Fermi National Accelerator Laboratory

CONTINUUM RESULTS FOR THE DETERMINATION OF HEAVY MESON DECAY CONSTANTS *

Brian Hill

Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106

and

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

Several perturbative results relevant for the determination of heavy meson decay constants have been obtained. This talk is a summary of some recent continuum results, including $1/m$ corrections to the heavy quark Lagrangian, which are relevant for $1/m$ corrections to other matrix elements.

1. INTRODUCTION

For brevity and clarity, in these proceedings I will summarize some perturbative results for the continuum static effective field theory without derivation. I refer to the papers on which this talk is based[1, 2] for more detail and references to related work.[3] In the following three sections, I discuss the tree level static effective field theory Lagrangian, $1/m$ corrections to it, and $1/m$ corrections to the current determining the B meson decay constant, f_B. One loop results are given in section 5, and I conclude with a discussion of lattice applications.

2. STATIC LAGRANGIAN

The degrees of freedom of a heavy quark undergoing interactions with momentum transfer much less than its mass are described by a two-component field, φ. In the rest frame of the heavy quark, the field is a doublet under the rotational group. Only two invariants of dimension four or less can be built from it. They are $\varphi^\dagger i\partial^0\varphi$ and $\varphi^\dagger\varphi$. In Minkowski space, at zeroth order in the $1/m$ expansion, the static effective field theory Lagrangian is therefore

$$\mathcal{L} = Z\varphi^\dagger i\mathcal{D}^0\varphi - Z\delta m\varphi^\dagger\varphi \ , \tag{2.1}$$

where the derivative $i\partial^\mu$ has been replaced by the gauge-covariant derivative, $i\mathcal{D}^\mu = i\partial^\mu + gA^\mu$. Z is the wave function renormalization of the heavy quark field, and δm is the mass counterterm. At tree level, $Z=1$ and $\delta m = 0$.

3. POWER CORRECTIONS TO THE ACTION

There are two dimension-five operators which incorporate the $1/m$ corrections to the static effective theory Lagrangian, the nonrelativistic kinetic energy,

$$\mathcal{O}_{kin} = \frac{1}{2m}\varphi^\dagger\mathcal{D}^i\mathcal{D}^i\varphi \ , \tag{3.1}$$

and the chromomagnetic moment operator,

$$\mathcal{O}_{mag} = \frac{i}{2m}\varphi^\dagger\epsilon_{ijk}\mathcal{D}^i\mathcal{D}^j\sigma_k\varphi \ . \tag{3.2}$$

In the Minkowski space Lagrangian, these operators appear with coefficient ZZ_{kin} and ZZ_{mag} respectively. Their normalization has been chosen so that $Z_{kin} = Z_{mag} = 1$ at tree level.

The operators $\mathcal{O}_{kin}$ and $\mathcal{O}_{mag}$ are the same as the operators that appear in the nonrelativistic effective field theory. The essential difference between the static and nonrelativistic theories is that in the former, these operators are treated perturbatively. These operators are relevant to $1/m$ corrections to various matrix elements, including the one which determines f_B.

4. POWER CORRECTIONS TO CURRENTS

The current whose matrix element determines f_B is $\bar{q}\gamma_5\gamma^0 b$. At zeroth order in $1/m$, the corre-

*Talk presented at LATTICE 90, Tallahassee, Florida, October 8-12, 1990. FERMILAB-CONF-90/259-T, 12/90, to appear in LATTICE 90: Proceedings of the International Conference on Lattice Field Theory, Nucl. Phys. B, Proceedings Supplements Section, ed. by U. M. Heller, A. D. Kennedy and S. Sanielevici.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

sponding effective field theory operator is proportional to

$$\mathcal{O} = \bar{q}\begin{pmatrix} 0 \\ I \end{pmatrix}\varphi \, . \tag{4.1}$$

The matrix appearing between the four-component light quark field, $\bar{q}$, and the two-component heavy quark field, φ, is four-by-two. In a Dirac basis, this four-by-two matrix is easily expressible in terms of two-by-two blocks. Let the coefficient of this operator in the effective field theory expansion of the full theory current be denoted D. At tree level, $D = 1$.

