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THE INFLATON SECTOR OF EXTENDED INFLATION

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ABSTRACT. In extended inflation the inflationary era is brought to a close by the process of percolation of true vacuum bubbles produced in a first-order phase transition. In this paper I discuss several effects that might obtain if the Universe undergoes an inflationary first-order phase transition.

1. Baryogenesis [1]

One of the most important results in particle astrophysics is the development of a framework that provides a dynamical mechanism for the generation of the baryon asymmetry. The baryon number density is defined as the number density of baryons, minus the number density of antibaryons: $n_B \equiv n_b - n_{\bar{b}}$. Today, $n_B = n_b = 1.13 \times 10^{-5}(\Omega_B h^2)$ cm^{-3}. Of course, the baryon number density changes with expansion, so it is most useful to define a quantity B, called the *baryon number of the universe*, which is the ratio of the baryon number density to the entropy density s. Assuming three species of light neutrinos, the present entropy density is $s = 2970$ cm^{-3}, and the baryon number is $B = 3.81 \times 10^{-9}(\Omega_B h^2)$. Primordial nucleosynthesis provides the constraint $0.010 \leq \Omega_B h^2 \leq 0.017$,[2] which implies $B = (3.81 \text{ to } 6.48) \times 10^{-11}$.

A key feature of inflation is the creation of a large amount of entropy in a volume that was at one point in causal contact. The creation of entropy in inflation would dilute any pre-existing baryon asymmetry, so it is necessary to create the asymmetry after, or very near the end of, inflation. In order for the baryon number to arise after inflation in the usual picture, it is necessary for three criteria to be satisfied: baryon number (B) violating reactions must occur, C and CP invariance must be broken, and non-equilibrium conditions must obtain. There are two standard scenarios for baryogenesis:[3] In the first picture the baryon asymmetry is produced by the "out of equilibrium" B, C, and CP violating decays of some massive particle, while the second scenario involves the evaporation of black holes.

In the out of equilibrium decay scenario, the most likely candidate for the decaying particle is a massive boson that arises in Grand Unified Theories (GUTs). In the simplest models, the degree of C and CP violation is larger for Higgs scalars than for the gauge vector bosons, so we will assume that the relevant boson is a massive Higgs particle. This Higgs is also taken to be different from the inflaton. The Higgs of GUTs naturally violate B. The origin of the C and CP violation necessary for baryogenesis is uncertain. It is practical simply to parameterize the degree of C and CP violation in the decay of the particle. To illustrate such a parameterization, imagine that some Higgs scalar H has two possible decay channels, to final states f_1, with baryon number B_1, and f_2, with baryon number B_2. Consider the initial condition of an equal number of H and its antiparticle, $\bar{H}$. The H's decay to final states f_1 and f_2 with decay widths $\Gamma(H \to f_1)$ and $\Gamma(H \to f_2)$, while the $\bar{H}$'s decay to final states $\bar{f}_1$ and $\bar{f}_2$ with decay widths $\Gamma(\bar{H} \to \bar{f}_1)$ and $\Gamma(\bar{H} \to \bar{f}_2)$. The decays produce a net baryon asymmetry per H–$\bar{H}$ given by

$$\epsilon \equiv \sum_{i=1,2} B_i \frac{\Gamma(H \to f_i) - \Gamma(\bar{H} \to \bar{f}_i)}{\Gamma_H}, \tag{1}$$

where Γ_H is the total decay width. Of course ϵ can be calculated if one knows the masses and couplings of the relevant particles. Reasonable upper bounds for ϵ are in the range of 10^{-2} to 10^{-3}, but it could be much smaller. For more details, the reader is referred to Ref. (3).

The non-equilibrium condition is most easily realized if the particle interacts weakly enough so that by the time it decays when the age of the Universe is equal to its lifetime, the particle is nonrelativistic. Then the decay products will be rapidly thermalized, and the "back reactions" that would destroy the baryon asymmetry produced in the decay will be suppressed.

In most successful models of new inflation the reheat temperature is constrained to be rather low. This is due to the fact that new inflation requires flat scalar potentials in order for inflation to occur during the "slow roll" of the scalar field toward its minimum. In order to maintain the flatness of the potential, the inflaton field must be very weakly coupled to all fields so that one-loop corrections to the scalar potential do not interfere with the desired flatness of the potential. The feeble coupling of the inflaton to other fields means that the process of converting the energy stored in the scalar field to radiation ("re"heating) is inherently inefficient. Although it is possible to overcome this difficulty in several ways, it remains a concern for new inflation.

The thermalization process of bubble wall collision at the end of extended inflation provides a natural arena for baryogenesis in the early Universe, as it automatically creates conditions far from thermal equilibrium, exactly as required for B, C, and CP violating GUT processes to produce an asymmetry.

