clump. In order to describe all the deviations, the coordinates of images of an object with coordinate (x_1, x_2. x_3) should be revised as

$$(x_1+la_1+\delta\ ,\ x_2+ma_2+\delta\ ,\ x_3+na_3+\delta\ ) \tag{5}$$

where δ is a random number in the range $(0,\sigma)$, and σ is given by

$$\sigma = d/R(t_o) \tag{6}$$

where d is the length scales of the size of clumps and/or peculiar motion.

From Eqs.(1) and (5), one can generate a redshift sample from a given set of coordinates of objects in basic cell. Let us now consider the T-3 universe with $a_1=a_2=a_3=a$, and a can be taken to be equal to 1 by re-normalizing R(t). Fig. 1 plots a redshift histogram of a sample, which includes all z

< 3 redshifts of images generated by 10 randomly distributed objects in the basic cell in an a = 1 universe with P = 2.5 (or L = 1200h^{-1} Mpc) and $\sigma = 0$. No periodicity can be found from Fig.1. In fact, this result has already been dicussed by Ellis (1987).

If, however, we let at least one object be located at the observer's place, i.e. the origin, the redshift histogram shows the expected periodicity. In Fig. 2 the sample is the same as that of Fig. 1, but one object has been placed at the origin of basic cell. The sample of Fig. 3 differs from Fig. 2 only by $\sigma = 0.05$ or d = 60h^{-1}Mpc. It can easily be seen from Figs. 2 and 3 that several significant peaks are superimposed on a random background. This feature is quite similar to the observed redshift samples of the quasar's emission lines and absorption lines. Histograms of both Fis. 2 and 3 also even have the most remarkable peak at $z \sim 2$, as in quasar samples. The failure to find periodicity in the simulations done by Ellis (1987) is just due to overlooking the contribution of objects within the region near the observer. In the following sections we will show that it is necessary to consider the contribution of the nearby objects.

3. Power spectrum analysis

As in the case of quasars, we adopt the method of power spectrum (PS) to analyze the periodicity in redshift distribution. Also as in the case of quasars, we study the periodicity with respect to the argument $w=\ln(1+z)$ in a redshift range of $z_1 < z < z_2$ (Fang et al., 1982).

The power spectrum P(k) is then given by

$$P(k) = \frac{1}{N_T} \left[\sum_{i=1}^{N_T} \cos(2\pi k w_i/W)\right]^2 + \left[\sum_{i=1}^{N_T} \sin(2\pi k w_i/W)\right]^2 \qquad (7)$$

where N_T is the total number of redshifts, $w_i = \ln(1+z_i)$, $W = \ln(1+z_2) - \ln(1+z_1)$ and k the wave number. If the points w_i are randomly distributed with uniform probability in the interval (w_1, w_2), the power spectrum P(k) should have both average and variance to be equal to 1. The probability that a given peak in a random spectrum has a power $P > P_o$ is given by $P_r(P>P_o)=\exp(-P_o)$. If one performs K independent experiment, the joint probability that at least one experiment will yield a spectral power exceeding P_o is

$$P_r(P > P_o) = 1 - (1-\exp(-P_o))^K \qquad (8)$$

Three PSs are plotted in Fig.4. In Fig. 4 (a), the sample is generated from one object located at the origin and 9 randomly distributed objects in the basic cell of a T-3 universe with P = 2.5. In (b) the sample is generated from 10 randomly distributed objects in the basic cell, and none is located near the origin. The samples of Fig. 4 (c) are taken from a simply connected flat universe, which includes 125X10 =1250 randomly distributed objects in the region $0 < x_1, x_2, x_3 < 2.5$. In all three PSs we take $z_1 = 0.5$ and $z_2 = 3.0$. Each PS in Fig.4 is the result of average of 10 samples.

The PS(a) is quite similiar to that of quasars. It has a clear peak at $k = 6$, which corresponds to a periodic component in the redshift distribution with respect to w by wavelength $\lambda = W/6 = 0.163$. The higher power near $k = 1, 2$ is due to global inhomogeneous, but not periodicity.

PSs (b) and (c) are obviously different from (a). No confirmable peaks exist in the PSs (b) and (c). Namely, no periodicity exists in the redshift distributions of samples (b) and (c). It is very interesting to note that it is very difficult to distinguish directly the pattern of the spatial distributions of samples (a) from those of (b) and (a). All distributions of (a), (b) and (c) appear to be isotropic. In fact, the peak P(6) in PS (a) result from only about $N_C = (4\pi/3)(2.5) \sim 65$ redshifts, which is much less than N_T 650. Therefore, PS is a very strong tool to pick up periodic components from a random background.

The influence of peculiar motion and clustering can be found from Fig. 5, in which the sample's parameters are still the same as that in PS (a) of Fig. 4, but taking $\sigma = 0.05$, 0.10 and 0.20, respectively. It is obvious that the significance of confidence of the k=6 peak is still as large as 98% for at least $\sigma = 0.1$, i.e. $d = 120h^{-1}$Mpc, which is larger than the length scales of clusters and peculiar motion. Therefore, both effects do not erase the peak given by the T-3 multiple connectivity of the topology of the universe.

The k=6 peak in power spectrum is given by the redshifts of the object at the origin and its N_c images. These redshifts distribute regularly with respect to the observer. This means the distribution of these N_c redshifts is coherent. One knows from Eq. (6) that the contribution of N_c coherent redshifts to the power spectrum is roughly proportional to N_c^2 , and their peak in PS will not be erased unless the total number of randomly distributed objects is larger than N_c^2 . Therefore, we expect the k=6 peak will still exist as long as the number of randomly distributed objects in the basic cell is less than $N_c^2 / N_c \sim 60$ or the total number of the sample is less than about 4000. This inference can clearly be verified from the distinct k=6 peak in Figs. 6 and 7. Fig. 6 shows PSs of samples generated by one object at the origin plus 5 or 20 randomly distributed objects in the basic cell. The sample used in Fig.7 is constructed by the N_c coherent redshifts plus 2000 redshifts of randomly distributed objects in the range of $(x_1^2 + x_2^2 + x_3^2)^{1/2} < 2.5$.

