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SEMI-LEPTONIC DECAYS ON THE LATTICE *

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We present our recent results for the semi-leptonic decays $D \to Kl\nu$, $D \to \pi l\nu$, $D_s \to \eta l\nu$ and $D_s \to Kl\nu$. We report on work in progress for $D \to K^* l\nu$.

1. INTRODUCTION

We update the results from our ongoing efforts for the calculation of semi-leptonic decays of charmed mesons. We have finished an extensive study of exclusive decays into final states with pseudoscalar mesons . A detailed description of this work, including a rather elaborate analysis of systematic errors, can be found in Ref. 1. In the present paper, we also give some preliminary results for the vector case.

Let us briefly mention some of the common features for the calculation of $< K|V_\mu|D >$ and $< K^*|(V-A)_\mu|D >$: For compatibility with other studies being done by our group we use a local current fixed at the origin of the lattice. The vector current (in the pseudoscalar case) is renormalized nonperturbatively using a lattice Ward identity.[2] In the pseudoscalar case 3 - momenta are injected to the initial or final state mesons while the other meson is kept at rest. For the vector case only the K^* meson has nonzero 3 - momentum.

2. THE VECTOR CASE

Let us briefly formulate how to calculate the matrix element and the form factors for $D \to K^*$;[3] for the corresponding case of $D \to K$ see Ref. 1 and 4. The matrix element can be parametrized in terms of form factors:[5]

$$< K^*,\lambda|(V-A)_\nu|D >= \epsilon_\alpha^{(\lambda)} T_{\nu\alpha} \quad , \tag{2.1}$$

$$\begin{aligned} T_{\nu\alpha} = & \, 2\frac{V(q^2)}{m_D + m_{K^*}} \epsilon_{\nu\alpha\sigma\rho} p_D^\sigma p_{K^*}^\rho \\ & + A_1(q^2)(m_D + m_{K^*})\delta_{\nu\alpha} \\ & - \frac{A_2(q^2)}{m_D + m_{K^*}}(p_D + p_{K^*})_\nu p_{D\alpha} \\ & + \frac{2m_{K^*}}{q^2}[A_0(q^2) - A_3(q^2)](p_D - p_{K^*})_\nu p_{D\alpha} \quad , \end{aligned} \tag{2.2}$$

$$A_3(q^2) \equiv \frac{m_D + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_D - m_{K^*}}{2m_{K^*}} A_2(q^2) \quad ,$$

with $A_0(0) = A_3(0)$. $\epsilon_\alpha^{(\lambda)}$ is the polarization vector of the K^* meson with helicity $\lambda = 0, +, -$ and as usual $q \equiv p_D - p_{K^*}$ is the four momentum transfer. We used the helicity basis where the form factors can be related to the appropriate resonances under the assumption of pole dominance:

$$\begin{aligned} V(q^2) &= \frac{V(0)}{1 - q^2/m_{1^-}^2} \quad , \; A_0(q^2) = \frac{A_0(0)}{1 - q^2/m_{0^-}^2} \quad , \\ A_i(q^2) &= \frac{A_i(0)}{1 - q^2/m_{1^+}^2} \quad , \; i = 1,2,3 \quad . \end{aligned} \tag{2.3}$$

The form factors are defined to be dimensionless. Note that pole dominance implies that the ratio $A_2(q^2)/A_1(q^2)$ is constant in q^2.

On the lattice we define the 2-pt function for the vector meson (K^*) as follows and find in the large time limit (under the usual assumptions):

$$\begin{aligned} G_{K^*}(\vec{p}, t; \mu, \alpha) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0|\chi_{K^*}^\mu(x)\chi_{K^*}^{\alpha\dagger}(0)|0\rangle \\ &\to \frac{1}{2E_{K^*}} e^{-E_{K^*}t} C_{K^*} \sum_\lambda \epsilon_\mu^{(\lambda)} \epsilon_\alpha^{(\lambda)} \quad , \end{aligned} \tag{2.4}$$

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	$D \to K$		$D \to \pi$		$D_s \to \eta$		$D_s \to K$	
	$f_+(0)$	$f_0(0)$	$f_+(0)$	$f_0(0)$	$f_+(0)$	$f_0(0)$	$f_+(0)$	$f_0(0)$
result	0.90	0.70	0.84	0.62	0.89	0.70	0.84	0.67
stat. err.	0.08	0.08	0.12	0.06	0.09	0.08	0.08	0.07
sys. errs:								
extrapol.	0.08	0.04	0.06	0.02	0.06	0.03	0.04	0.00
a^{-1}	0.04	0.03	0.03	0.02	0.04	0.03	0.04	0.03
κ_{ch}	0.04	0.03	0.19	0.13	0.06	0.04	0.03	0.03
Z_V^{loc}	0.09	0.07	0.08	0.06	0.09	0.07	0.08	0.07
scal. vio.	0.16	0.22	0.27	0.30	0.25	0.22	0.15	0.21
SU(3)	0.05	0.05			0.10	0.08	0.05	0.05
tot. sys.	0.21	0.24	0.35	0.34	0.30	0.25	0.18	0.23

