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COHERENT X-RAY GENERATION VIA LASER PUMPING OF A RELATIVISTIC ION BEAM – FEASIBILITY ASSESSMENT

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The efficiency of most popular approach to generating short wavelength radiation involving particle beams, FEL[1], is limited by the Thomson scattering cross section. If one were to use a beam of particles having a resonant scattering cross section for some frequency, a greatly enhanced gain might be expected. A natural candidate is a hydrogenic positive ion, having $Z > 2$, with a single bound electron. The fact that the ion is charged allows the beam to be accelerated to relativistic energies ($\gamma >> 1$), therefore one can exploit the properties of relativistic kinematics which dictate that the back scattered radiation will have its wavelength shortened by a factor $(2\gamma)^2$.

Suppose the ion beam encounters a laser beam traveling in the *opposite* direction. In the rest frame of the ions the frequency of the light is enhanced by a factor 2γ (extreme relativistic case, $\gamma >> 1$). Assume that the rest frame frequency matches, say, the 1s–2p transition frequency, and that the interaction path length in the lab frame corresponds to a time in the rest frame necessary to invert the level population (a so-called π pulse). If we subsequently apply a different wave, traveling *parallel* to the ion beam, we may achieve stimulated emission. The wavelength of this signal would be reduced by a factor 2γ from that of the resonant radiation in the rest frame, and by a factor $(2\gamma)^2$ from the original laser wavelength. If the ions were at rest, there is no reduction in the wavelength since the frequencies of the pump and the stimulated emission would be identical; for a relativistic beam with $\gamma = 50$ the frequency of the stimulated radiation is higher by a factor 10^4! Feasibility, of course, rests on whether the necessary pump power is achievable and whether the gain is sufficient to overcome the losses associated with typical x-ray mirrors; both of these are governed by the spontaneous emission rate.

One can derive expressions for the time evolution of the 1s and 2p level occupations (in the rest frame) as a function of an applied oscillatory electric field of strength E and frequency ω for some initial occupation of the levels.The time dependence of a two-by-two density matrix, N, (in the interaction picture) is governed by the following equation[2]:

$$\frac{\partial}{\partial t} \mathrm{N} = i\hbar\, [\mathrm{U}, \mathrm{N}] - \frac{1}{\tau} (\mathrm{N} - \mathrm{N}_{eq}) \, . \tag{1}$$

where U denotes the perturbing Hamiltonian given by:

$$\mathrm{U} = |\mathrm{U}| \begin{bmatrix} 0 & e^{i\varepsilon t} \\ e^{-i\varepsilon t} & 0 \end{bmatrix}. \tag{2}$$

Here $|\mathrm{U}| = eE|\langle 1s|z|2p\rangle|/2$, $\langle 1s|z|2p\rangle = 4\sqrt{2}(2/3)^5(a_o/Z)$, $\varepsilon = \omega_{1s\to 2p} - \omega$, and $\hbar\omega_{1s\to 2p} = E_{2p} - E_{1s} = (3/4)RZ^2$, where R and a_o are the Rydberg and Bohr radius respectively. The last term in Eq.(1) accounts for spontaneous emission[2], where τ is the lifetime and N_{eq} is the equilibrium density matrix (at zero temperature).To describe the pumping stage we solve Eq.(1) with the initial population entirely in the ground state. The required evolution of the excited state population is given by the first diagonal element of the density matrix[3]

$$\mathrm{N}_{\uparrow\uparrow}(t) = \frac{1}{2}\, \frac{(\Omega\tau)^2}{1 + (\Omega\tau)^2} \left\{ 1 - e^{-t/\tau} \left[\frac{\sin \Omega t}{\Omega \tau} + \cos \Omega t \right] \right\} , \tag{3}$$

$$\Omega \equiv \sqrt{\Omega_o{}^2 + \varepsilon^2} \, ,$$

where $\Omega_o = 4|\mathrm{U}|/\hbar$. Therefore population inversion, $\mathrm{N}_{\uparrow\uparrow} = 1 - O(\Omega\tau)^{-1}$, requires $\Omega\tau >> 1$ (strong field limit) and occurs after a time $T_{rest} = \pi/\Omega$; (this corresponds to the π-pulse in NMR). Evaluating τ from the Einstein relation[4],

$$\tau = \frac{3\hbar c^3}{4e^2\omega_{1s\to 2p}{}^3} |\langle 1s|z|2p\rangle|^{-2} = 1.59 \times 10^{-9}\, Z^{-4}\ \mathrm{s}, \tag{4}$$

and using the explicit expression for Ω,

$$\Omega = 4\sqrt{2}(2/3)^5 \frac{eEa_0}{3\hbar Z} ; \tag{5}$$

one can rewrite the strong field condition ($\Omega\tau >> 1$) as an inequality on the required electric field (rest frame value):

$$E_{rest} >> 0.349 \times Z^5 \text{ st. V cm}^{-1} . \tag{6}$$

In the lab-frame the E-field is reduced by a factor 2γ and the pulse width , $T^{\pi}{}_{rest} = \pi/\Omega$, translates to an interaction length

$$L = \gamma c\, T^{\pi}{}_{rest} = 2.61 \times 10^1 \, Z/E_{lab} \tag{7}$$

and a power flux, $\langle S \rangle = (c/8\pi)\, E_{lab}{}^2$, of

$$\langle S \rangle >> 3.64 \times Z^{10}/\gamma^2 \text{ W/cm}^2 , \tag{8}$$

which is readily achievable in steady state for low Z. Were one to operate in the fundamental Gaussian mode of an optimized confocal resonator, the condition on the *total power flux*, P, can be written as