Our only assumption about first-order inflation is that the parameter that determines the efficiency of bubble nucleation, $\epsilon(t) = \Gamma(t)/H^4(t)$, where Γ is the nucleation rate per volume and H is the expansion rate of the Universe, has a time dependence that suppresses bubble nucleation early in inflation, then rapidly increases so inflation is brought to a successful conclusion in a burst of bubble nucleation.

In order to keep the discussion as general as possible, consider the salient features of the potential in terms of a few parameters that can be easily identified with any scalar potential that undergoes spontaneous symmetry breaking. The parameters of the potential are assumed to be: 1.) σ_0, the energy scale for SSB, i.e., the VEV of the scalar field. 2.) λ, a dimensionless coupling constant of the inflaton potential. We will assume that the potential is proportional to λ. 3.) ξ, a dimensionless number that measures the difference between the false and the true vacuum energy density via $\rho_V = \xi\lambda\sigma_0^4$. ξ must be less than unity for sufficient inflation to occur.

From these few parameters it is possible to find all the information required about the bubbles formed in the phase transition. For instance, an important parameter is the size of bubbles nucleated in the tunnelling to the true vacuum. In the thin-wall approximation, the size of a nucleated bubble is given by $R_C \sim 3(\xi\lambda^{1/2}\sigma_0)^{-1}$. Bubbles smaller than this critical size will not grow, and it is exponentially unlikely to nucleate bubbles larger than this critical size. We will assume that all the true-vacuum bubbles are initially created with size $R = R_C$.

Another interesting parameter is the thickness of the bubble wall separating the true-vacuum region inside from the false-vacuum region outside the bubble. For the potential described above, the bubble wall thickness is $\Delta \sim (\lambda^{1/2}\sigma_0)^{-1}$. Note that the ratio of the bubble thickness to its size is $\Delta/R_C \sim \xi$; as advertised, if $\xi \ll 1$, the thin-wall approximation is valid. We note here that the results are (probably) valid even in the absence of the thin-wall approximation. Finally, the energy per unit area of the bubble wall is $\eta \sim \lambda^{1/2}\sigma_0^3$.

It is necessary to have some idea of the size of bubbles at the end of inflation, when bubbles of true vacuum percolate, collide, and release the energy density tied up in the bubble walls. The bubbles of true vacuum are nucleated with size $R = R_C$. After nucleation the bubble will grow until it collides with other bubbles.

Bubbles nucleated at late time will have little growth in coordinate radius, and any increase in the physical size of such a bubble is due solely to the growth in the scale factor between the time the bubble is nucleated and the end of inflation.

The physical size of a bubble nucleated at time $t_{\rm NUC}$ is related to its coordinate size by $R(t_{\rm NUC}) = r(t_{\rm NUC})a(t_{\rm NUC}) = R_C$. If there is negligible growth in the coordinate size of the bubble between the $t_{\rm NUC}$ and end of inflation $t_{\rm END}$, then at the end of inflation the bubble will have a physical size $R(t_{\rm END}) \equiv R = r(t_{\rm NUC})a(t_{\rm END}) = R_C[a(t_{\rm END})/a(t_{\rm NUC})]$. Assume that the burst of bubble nucleation at the end of inflation leads to bubbles all of the same size, $R = \alpha R_C$, where $\alpha \equiv a(t_{\rm END})/a(t_{\rm NUC})$.

Now we have the picture of the Universe at the end of extended inflation. To a good approximation the Universe is percolated by bubbles of true vacuum of size $R = \alpha R_C$, with all the energy density residing in the bubble walls. The next step is to examine how the release of energy from the bubble walls into radiation via bubble wall collisions takes place.

Now concentrate on a single bubble of radius $R = \alpha R_C$. The collisions of the bubble walls produce some spectrum of particles, which are subsequently thermalized. We need to estimate the typical energy of a particle produced in these collisions. When a bubble forms, the energy of the false vacuum has been entirely transformed into potential energy in the bubble walls, but as the bubbles expand, more and more of their energy becomes kinetic and the walls become highly relativistic. A simple calculation shows that if the bubble has expanded by a factor of α since nucleation, then only $1/\alpha$ of its energy remains as potential energy. The numerical simulations of bubble collisions by Hawking, Moss, and Stewart[4] demonstrate that during collisions the walls oscillate through each other, and it seems reasonable that the kinetic energy is dispersed at an energy related to the frequency of these oscillations (see their discussion of phase waves). The kinetic energy is presumably dispersed into lower energy particles, and does not participate in baryogenesis. We are more interested in the fate of the potential energy. The bubble walls can be imagined as a coherent state of inflaton particles, so that the typical energy of the products of their decays is simply the mass of the inflaton. This energy scale is just equal to the inverse thickness of the wall. Note that by the time the walls actually disperse, most of the kinetic energy has been radiated away,[4] so the walls are probably no longer highly relativistic.