4. Conditions for a detectable periodicity

With all the above-mentioned preparations, we can now discuss a more general situation. If there are a total of N objects distributed randomly among the basic cell with uniform probability, the number of objects within the box σ^3 with its center at the observer is then on average going to be $N\sigma^3$. The total $N_c N\sigma^3$ redshifts of such neighboring objects and their images are coherent. Therefore, the k=6 peak given by these coherent redshifts will not be erased until the number of redshifts of the random background is larger than $(N_c N\sigma^3)^2$. This means that the k=6 peak will definitely appear in the PS if

$$N \quad N_o = 1/N_c \sigma^6 \tag{9}$$

Therefore, one can conclude that the periodicity of redshift ditribution in a T-3 universe will certainly be detectable as long as the number density of the considered objects is large so that

$$N/L^3 > N_o/L^3 = \frac{3}{4} \sigma^{-6} (c/H_o)^{-3} \tag{10}$$

or the mean distance D of the considered objects is not large as

$$D < (4\pi/3)^{1/3}\sigma^2 \ (c/H_o) \tag{11}$$

For instance, when $\sigma = 0.1$, we have $D < 50h^{-1}$ Mpc. This result would also consistent with quasars. From some incomplete catalogs of quasars, it has already been shown that the mean distance between neighboring quasars is about 100 Mpc (Chu and Fang, 1987).

More precisely, the value of N_o shouuld be found from the solution of the equation as

$$F(N_o\sigma^3, \sigma) = N_c N_o \tag{12}$$

The function $F(n,\sigma)$ is given by

$$F(n, \sigma) = [\sum_{i=1}^{N_T} \cos(12\pi w_i/W)]^2 + [\sum_{i=1}^{N_T} \sin(12\pi w_i/W)]^2 \tag{13}$$

where the sum takes over N_T redshifts in the sample, which is generated by n objects distributed randomly within the box σ^3 with the center at the observer. Fig. 8 shows some results of $F(n, \sigma)$.

If the considered objects are clustered, the number of nearby objects may be much less than the average $N\sigma^3$, so that both conditions (9) and (10) will not be applicable. However, we (the observer) are, fortunately, also to be located in a cluster of objects. Therefore, the clustering will give more nearby objects than the average $N\sigma^3$. This is, clustering even more conducive to the observable periodicity in redshift distribution given by a T-3 universe.

Finally, we consider the periodicity in a 3-dimension torus universe with different sizes a_i in different direction. Fig. 9 shows the dependence of the K=6 power on the ratio of $a_2/a_1$and a_3/a_1 . The samples used in Fig. 9 are generated from only one object located at the origin. Therefoe P(6) is, in fact, the number of coherent redshifts in the sample. It can be seen that P(6) is sensitively dependent on the ratios of sizes of the universe. This means that the periodicity of redshift distribution is one of the sensitive features related to the topology of the universe. One can then conclude that the periodicity in the distribution of large redshift objects, like quasars and Ly clouds, may play an important role in the observational cosmology of determining the topology of the universe.

References

Arp, H., Chu, Y., Zhu, X.: 1989, preprint

Burbidge, G.: 1968, Astrophys. J. Lett. 154, L41

Burbidge, G., O'Dell, S.L.: 1972, Astrophys. J. 178, 583

Chu, Y., Fang, L.Z.: 1987, Observational Cosomology, eds. A. Hewitt, G. Burbidge, L.Z. Fang, Reidel

Chu, Y., Zhu, X.: 1989, Astron. Astrophys. 222, 1

Cowan, C.L.: 1969, Nature 224, 655

Ellis, G.F.R.: 1987, Theory and Observational Limits in Cosmology, ed. W.R. Stoeger, Specola Vaticana

Ellis, G.F.R., Schreiber, G.: 1986, Phys. Lett. A, 115, 97

Fang, L.Z., Chu, Y., Liu, Y., Cao, C.: 1984, Astron. Astrophys. 106, 287

Fang, L.Z., Liu, Y.L.: 1988, Mod. Phys. Lett. A, 13, 1221

Fang, L.Z., Mo, H.J.: 1987, Observationl Cosmology, eds. A. Hewitt, G. Burbidge, L.Z.Fang, Reidel

Fang, L.Z., Sato, H.: 1983, Acta Astr. Sinica 24, 410

Plagemann, S.H., Feldman, P.A., Gribbin, J.R.: 1969, Nature 224, 875

Wills, D., Ricklefs, L.: 1976, Mon. Not. R. Astr. Soc. 175, 81

Wills, D., Wills, B.J.: 1987, Observational Cosmology, eds. A. Hewitth, G. Burbidge, L.Z.Fang, Reidel

Figure Captions

Fig.1 Histogram of a redshift samples generated by 10 randomly dostributed objects in the basic cell.

Fig.2 Histogram of a redshift sample generated by one object located at the origin and 9 random distributed in the basic cell and $\sigma = 0$.

Fig.3 Histogram of a redshift sample as that in Fig. 2, but $\sigma = 0.05$.

Fig.4 10 time average of power spectra of samples generated by (a) one object at the origin and 9 random distributed objects in the basic cell; (b) 10 random distributed objects in the basic cell; (c) 1250 objects distributed randomly in the range $0 < x_1, x_2, x_3 < 2.5$ in a simply connected flat universe.

Fig.5 10 time average of power spectra of samples generated by one object at the origin and 9 randomly distributed objects in the basic cell and (a) $\sigma = 0.05$; (b) $\sigma = 0.10$; (c) $\sigma = 0.20$.