Table 1: The results for the form factors at $q^2 = 0$ for various processes

$$| \langle 0|\chi^{\mu}_{K^*}(0)|K^*, \lambda\rangle | = C^{1/2}_{K^*}\epsilon^{(\lambda)}_{\mu} \quad .$$

Similarly we have for the 3-pt. function:

$$\begin{aligned} G_3(\vec{p}, t_{K^*}, t_D; \mu, \nu) = \qquad (2.5) \\ \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}} \langle 0|\chi^{\mu}_{K^*}(x)(V-A)_{\nu}(0)\chi^{\dagger}_{D}(y)|0\rangle \\ \to \frac{1}{2E_{K^*}2m_D} e^{-E_{K^*}t_{K^*}-m_D t_D}\sqrt{C_{K^*}C_D} \\ \cdot \sum_{\lambda} \epsilon^{(\lambda)}_{\mu} \langle K^*, \lambda|(V-A)_{\nu}|D\rangle \quad . \end{aligned}$$

Defining the ratio:

$$\begin{aligned} R(\vec{p}; \mu, \nu, \beta) = \qquad (2.6) \\ \left| \frac{G_3(\vec{p}, t_{K^*}, t_D; \mu, \nu)}{G_{K^*}(\vec{p}, t_{K^*}; \mu, \beta) G_D(0, t_D)} \right| \sqrt{C_{K^*}C_D} \end{aligned}$$

we find that

$$M_{\mu\nu} \equiv \sum_{\lambda} \epsilon^{(\lambda)}_{\mu}\epsilon^{(\lambda)}_{\alpha} T_{\nu\alpha} = \sum_{\lambda} \epsilon^{(\lambda)}_{\mu}\epsilon^{(\lambda)}_{\beta} R(\vec{p}; \mu, \nu, \beta) \qquad (2.7)$$

(sum on α; no sum on β or μ). The lattice calculation of the ratio R enables us to use this equation to solve for the form factors.

3. RESULTS

3.1. THE PSEUDOSCALAR CASE

Our results are illustrated in figures 1 and 2 for the decay $D \to K$. Table 1 shows our results for $f_+(0)$ and $f_0(0)$ for the decays $D \to K, \pi$ and $D_s \to \eta, K$ with a list of the systematic error estimates. (We refer the reader to Ref. 1 for a detailed description of this table and all our results.) Let us highlight here just a few important points. The most significant systmatic error source is scaling violation. The error listed in the table was obtained