The probable first step in the reheating process is converting this coherent state of Higgs into an incoherent state. The next step would be the conversion of the incoherent state of Higgs into other particles either through decay of the Higgs, or through inelastic scattering.

We are assuming that baryon-number violating bosons H will be produced in the process. The σ field is typically in the adjoint representation of the gauge group, while H is typically in the fundamental or some other representation. It is possible to en[illegible]on some symmetry forbidding a direct σ–H coupling, or that the coupling is very small compared to other couplings. If this is the case, production of H relative to other particles will be suppressed by some power of the small coupling constant. However in the generic case where all couplings are of the same magnitude there will be no suppression. Of course the ultimate answer is model dependent but calculable.

As discussed earlier, bubbles do not grow substantially before percolation in our idealized extended inflation model. Hence α remains not too far from 1, although a growth by a factor of 1000 even will not necessarily rule out the model. The bubble wall collisions yield a significant amount of the original false-vacuum energy in the form of potential energy, giving rise to high energy particles. The potential energy in the bubble walls is given by $M_{\rm POT} = 4\pi\eta R^2 \sim 4\pi\lambda^{1/2}\sigma_0^3 R^2$. Taking the mean energy of a particle produced in the collisions to be of the order of the inverse thickness of the wall, $\langle E\rangle \sim \Delta^{-1}$, the mean number of particles produced in the collisions from the wall's potential energy is $\langle N\rangle \simeq M_{\rm POT}/\langle E\rangle \sim 4\pi\Delta\lambda^{1/2}\sigma_0^3 R^2$.

In general, the bubble collisions will produce all species of particles, at least all species with masses not too large compared to $\langle E\rangle$. In the following we will assume that this is the case for the baryon-number violating Higgs particles. If the Higgs mass exceeds Δ^{-1} by a significant amount, we can expect some suppression, presumably exponential, in the number of Higgs formed. This possibility will be discussed later. For now, we simply parameterize the fraction of the primary annihilation products that are supermassive Higgs by a fraction f_H, which in general will depend on the masses and couplings of a particular theory in question. The typical number of Higgs particles produced per bubble is $\langle N_H\rangle \sim f_H\langle N\rangle \sim 4\pi f_H\Delta\lambda^{1/2}\sigma_0^3 R^2$.

Now assume that the only source of the supermassive Higgs is from the primary particles produced in the bubble-wall collisions. This will be true if the reheat temperature, T_{RH}, is below the Higgs mass.

The Higgs particles produced in the wall collisions decay, producing a net baryon asymmetry ϵ per decay, where ϵ is given in Eq. (1). Hence, the excess of baryons over antibaryons produced from a single bubble, $N_B = N_b - N_{\bar b}$, is given by

$$N_B = \epsilon\langle N_H\rangle \sim 4\pi\epsilon f_H\sigma_0^2 R^2, \tag{2}$$

where we have substituted in for the bubble thickness. This results in a baryon number density of

$$n_B = N_B/(4\pi R^3/3) = 3\epsilon f_H\sigma_0^2 R^{-1}. \tag{3}$$

Now calculate the entropy generated in bubble-wall collisions. As stated above, the potential energy of a bubble is $M_{\rm POT} = 4\pi\sigma_0^3\lambda^{1/2}R^2$. Including the (possibly dominant) kinetic energy contribution, the total mass of the bubble is $M_{\rm BUB} = 4\pi\sigma_0^3\lambda^{1/2}R^2\alpha$. Thermalization of the mass in the bubble walls will redistribute this energy throughout the bubble, resulting in a radiation energy density

$$\rho_R \sim M/(4\pi R^3/3) \sim 3\lambda^{1/2}\sigma_0^3\alpha/R = \xi\lambda\sigma_0^4, \tag{4}$$

which is just the false vacuum energy. The reheat temperature is related to the radiation energy density via $\rho_R = (g_*\pi^2/30)T_{RH}^4$, where g_* is the effective number of degrees of freedom in all the species of particles which may be formed in the thermalization process. From this we obtain the entropy density, s, produced by the thermalization of the debris from bubble-wall collisions:

$$s = \frac{2\pi^2}{45} g_* T_{RH}^3 \sim g_*^{1/4} \xi^{3/4} \lambda^{3/4} \sigma_0^3. \tag{5}$$

Eqs. (3) and (5) give $B \equiv n_B/s = \epsilon f_H \, \alpha^{-1} g_*^{-1/4} \lambda^{-1/4} \xi^{1/4}$.