Fig.6 10 time average of power spectra of samples generated by one object at the origin and (a) 5 or (b) 20 randomly distributed objects in the basic cell.

Fig.7 10 time average of power spectra of samples constructed with one object at the origin and its = 0.05 T-3 images plus 2000 objects randomly distributed in the range $(x_1^2 + x_2^2 + x_3^2)^{1/2} < 2.5$.

Fig.8 Function $F(n, \sigma)$. (a) $\sigma = 0$; (b) $\sigma = 0.05$; (c) $\sigma = 0.10$.

Fig.9 P(6) as a function of the ratios of the sizes of the universe.

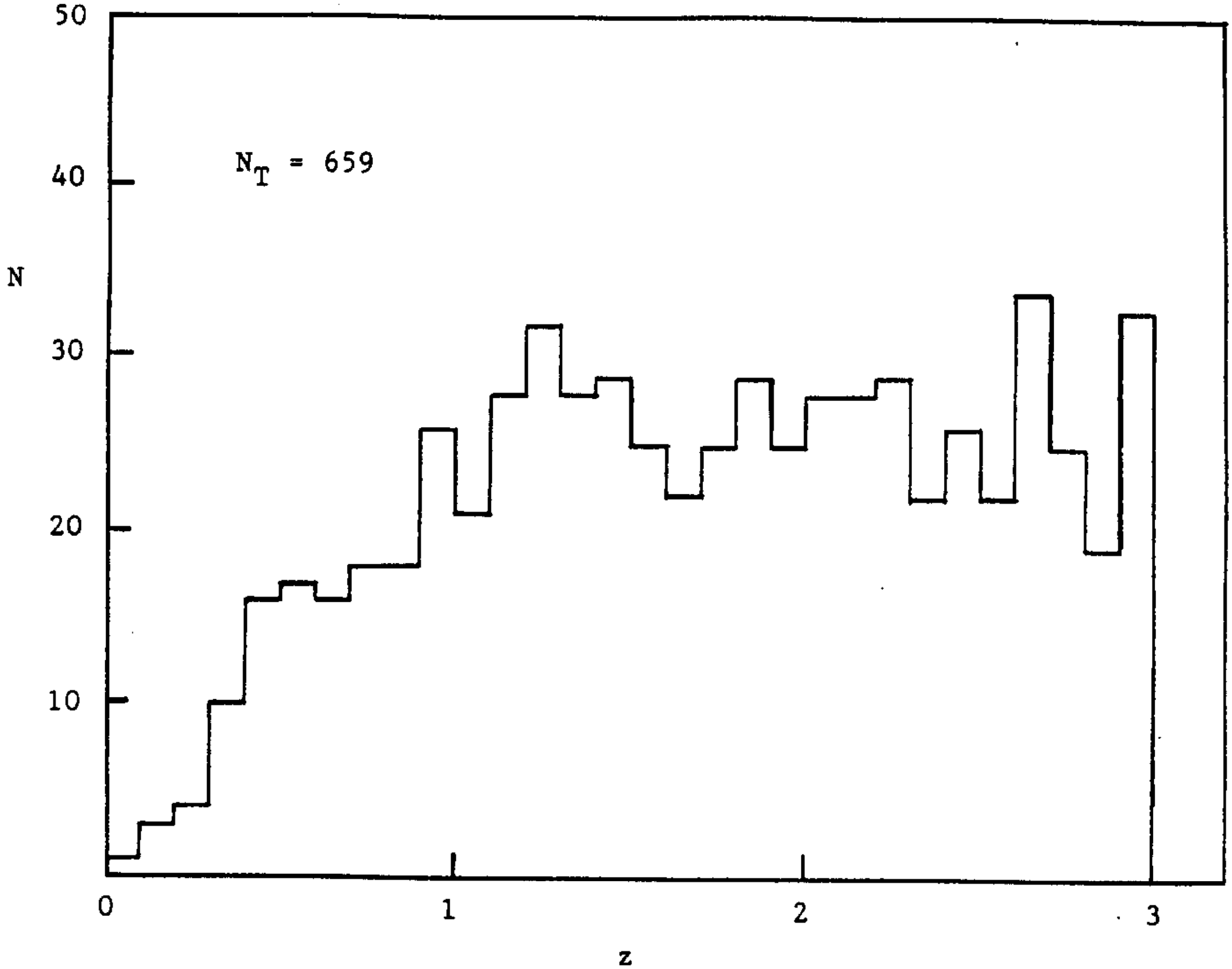

Fig. 1

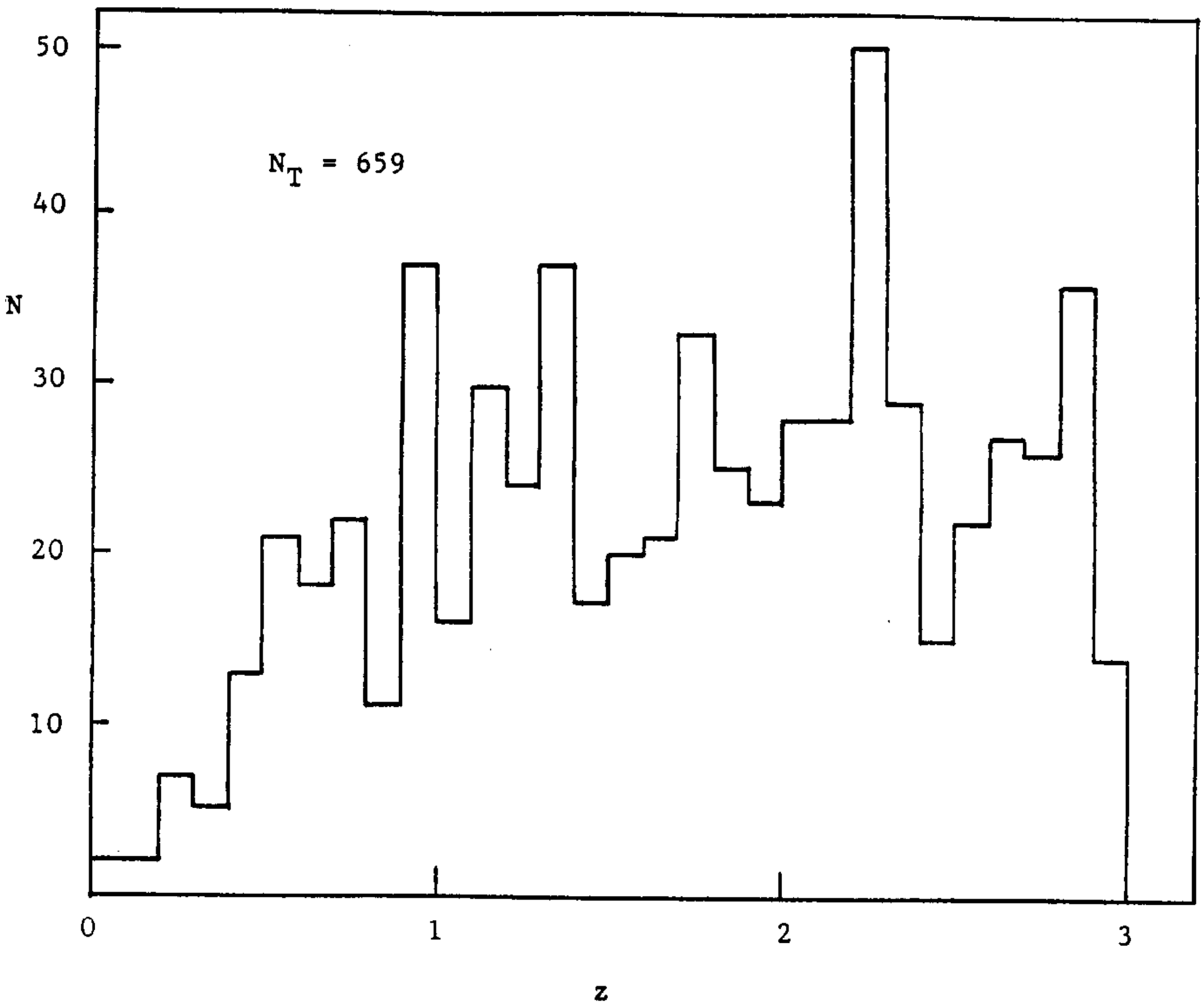

Fig. 2

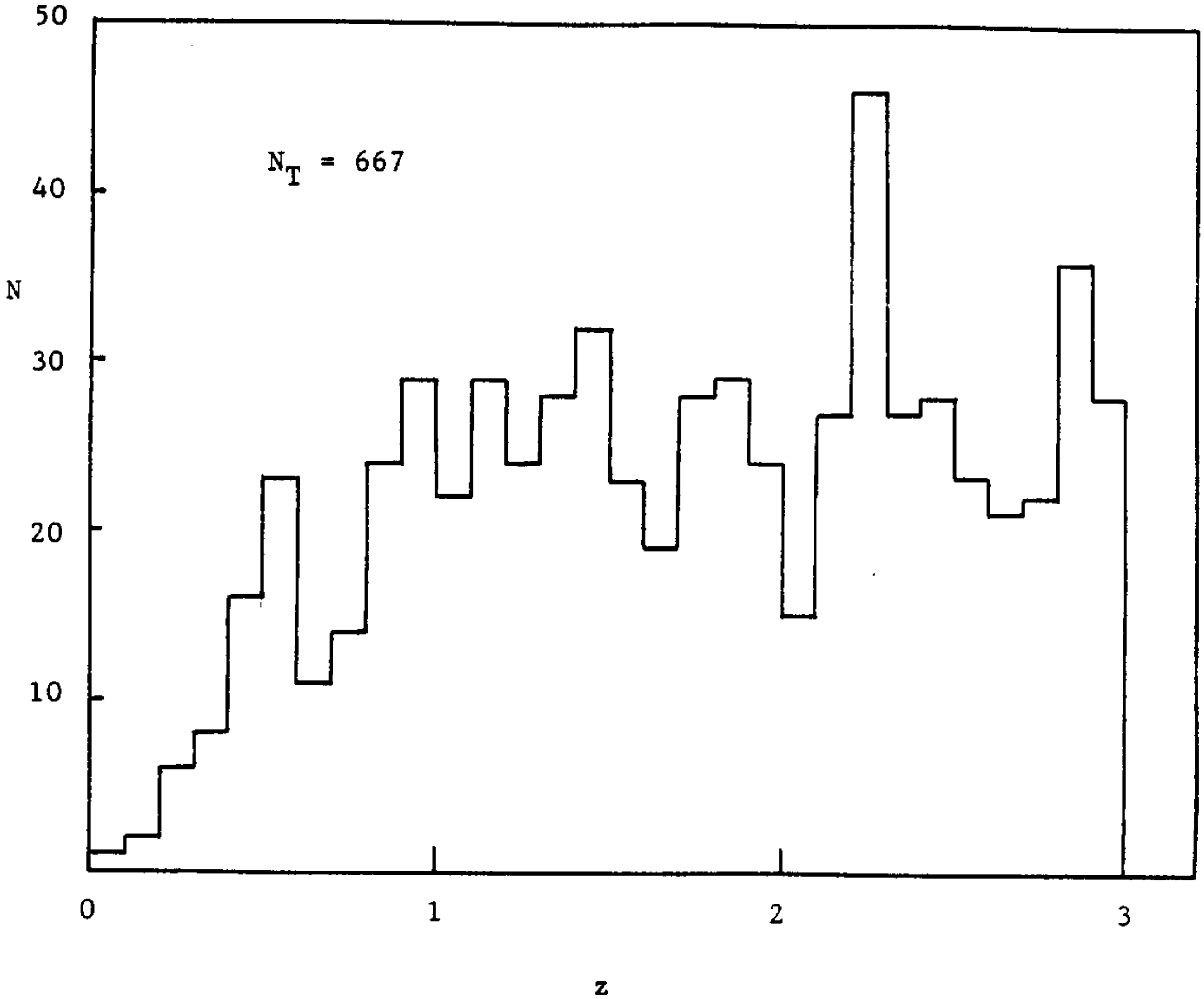