$$P >> 10^{-2} \times Z^4 \text{ [W]} \tag{9}$$

and may be enhanced over the drive power by a factor involving the Q of the resonator. Finally, the Doppler-shifted resonance condition $2\pi c/\lambda = \omega_{1s\rightarrow 2p}/(2\gamma)$ relates γ and λ as follows

$$\gamma = 4.12 \times 10^4 \, Z^2 \, \lambda . \tag{10}$$

We now examine the second stage of our model, where the inverted population ion beam is subjected to an incoming electromagnetic wave, which is to be amplified by stimulated emission. The field strength will be assumed small, i.e. $\Omega'\tau << 1$, where we use Ω' for the precession frequency associated with the amplified wave.

To introduce the gain mechanism we note that the rate of production of coherent photons, N_{coh}, is given by a solution of Eq.(1) with the assumed population inversion as the initial condition.

$$N_{coh}(t) = \frac{1}{2}\left(\frac{\Omega'_o}{\Omega'}\right)^2 \Omega' \left\{ e^{-t/\tau} \sin \Omega'\tau + \frac{1 + (\Omega'\tau)^2}{e^{-t/\tau}\,(\sin \Omega'\tau + \Omega'\tau \cos \Omega'\tau) - 1} \right\}, \tag{11}$$

$$\Omega' \equiv \sqrt{\Omega'_o{}^2 + \varepsilon^2} .$$

The above solution describes a general "off resonance" situation ($\varepsilon \neq 0$) which reflects frequency spread , $\Delta\omega/\omega = \Delta\gamma/\gamma$, due to a longitudinal momentum spread in the incoming ion beam ($\Delta\gamma$). Note that this effect could be neglected in the discussion of the pumping stage where the strong field limit ($\Omega_o\tau >> 1$) applies. However, the lasing stage is in the weak field regime and the off resonance frequency broadening has to be considered.

The total number of coherent photons emitted during some time t (to be discussed shortly) results in an increment to the energy density of the initial wave, ΔE, given by

$$\Delta E = \hbar n \, \omega_{1s\rightarrow 2p} \left\langle \int_0^t N_{coh}(t')dt' \right\rangle_\varepsilon ; \tag{12}$$

here n is the ion density (in the rest frame) and

$$\langle \ldots \rangle_\varepsilon \equiv \frac{1}{\Delta\varepsilon} \int_{-\Delta\varepsilon/2}^{\Delta\varepsilon/2} \ldots d\varepsilon$$

defines the ensemble average over momenta in the incoming beam. The right hand side of Eq.(12) is evaluated in two asymptotic regions: (a) cool beam limit ($\Delta\varepsilon\tau << 1$); (b) hot beam limit ($\Delta\varepsilon\tau >> 1$).

Evaluating appropriate ε–averages in the above two regimes, one can rewrite Eq.(12) as follows:

$$\Delta E = \frac{\hbar}{2} n\, \omega_{1s\rightarrow 2p}\, (\Omega_0'\tau)^2 A\ , \tag{13}$$

where

$$A = \begin{cases} 0.17 & ;\ \Delta\varepsilon\tau << 1 \\ 0.39\pi/\Delta\varepsilon\tau & ;\ \Delta\varepsilon\tau >> 1. \end{cases} \tag{14}$$

The relationship between Ω'_o and the energy density E ,

$$(\Omega'_o)^2 = 2^5\pi^3\, e^2 |\langle 1s|z|2p\rangle|^2\, E/h^2\ , \tag{15}$$

allows us to rewrite Eq.(13) as

$$\Delta E = (2\pi)^2 A n\, e^2\, \omega_{1s\rightarrow 2p}\, \tau^2\, E/h\ . \tag{16}$$

The total gain, G, is defined as

$$G = (E + \Delta E)/E \tag{17}$$

and lasing requires $GR^2 > 1$; here R is the mirror reflectivity of the second (lasing) cavity. Finally the gain coefficient can be written as :

$$G = 1 + (2\pi)^2 A n\, e^2\, \omega_{1s\rightarrow 2p}\, \tau^2/h\ . \tag{18}$$

Assuming mirrors with an intensity reflection efficiency of 50% and a longitudinal momentum spread in the incoming ion beam of 1% ($\Delta\gamma/\gamma = 10^{-2}$) one can estimate the ion concentration required for lasing as

$$n_{rest} = 6.84 \times 10^9\, Z^8\, cm^{-3}\ . \tag{19}$$

In the lab frame this requires an ion beam current density of

$$j_{lab} = (Z - 1)e\, n_{rest}\, c\gamma\ . \tag{20}$$

For the case of Li^{++} ions the threshold current density would be

$$j_{lab} = 4.31 \times 10^5\ A\ cm^{-2}\ . \tag{21}$$

Particle beams having the requisite current density, momentum spread and energy are within the scope of present generation high current storage rings. The discussion in this paper centers on the 1s–2p transition in a hydrogenic atom. However, other types of transitions may also be of interest; particularly those with longer lifetimes (eg., transitions to various dipole-forbidden or metastable states). It may also be possible to exploit a nuclear transition as well as various multilevel systems.

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