Provided the mass of the Higgs is less than T_{RH}, one might conjecture that f_H is given simply by g_H/g_*, where g_H is the number of Higgs degrees of freedom; that is, all suitably light particles are produced equally. In general the situation will be more complex, and the fraction of Higgs produced will depend on the various couplings in the theory. This introduces a model dependence into the picture, though in fact one can always regard ϵf_H as a single unknown parameter. For simplicity, we assume here that all particles are indeed produced equally. Substituting this gives the final result $B = \epsilon g_H \, \alpha^{-1} g_*^{-5/4} \lambda^{-1/4} \xi^{1/4}$. This allows us to make numerical estimates of B based on sample values of these parameters. Notice that the dependence on both λ and ξ, which are the two parameters on which the inflaton's potential depends, is very weak. The important contributions are the degree of asymmetry in CP violating Higgs decays, the number of particle species available for production in the wall collisions and the factor α by which bubbles expand before colliding.

It is also interesting to note the possibility of isothermal perturbations arising from the thermalization process. While we have assumed throughout this paper that at percolation all the true vacuum bubbles have the same size, the full picture is somewhat more complicated, as bubbles formed earlier in inflation will grow to larger sizes than those formed right at the end. While homogeneity of the microwave background requires large bubbles to be suppressed, one would still expect to see a range of sizes of small bubbles, and hence spatial variations in the ratio of baryon number density to entropy density from point to point.

In conclusion then, we have seen that the result of the first-order phase transition bringing extended inflation to an end is an environment well out of thermal equilibrium. In such conditions baryogenesis via the decay of baryon number violating Higgs particles can proceed, and we have demonstrated a means by which the baryon number can be estimated. The mechanism has further been shown to work for a large range of model parameters and to have the capability of predicting a baryon asymmetry of the required magnitude.

For more details on baryogenesis, the reader is referred to the original paper of Barrow, Copeland, Kolb, and Liddle.[1]

2. Black Holes [5]

There are three possible sources for the formation of small primordial black holes after extended inflation. Holes may form via the gravitational instability of inhomogeneities formed during the thermalization phase; there is the possibility of trapped regions of false vacuum (within their Schwarzschild radii) caught between bubbles of true vacuum;[6] and there is the possibility that black holes are formed in the collision process.[4]

Unfortunately, the technical details of even estimating the typical number density and mass of the black holes formed by these processes are quite difficult. Some progress in this direction was made by Hawking, et al.,[4] in the context of the original inflationary scenario, and more recently Hsu[7] has examined black hole production from false vacuum regions in extended inflation. In order to keep the discussion on a more general footing, for now simply assume that some fraction β of the energy after collisions is in black holes, while the remaining $1-\beta$ is in radiation,[8] and later consider the various outcomes implied by the differing values of β.

The total energy density at the end of extended inflation is partitioned between the energy density of radiation, ρ_R, and black holes, ρ_{BH}:

$$\begin{aligned} \rho(t_{\mathrm{END}}) &= \rho_R(t_{\mathrm{END}}) + \rho_{BH}(t_{\mathrm{END}}) \\ \rho_R(t_{\mathrm{END}}) &= (1-\beta)\rho(t_{\mathrm{END}}) = \frac{\pi^2}{30} g_* T_{RH}^4 \\ \rho_{BH}(t_{\mathrm{END}}) &= \beta\rho(t_{\mathrm{END}}) = M_0 n_{BH}(t_{\mathrm{END}}), \end{aligned} \tag{6}$$

where T_{RH} is the reheat temperature, M_0 is the initial mass of the black holes formed (for convenience we will assume that they all have the same mass), and n_{BH} is the number density of black holes. The time t_{END} can also be expressed in terms of $\rho(t_{\mathrm{END}})$:

$$t_{RH}^2 \equiv \left(\frac{3}{32\pi}\right) \frac{m_{Pl}^2}{\rho(t_{\mathrm{END}})}. \tag{7}$$

(For matter domination, the factor $3/32\pi$ is replaced by $1/6\pi$.) From H_{END} and ρ we also define a "horizon mass" at the end of inflation:

$$M_{\mathrm{HOR}} = \frac{4\pi}{3}\rho(t_{\mathrm{END}})(2t_{RH})^3 = \left(\frac{3}{32\pi}\right)^{1/2} \frac{m_{Pl}^3}{\rho^{1/2}(t_{\mathrm{END}})}. \tag{8}$$

(The right hand side is the same in the matter dominated case.) M_{HOR} represents the mass within the "physics horizon," at the end of inflation, and plays the same role as the mass within the horizon in the standard FRW model.

Once formed, the black holes evaporate at a rate given by

$$\dot{M}_{BH} = -\frac{g_*}{3}\frac{m_{Pl}^4}{M_{BH}^2}, \tag{9}$$

which leads to a time dependence of the black hole mass of

$$M_{BH}^3(t) = M_0^3 - g_* m_{Pl}^4 (t - t_{\mathrm{END}}). \tag{10}$$

It is convenient to define a black hole lifetime,

$$\tau \equiv M_0^3 / g_* m_{Pl}^4, \tag{11}$$

and the expression for the mass as a function of time becomes $M(t) = M_0[1-(t-t_{\mathrm{END}})/\tau]^{1/3}$. The evaporation ends at time $t_{BH} = t_{RH} + \tau$.