Fig. 3

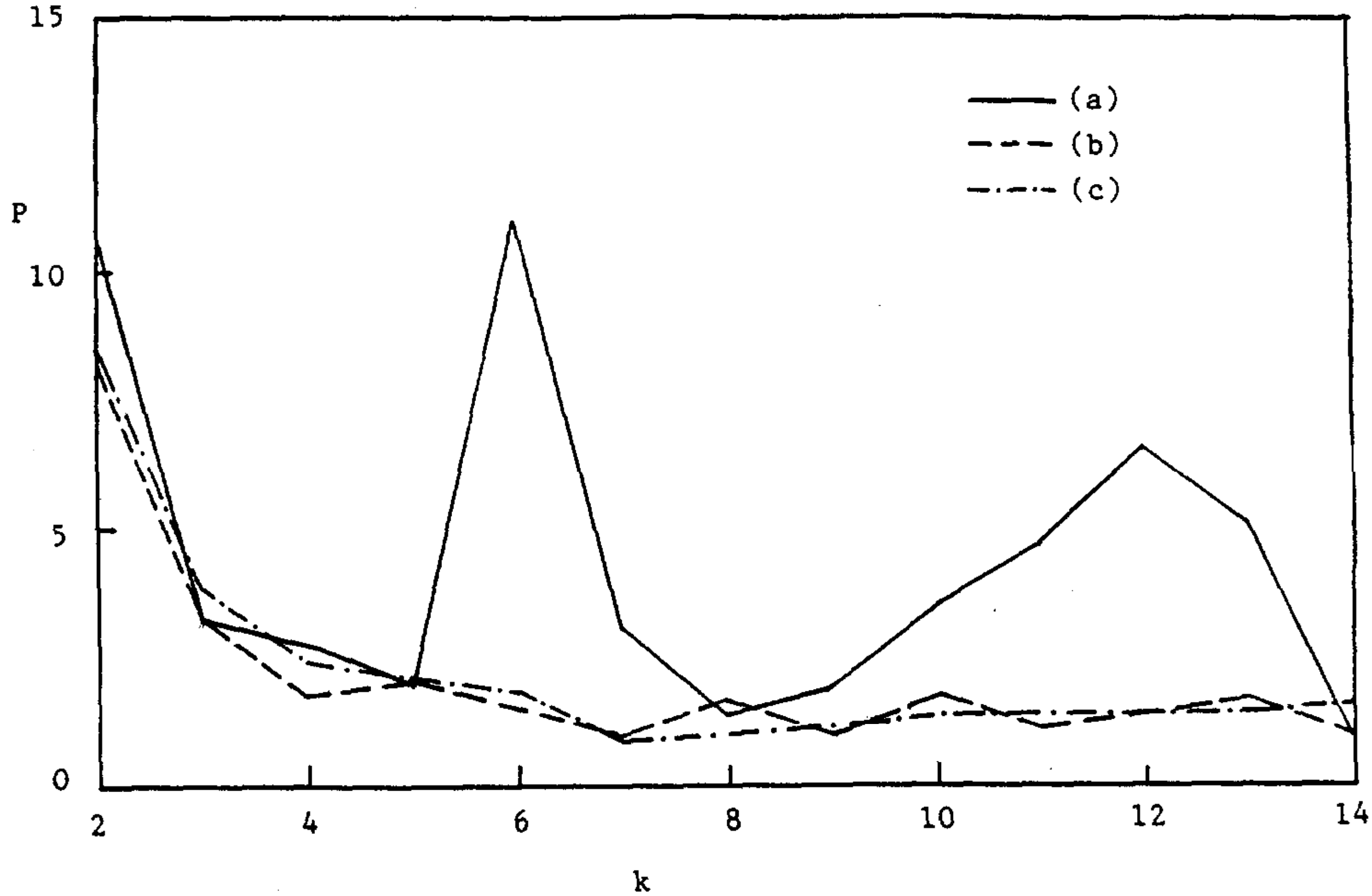

Fig. 4

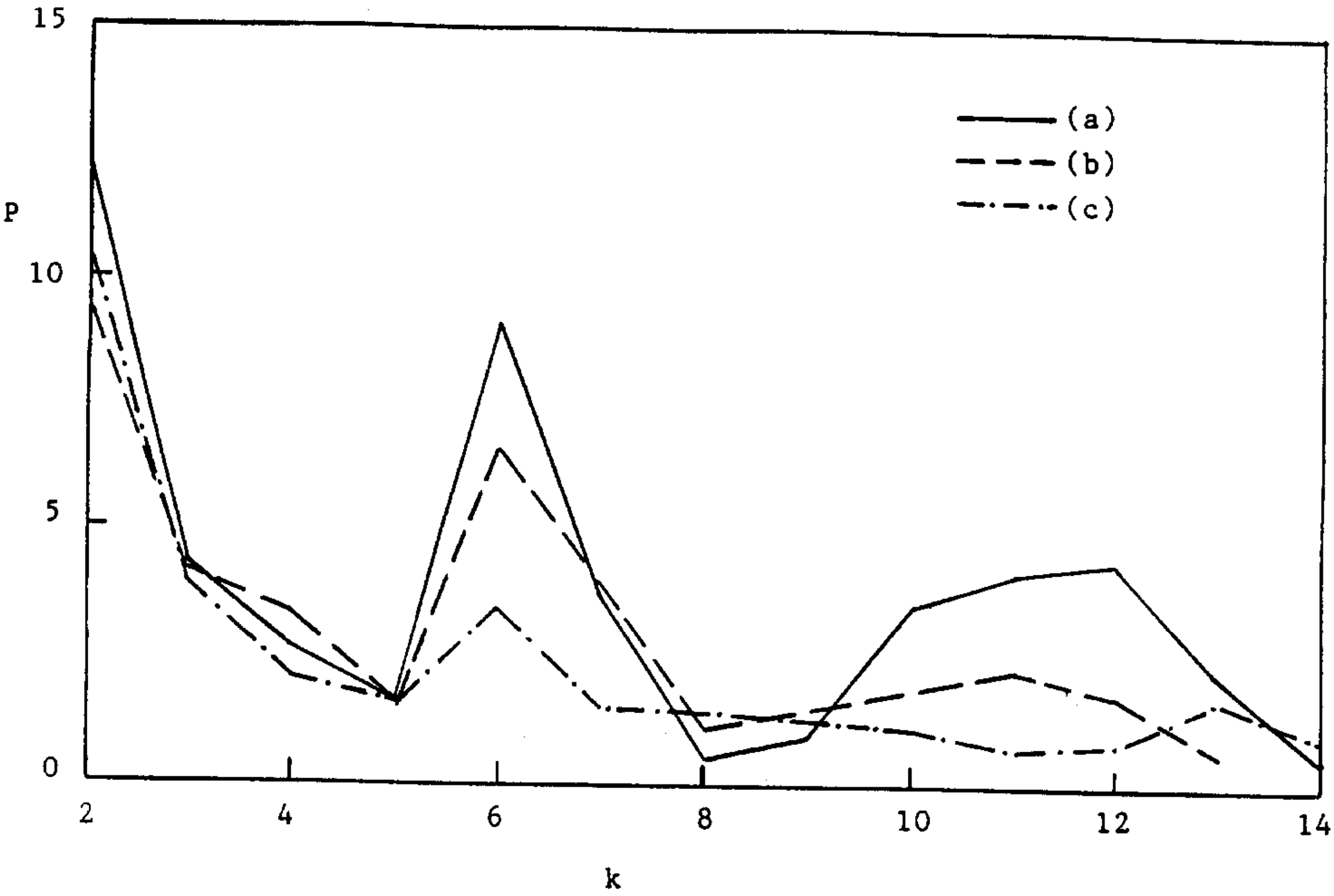

Fig. 5

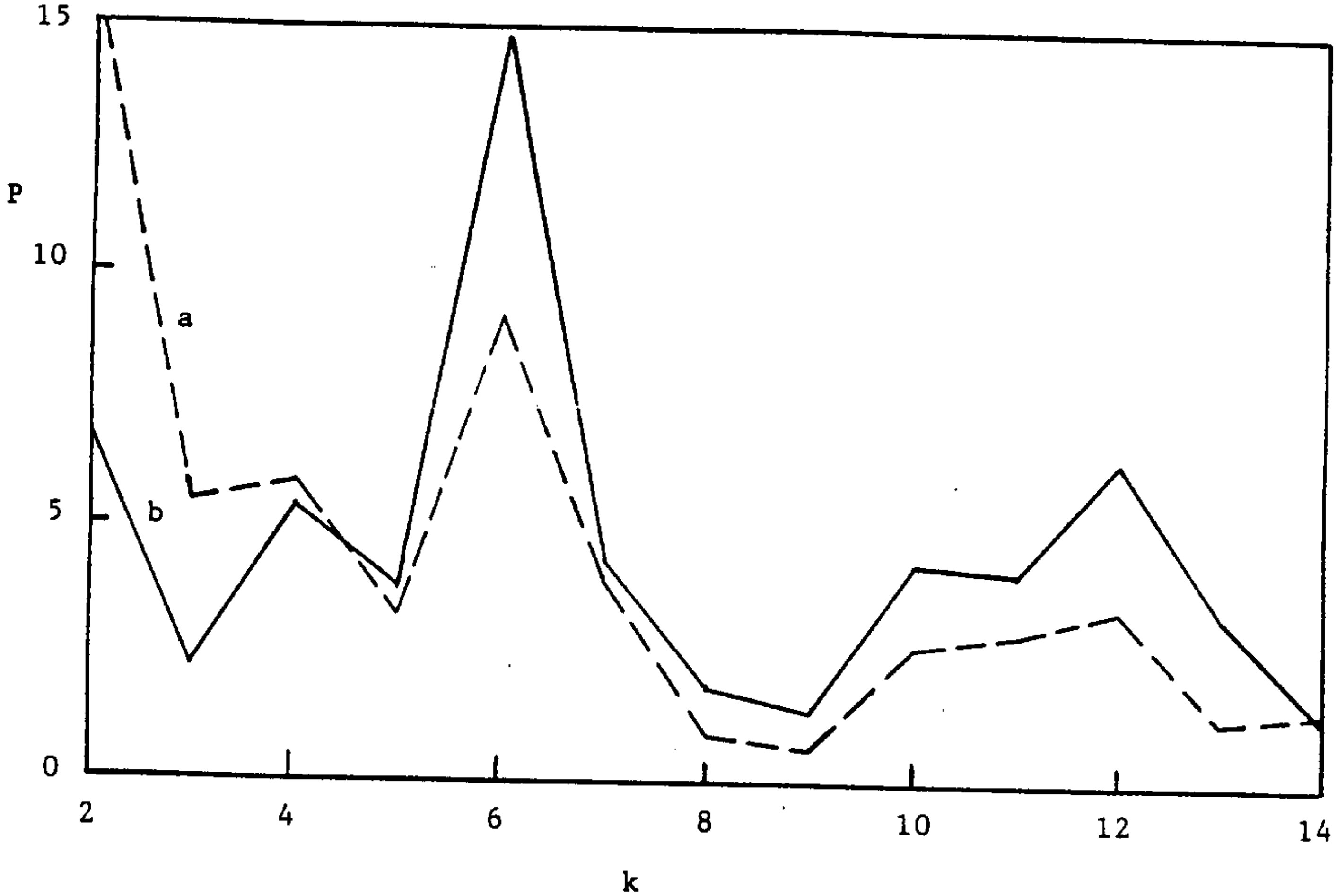