There are three $1/m$-suppressed operators that can appear in the expansion of $\bar{q}\gamma_5\gamma^0 b$. They are

$$\begin{aligned} \mathcal{O}_1 &= -\frac{m_l}{m}\bar{q}\begin{pmatrix} 0 \\ I \end{pmatrix}\varphi \, , \\ \mathcal{O}_2 &= \frac{1}{m}\bar{q}\begin{pmatrix} \sigma_j \\ 0 \end{pmatrix} i\mathcal{D}^j\varphi \, , \\ \mathcal{O}_3 &= \frac{1}{m}\bar{q}(-i\overleftarrow{\mathcal{D}}^j)\begin{pmatrix} \sigma_j \\ 0 \end{pmatrix}\varphi \, . \end{aligned} \tag{4.2}$$

Any $1/m$-suppressed operators with these rotational and discrete symmetry transformations can be expressed in terms of these three operators to this order in $1/m$ by using the equations of motion. Let the coefficients of $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_3$ in the expansion of the full theory current be D_1, D_2 and D_3. At tree level, $D_1 = 0$, $D_2 = -\frac{1}{2}$ and $D_3 = 0$.

5. ONE LOOP RESULTS

To one loop order, the renormalizations of the $1/m$-suppressed operators in the Lagrangian are given by $Z_{kin} = 1$ and

$$Z_{mag} = 1 + \frac{\alpha_S}{3\pi}\left(\frac{9}{4}\ln\frac{\mu^2}{m^2} + \frac{13}{2}\right) \, . \tag{5.1}$$

The coefficient of the part of the effective theory current that appears at zeroth order in the $1/m$ expansion is

$$D = 1 + \frac{\alpha_S}{3\pi}\left(-2 - \frac{3}{2}\ln\frac{\mu^2}{m^2}\right) \, . \tag{5.2}$$

The coefficients of the $1/m$-suppressed terms in the expansion of the current are

$$\begin{aligned} D_1 &= \frac{\alpha_S}{3\pi}\left(-3\ln\frac{\mu^2}{m^2} + 2\right) \, , \\ D_2 &= -\frac{1}{2} + \frac{\alpha_S}{3\pi}\left(\frac{3}{4}\ln\frac{\mu^2}{m^2} + 3\right) \, , \\ D_3 &= \frac{\alpha_S}{3\pi}(-6) \, . \end{aligned} \tag{5.3}$$

6. CONCLUSIONS

There are two sources of $1/m$ corrections to f_B. Both the matrix elements of the time ordered products of the $1/m$ suppressed operators in the Lagrangian with the zeroth order current,

$$i\int d^4x T((Z_{kin}\mathcal{O}_{kin}(x) + Z_{mag}\mathcal{O}_{mag}(x))D\mathcal{O}(y)) \, , \tag{6.1}$$

and the matrix elements of the local $1/m$ suppressed operators in the current,

$$D_1\mathcal{O}_1(y) + D_2\mathcal{O}_2(y) + D_3\mathcal{O}_3(y) \, , \tag{6.2}$$

are needed. These corrections may account for part of the discrepancy between D and B meson decay constants which have been measured by two different lattice gauge theory methods.

A lattice determination of these matrix elements would require a choice of discretization of the dimension-five operators appearing in the time-ordered products and of the three local operators in equation (4.2). Unfortunately, the renormalization of these operators gives power divergences which are difficult to subtract perturbatively.

Corrections of order $1/m$ nicely account for the ratio of D^*–D to B^*–B splitting. A lattice calculation of the matrix element of $\mathcal{O}_{mag}$ between two heavy mesons would determine the coefficient and provided a stringent test of this method. There is no mixing with lower dimensional operators for the operator determining B^*–B splitting.

Further perturbative computations relevant to this program are the computation of the matching of the discretized operators to their continuum counterparts.

ACKNOWLEDGEMENTS

This work was supported in part by NSF Grant PHY89-04035.

REFERENCES

1. E. Eichten and B. Hill, Phys Lett. B. **243** (1990) 427.
2. M. Golden and B. Hill, Fermilab Preprint, FERMILAB–PUB–90/216–T.
3. A. F. Falk, B. Grinstein and M. E. Luke, Harvard University Preprint, HUTP–90/A044.