Black holes radiate as blackbodies with temperature $T_{BH} = m_{Pl}^2/8\pi M_{BH}$. This allows us to calculate what is, for our purposes, the most important quantity—the number of particles emitted during the course of the evaporation. Let us first calculate the number of particles emitted while the black hole is between the temperatures T and $T + dT$. The change in mass of the black hole, dM, which is the amount of energy radiated as particles, is given by

$$dM = \frac{m_{Pl}^2}{8\pi}\left(\frac{1}{T} - \frac{1}{T+dT}\right). \tag{12}$$

Each emitted particle has energy $3T$ (the mean energy of a particle in a Maxwell-Boltzmann distribution at temperature T), so the number of particles emitted between those temperatures is just

$$dN = \frac{m_{Pl}^2}{24\pi T}\left(\frac{1}{T} - \frac{1}{T+dT}\right) = \frac{m_{Pl}^2}{24\pi T^3}dT. \tag{13}$$

Integrating this, we find that the number of particles emitted as the black hole temperature increases from its initial temperature T_0 to ∞ is

$$N = \frac{4\pi M_0^2}{3m_{Pl}^2}. \tag{14}$$

Note that this gives the total number of particles emitted.

It is interesting to consider the possibility that amongst the particles radiated are Higgs bosons, again denoted as H, whose decay can lead to the baryon asymmetry. Again, B will depend upon the fraction of the particles emitted as H, denoted as f_H. To determine the appropriate form for f_H, the initial temperature of the black hole at formation may be important. If it is less than the mass of the Higgs boson, m_H, then the thermal spectrum in the initial phase of the evaporation will not include Higgs as the typical energy is not high enough to produce so massive a particle. Only when the black hole temperature has increased to m_H will the thermal radiation include a significant fraction of Higgs. This can lead to an overall suppression in the number of Higgs produced during the complete course of the evaporation. Once the temperature is high enough to radiate Higgs, we expect that the energy of radiated particles will be distributed evenly amongst all radiated species, so that f_H is a constant given by g_H/g_*.

Black hole evaporation affects the evolution of both components of the total mass density. Since the hole mass is decreased by evaporation, the evolution of the black hole energy density, which in the absence of evaporation would be that of nonrelativistic matter ($\rho_{NR} \propto a^{-3}$, where a is the scale factor), is altered. The production of radiation from the hole evaporation also modifies the evolution of radiation energy density, which normally scales as a^{-4}. Of course, the departure of the energy densities from the normal evolution is most pronounced around the time $t \sim t_{RH} + \tau$. An exact treatment of this effect is given in Ref. (5), where a network of equations is derived describing the evolution of the different components of the energy density and also the evolution of the baryon asymmetry. In order to understand the general results, let us for the moment ignore the complication resulting from the decrease of the hole mass.

Two different situations arise, depending on whether black holes or radiation dominate the energy density of the Universe at the time the holes evaporate.[8] If $\beta < 1/2$, then the evolution of the scale factor is that appropriate to a radiation-dominated Universe, i.e., $a(t) \sim t^{1/2}$, and the energy density of black holes goes as $a^{-3} \propto t^{-3/2}$, while that of radiation goes as $a^{-4} \propto t^{-2}$. Therefore, provided their lifetime is sufficiently long, black holes will come to dominate the Universe at a time $t_* = t_{\rm END}(1-\beta)^2/\beta^2$, and hence if $\tau > t_* - t_{\rm END}$, they will come to dominate before their evaporation. If $\beta > 1/2$, black holes dominate even initially. If the black holes dominate before evaporation, then their evaporation produces not only the baryons, but also the entropy.

For the details of the calculations the reader is referred to Ref. (5). Here I shall simply summarize the results.

First consider the case where black hole evaporation occurs before domination. This corresponds to small β and initially light black holes. Since the black holes never dominate, the Universe expands like a radiation-dominated Universe, with $a \propto t^{1/2}$. If the black holes evaporate before domination, their radiation will not significantly change the background entropy density.

In this case the final baryon asymmetry is

$$B_A \equiv \frac{n_B}{s} = \frac{1}{2}\epsilon f_H \left(\frac{45\pi}{g_*}\right)^{1/4} \left(\frac{M_0}{m_{Pl}}\right)^{1/2} \left(\frac{M_0}{M_{\rm HOR}}\right)^{1/2} \frac{\beta}{(1-\beta)^{3/4}}, \tag{15}$$

where we have used Eq. (8). Note that the penultimate factor gives the initial black hole mass as a fraction of the horizon mass.