Fig. 6

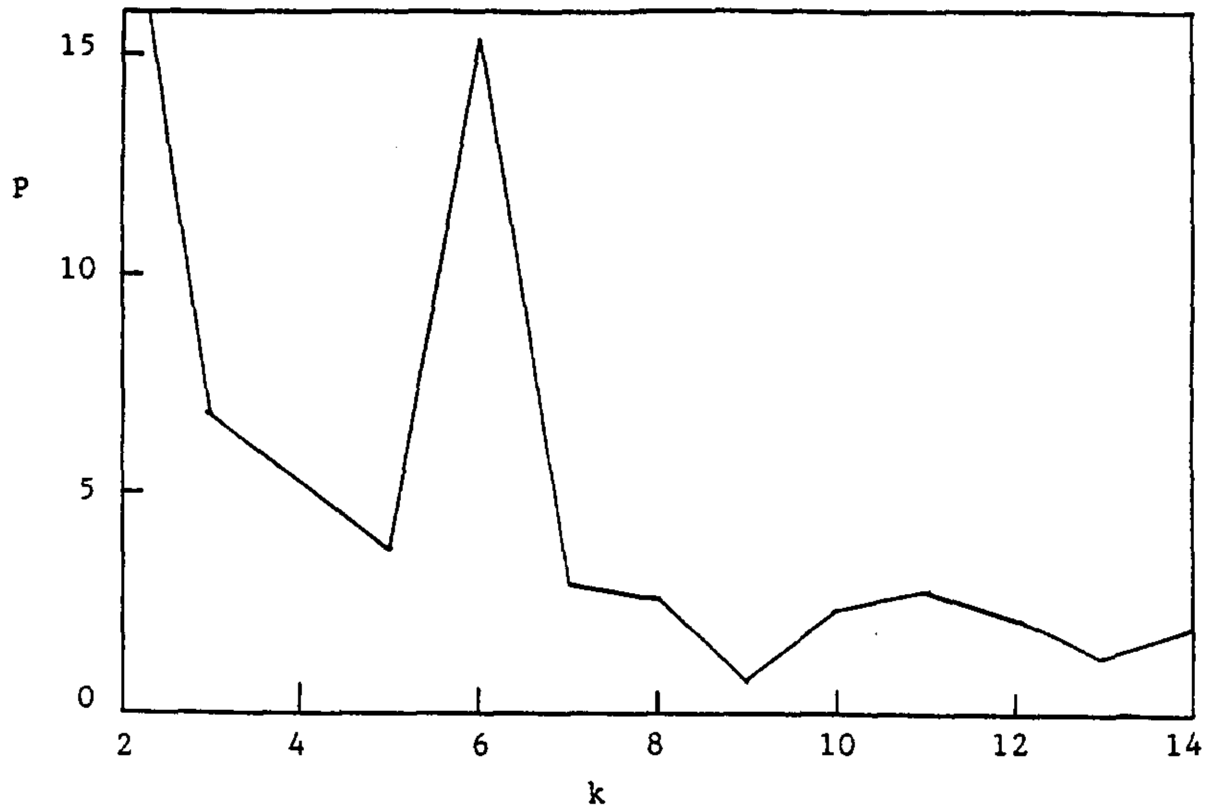

Fig. 7

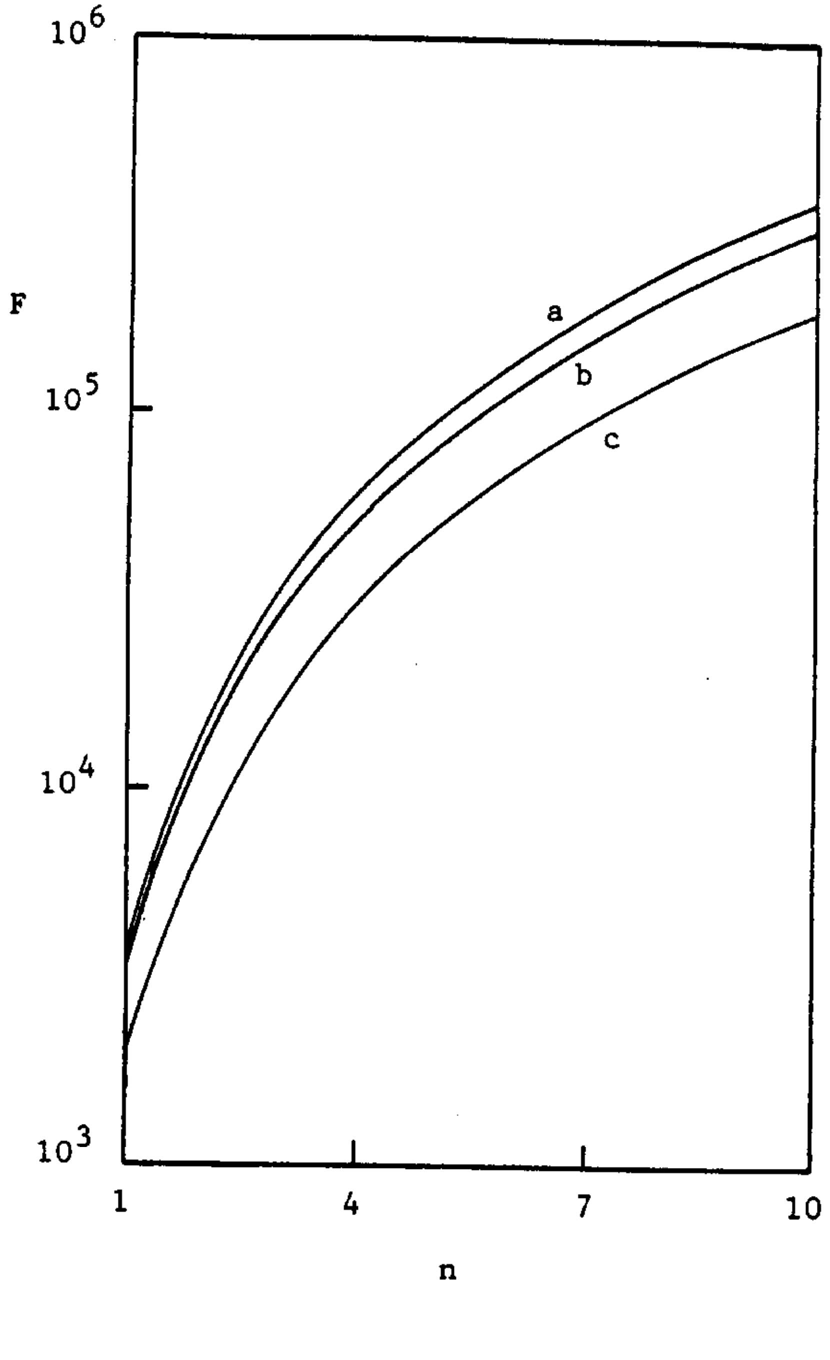

Fig. 8

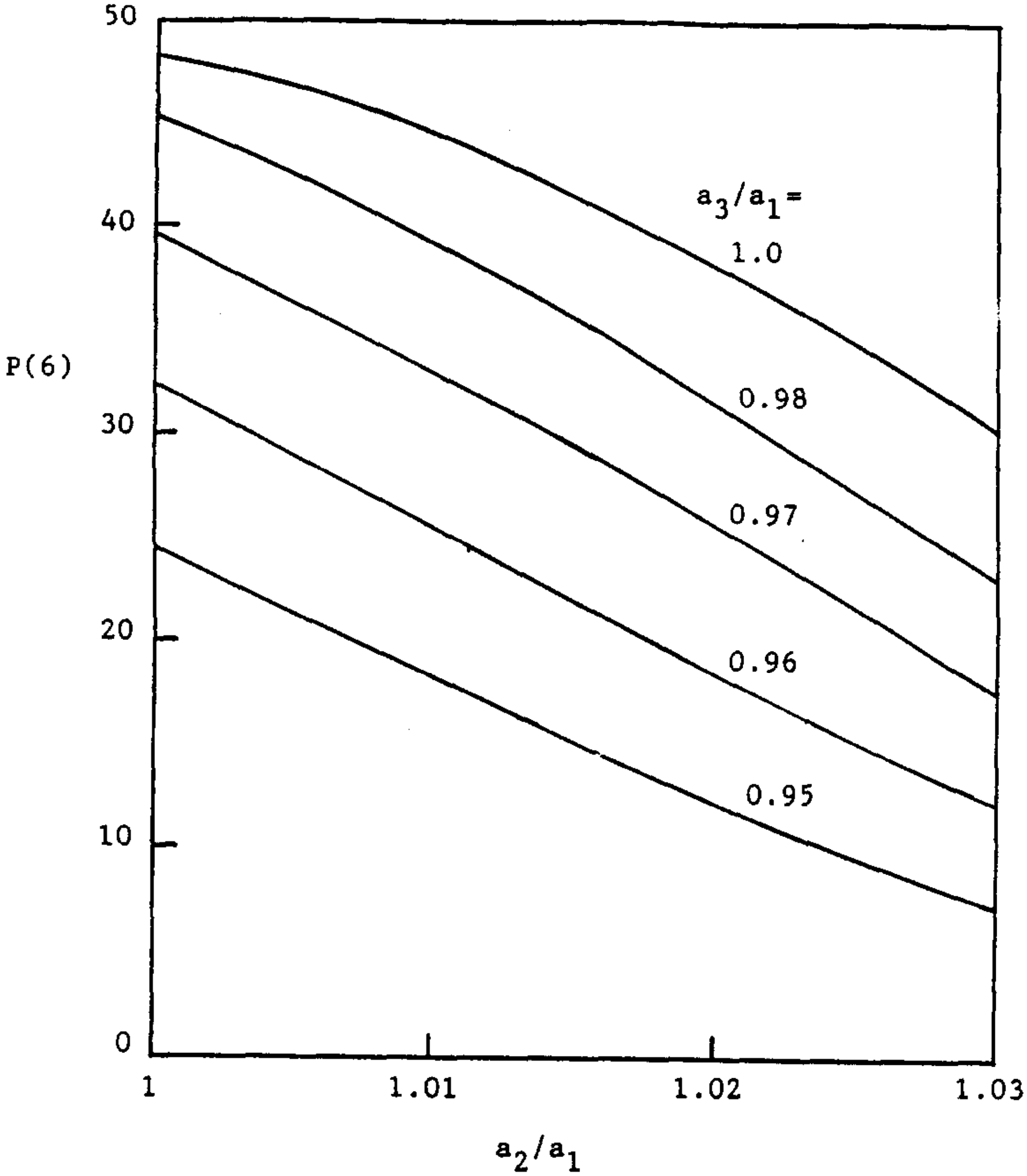

Fig. 9