Now consider the second possibility, that holes evaporate after they dominate the energy density. This divides into two further sub-cases; in the former, black holes come to dominate at time t_* as defined earlier, while in the latter black holes dominate immediately after formation.

In the first of these sub-cases, once $t > t_*$ the scale factor evolves as appropriate for a matter-dominated Universe, $a(t) \sim t^{2/3}$, and so $\rho_{BH}(t) = \rho_{BH}(t_*)(t_*/t)^2$ and $\rho_R(t) = \rho_R(t_*)(t_*/t)^{8/3}$, with the energy densities equal at t_*.

The evaporation of a single black hole gives a baryon number $n_B = \epsilon f_H N \, n_{BH}(t_{BH})$. This time, though, the entropy is also determined by the other black hole evaporation products, as they provide the dominant contribution. The result for the baryon number is

$$B_{B1} = \frac{1}{2}\epsilon f_H \left(\frac{45\pi}{g_*}\right)^{1/4} \left(\frac{M_0}{m_{Pl}}\right)^{1/2} \left(\frac{M_0}{M_{\rm HOR}}\right)^{1/2} (1-\beta)^{1/4} \left(1+\frac{\tau}{t_{RH}}\right)^{-1/2}. \tag{16}$$

This expression is very similar to that obtained in the "evaporation before domination" scenario; in particular the black hole mass appears in the same functional form, and the prefactors are all the same with the exception of the β term, which naturally has changed as we move to a different physical situation. The last factor demonstrates how a long black hole lifetime dilutes the baryon asymmetry obtained; if τ is very small this factor is just equal to one, while for $\tau \gg t_{RH}$ we get a reduction in the baryon asymmetry by a factor of about $\sqrt{M_0^3/M_{\rm HOR} m_{Pl}^2 g_*}$. Clearly, this factor can be important for long-lived (initially massive) black holes. These are also exactly the type of holes that one might expect to survive long enough to come to dominate even if β is originally substantially less than 1/2.

We now examine the second sub-case of black hole domination—that in which the black holes dominate even initially. The black hole energy density is now given by $\rho_{BH}(t) = \rho_{BH}(t_{RH})(t_{RH}/t)^2$, and

$$B_{B2} = \frac{1}{2}\epsilon f_H \left(\frac{45\pi}{g_*}\right)^{1/4} \left(\frac{M_0}{m_{Pl}}\right)^{1/2} \left(\frac{M_0}{M_{\rm HOR}}\right)^{1/2} \beta^{1/4} \left(1+\frac{\tau}{t_{RH}}\right)^{-1/2}, \tag{17}$$

which is just Eq. (16) multiplied by $(\beta/(1-\beta))^{1/4}$. This factor represents the dilution of the black hole energy density up to domination. As expected, Eqs. (16) and (17) match in the case of marginal domination where $\beta = 1/2$. The β dependence in Eq. (17) simply reflects the fraction of the horizon mass contributed by black holes. It differs from Eq. (16) because here there is no evolution in the initial radiation-dominated phase, hence no era of dilution before domination. In the case of Eq. (16) an extra multiplier of $[(1-\beta)/\beta]^{1/4}$ is needed to account for the evolution in the initial radiation-dominated phase.

This completes the set of results for the different regions of domination, and is summarized in Table I. Many more details are to be found in the paper of Barrow, Copeland, Kolb, and Liddle.[5]

Table I. Results for the baryon number produced by black hole evaporation depend upon β (the fraction of the energy of the Universe in black holes at $t = t_{\rm END}$, where $t_{\rm END}$ is taken to be the end of inflation), t_* (the time at which the black holes dominate the mass of the Universe), and $\tau = M_{BH}^3/g_* m_{Pl}^4$ (the lifetime of a black hole of mass M_{BH}).

β	τ	$B \equiv n_B/s$
$\beta < 1/2$	$\tau < t_* - t_{\rm END}$	Eq. (15)
$\beta < 1/2$	$\tau > t_* - t_{\rm END}$	Eq. (16)
$\beta > 1/2$	independent of τ	Eq. (17)

3. Topological Defects [9]

I have already discussed the generation of adiabatic density fluctuations during extended inflation. However there might very well be a different mechanism for the formation of structure after extended inflation, namely the formation of topological defects in the inflaton field formed as it passes through the phase transition. Calculations of the false-vacuum decay rate made so far consider the evolution from a false-vacuum state to a *unique* true-vacuum state. However, the inflaton is far more likely to have degenerate minima, especially if it is part of a grand-unified Higgs sector.

Recall the picture of defect formation in a smooth second-order phase transition.[10] At early times the universe was very hot and the fields describing interactions were in a highly symmetric phase. However as the universe expanded and cooled, symmetry breaking occurred, which may have left behind remnants of the old symmetric phase, possibly in the form of strings, domain walls or monopoles. Here, we concentrate on strings.

Strings appropriate to galaxy formation are required to have a line density of $G\mu \sim 10^{-6}$, where $\mu \sim \sigma_0^2$, corresponding to a breaking scale of 10^{-3} Planck masses. Unfortunately, generic new and chaotic inflationary scenarios occur at or below this energy scale, and hence the strings form before or early in the inflationary epoch and are rapidly inflated away. It has been demonstrated that the universe cannot be made to reheat after inflation to sufficiently high temperatures as to restore the symmetry of the string-forming field and allow a new phase of string formation after inflation.[11,12] This leads to the incompatibility of cosmic strings with new or chaotic inflation. These arguments apply whether the inflaton and the cosmic string fields are the same field or different ones (in chaotic inflation the inflaton field can never be identified with the cosmic string field as the symmetry is broken even initially). In the case where the cosmic string field is distinct from the inflaton field, models have been proposed which resolve the conflict. The model of Vishniac, Olive, and Seckel[12] couples the inflaton and the string field in a particular way, but the only motivation for doing this is to solve the strings-inflation problem, so their solution appears unnatural. More recently, Yokoyama[13] has suggested that a non-minimal coupling to gravity of the string field can hold it in its symmetric phase during inflation, and allow strings to form at the end of inflation.

Now consider the picture of string formation in extended inflation, where the fact that the transition is first order has crucial consequences. As the Universe cools from high temperatures, a complex scalar field is trapped in a false-vacuum state and the Universe enters a phase of rapid power-law expansion. Bubbles of true vacuum then begin to nucleate and grow at the speed of light. Due to the presence of event horizons in the inflating Universe they grow to a constant comoving volume which depends on their time of formation. The important ingredient to our scenario is that each bubble forms independently of the rest, and so there is no correlation between the choice of true vacuum made in each bubble from the selection of degenerate true vacua. Eventually the bubbles grow and collide, finally percolating the Universe and bringing the inflationary era to an end.

At the end of inflation, the collision of bubble walls (in which all the energy is held) produces particles and causes thermalization of the energy. However, because the scalar field is only correlated on the scale of a bubble, we can expect topological defects to be present. The usual arguments state that there is typically of order one cosmic string per correlation volume of the scalar field, and hence we expect roughly one string per mean bubble size at the end of inflation.

This model for the formation of strings allows for the existence of large voids, which would be a consequence of the rare large bubbles. Although the typical string separation at the end of inflation is ξ_{eff}, extended inflation allows for the possibility of rare large bubbles, formed by quantum tunnelling early in inflation. The true vacuum formed inside bubbles contains no matter (any matter originally in that volume is assumed to be inflated away while the scalar field dominates the energy density). All the energy of the Universe after inflation is contained in the walls of the expanding bubbles which collide to form matter and to cause thermalization of the energy density. After collisions, matter will flow back into the void, though as it cannot travel faster than light, we can calculate the minimum time the bubble will require to thermalize. A large bubble will have a coherent scalar field vacuum and hence no strings will be formed within it—we can thus expect the interior of the bubble to evolve into a large region void of strings. If cosmic strings are to provide the seeds for galaxy formation, then we can expect to see few or no galaxies within the void. The presence of voids is an additional property of this model which may help explain observed large-scale structure.

In fact, at the time of percolation the bubbles may have a range of sizes, which can lead to the formation of an initial string network differing from the usual one. As the correlation length is essentially just the bubble size, and because there would appear to be no *a priori* reason why bubbles everywhere should be *exactly* the same size (at small sizes the assumption of a scale-invariant bubble size distribution would seem more reasonable), the strings will be formed with a randomly spatially varying correlation length. This will presumably lead to higher densities of strings in some regions than others, which again may have implications for structure formation, depending on how much the effects of the initial string distribution might be wiped out by the future evolution and decay of strings. One desirable effect of a more dilute string network would be to avoid the uncomfortable bounds from gravitational wave production from small string loops.[14] The fact that the correlation length will generically be greater (and in some models perhaps much greater) than that of the Kibble mechanism may also have important implications, though perhaps not as great as one might naively suppose if the small strings rapidly disappear from the network once string evolution commences.

These formation arguments can be equally well applied to the cases of domain walls and monopoles, again giving rise to an estimate of order one defect per bubble at the time of bubble collision. In the case of domain walls this will give rise to an excessive number, and will be disallowed on cosmological grounds. Hence, any extended inflation model featuring a potential with domain wall solutions (i.e., a disconnected vacuum manifold) can be ruled out. The situation is less clear for monopoles, because the correlation length may well be substantially greater than that of the Kibble mechanism and hence proportionally fewer monopoles are expected. However, standard estimates of the cosmological monopole abundance[15] give values of perhaps twenty orders of magnitude in excess of the Parker limit,[16] so the correlation length would have to be increased by seven or eight orders of magnitude before being within experimental limits—such an increase seems very unlikely.

If we consider the unification to be part of a grand-unified theory, the problem of monopole overproduction must be addressed, as any breaking to the symmetry of the standard model must produce monopoles at some stage. The simplest method is to arrange for monopoles to be formed in a partial symmetry breaking and then later inflated away in a second transition.

For more details, the reader should see the original paper of Copeland, Kolb, and Liddle.[9]

4. Gravity Waves [17]

One of the most interesting new features of a completed first-order phase transition is the observation by Turner and Wilczek that a significant amount of gravity waves might be produced in the reheating process.[17]

The beauty of this observation is that it is largely independent of the details of the particular extended inflation model. In the picture of reheating I have been describing here, bubbles of size R and mass $M_{\rm BUB}$ collide. Furthermore, the bubbles are most likely relativistic, or at least semi-relativistic, when they collide. The luminosity in gravity waves emitted in such a close encounter can be estimated from the quadrupole formula:

$$L_{GW} \sim G_N \left(\frac{d^3Q}{dt^3}\right)^2 \sim G_N \frac{M^2_{\rm BUB}}{R^2}. \tag{18}$$

Thus a bubble collision releases an energy E_{GW} given by

$$E_{GW} \sim RL_{GW} \sim G_N \frac{M^2_{\rm BUB}}{R}, \tag{19}$$

in the form of gravitational radiation with wavelength R.

Of course it is most useful to compare this energy with the total energy released in the bubble collision. Since the total mass of the bubble, $M_{\rm BUB}$, is eventually released into radiation, then the ratio of the gravitational wave energy density to the radiation energy density is

$$\epsilon_{GW} \equiv \frac{E_{GW}}{M_{\rm BUB}} \sim G_N \frac{M_{\rm BUB}}{R}. \tag{20}$$

If this is true after extended inflation, then the present ratio would be approximately $g_*(\text{today})/g_*(T_{RH}) \sim 0.01$ times this value. Since the contribution to Ω in radiation is today about $3 \times 10^{-5} h^{-2}$, and ρ_{GW} and ρ_R both decrease in expansion as a^{-4}, this implies that today $\Omega_{GW} h^2 \sim 10^{-5} \epsilon_{GW}$.

The wavelength of the gravitational radiation today would simply be the wavelength at creation, $\lambda(T_{RH}) \sim R$, redshifted by the expansion of the Universe:

$$\lambda(\text{today}) = R[a(\text{today})/a(T_{RH})] \sim RT_{RH}/2.7\text{ K} \sim 4 \times 10^{26} R(M/10^{14}\text{GeV}), \tag{21}$$

where again we have assumed that the re-heat temperature is comparable to the mass scale of symmetry breaking M.

Now the question is what to use for R. Turner and Wilczek make the reasonable assumption that the size of the bubble is the particle horizon at the end of extended inflation. If this is true, then $G_N M_{\rm BUB}/R$ is about unity, $\epsilon_{GW} \sim 0.01$, and $\Omega_{GW} h^2 \sim 10^{-5}$. The fact that ϵ_{GW} is about unity in this case is easy to understand: masses the size of the horizon are moving about with velocities of about the velocity of light! This choice for R also predicts $R \sim H^{-1} \sim m_{Pl}/M^2 \sim 2 \times 10^{-19}(10^{14}\text{GeV}/M)^2$cm, which leads to a present wavelength for the gravity waves of $\lambda(\text{today}) = 8 \times 10^3 (10^{14}\text{GeV}/M)$ cm. This is quite interesting because it is within the sensitivity and wavelength range of LIGO II and other large second-generation interferometric detectors.

However it might be equally possible that the bubbles are much smaller. The smallest they might be is R_C, their critical size. Let's take the pessimistic view that $R \sim M^{-1}$. If this is true, then $\epsilon_{GW} \sim G_N M_{\rm BUB}/M = M_{\rm BUB} M/m_{Pl}^2$. If the bubble size is M^{-1} and the false-vacuum energy is of order M^4, then $M_{\rm BUB} \sim M$, and $\epsilon \sim M^2/m_{Pl}^2 \sim 10^{-10}(M/10^{14}\text{GeV})^2$. This would lead to a present value of $\Omega_{GW} h^2 \sim 2 \times 10^{-15}(M/10^{14}\text{GeV})^2$. Another price to be paid is that the present wavelength of the gravity waves would be much smaller: $\lambda(\text{today}) = 8 \times 10^{-2}$ cm. This is too small in magnitude and wavelength for interferometric detectors.

Clearly the correct answer is model dependent. The latter assumption is most likely far too pessimistic, while the former assumption may turn out to be somewhat optimistic.

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