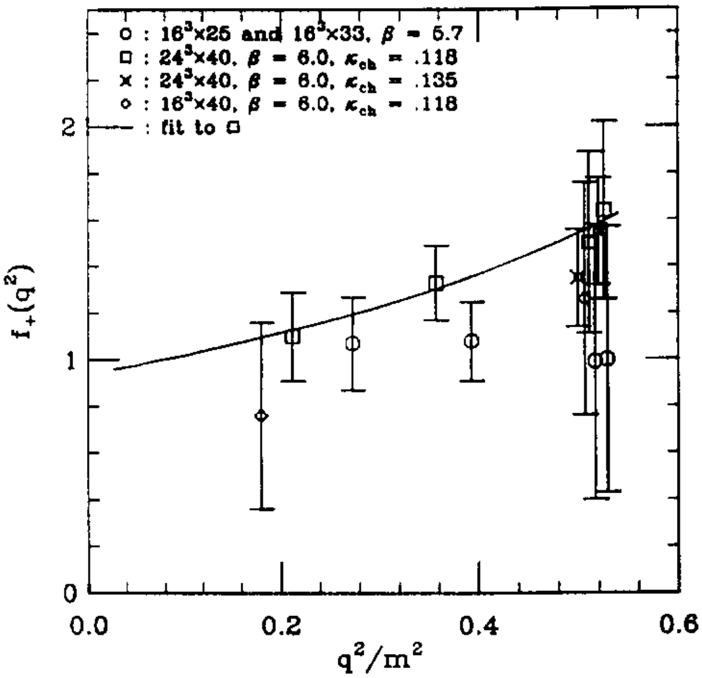

Figure 1: The form factor $f_+(q^2)$ vs. q^2/m_D^2 for $D \to K$

by comparing results from our two largest lattices, $16^3 \times 25$ at $\beta = 5.7$ and $24^3 \times 40$ at $\beta = 6.0$ (We take the results from the latter lattice as our final answer). These two lattices are similar in spatial volume. This is clearly only a rough estimate of the scaling violations for the $\beta = 6.0$ lattice; simulations at higher coupling have to be performed for a more precise determination of this systematic error. It is remarkable, however, that these scaling deviations almost disappear, if we compare the ratio f_0/f_+ instead of the form factors on the two lattices. This is illustrated in figure 2. We conclude that a ratio of form factors gives us a more reliable answer than the form factors themselves, because it is less sensitive to systematic errors.

Our final result for $f_+(0)$ for the decay $D \to K$ is:

$$f_+(0) = 0.90 \pm 0.08 \pm 0.21 \qquad (3.1)$$

Table 2 lists our results for all the decays studied in comparison with various model predictions and in the case of $D \to K$ with experimental measurements. The experimental result for $f_+^K(0)$ in the table is an average[11] of the Mark III,[12] the E 691[13] and the E 653[14] experiments. Our result agrees with the experimental number within the uncertainties.

Our numerical result (3.1) also agrees within errors with the result from the ELC collaboration,[15] $f_+^K(0) = 0.66 \pm 0.07$. However, at this point, the

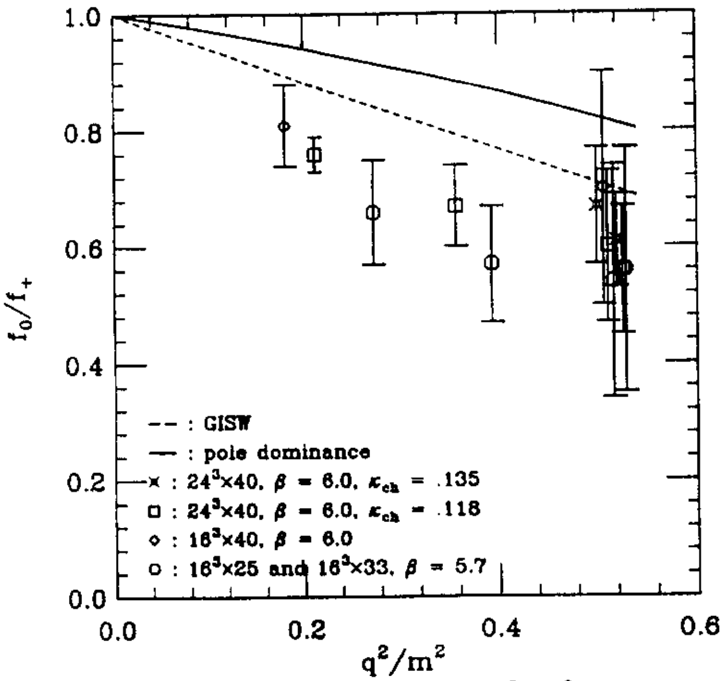

Figure 2: The ratio $f_0/f_+(q^2)$ vs. q^2/m_D^2 for $D \to K$

process	*model*	$f_+(0)$	$f_+(q_m^2)$	$f_0(0)$	$f_0(q_m^2)$
$D \to K$	BSW, KS	0.76	1.32	0.76	1.05
	GISW, AW	0.77	1.15	0.77	0.80
	AEK, DP	$0.6 - 0.75$		$0.6 - 0.75$	
	Exp.	0.69 (04)			
	this work	0.90 (08)	1.64 (36)	0.70 (08)	0.95 (11)
	sys. error	± 0.21		± 0.24	
$D \to \pi$	BSW, KS	0.69	2.62	0.69	1.35
	GISW, AW	0.51	1.28	0.51	0.53
	AEK, DP	$0.6 - 0.75$		$0.6 - 0.75$	
	this work	0.84 (12)	2.44 (80)	0.62 (06)	0.95 (10)
	sys. error	± 0.35		± 0.34	
$D_s \to \eta$	BSW	0.72	1.32	0.72	1.03
	this work	0.89 (09)	1.59 (36)	0.70 (08)	0.95 (12)
	sys. error	± 0.30		± 0.25	

Table 2: Various D decays in comparison

errors are too large for this agreement to have much significance.

3.2. THE VECTOR CASE

The decay $D \to K^*$ has received attention recently because the quark model predictions for this decay do not seem to reproduce the experimental results from the E 691 collaboration.[16] In particular the form factor $A_2(0)$ is measured to be consistent with zero, whereas model calculations predict this form factor to be $\mathcal{O}(1)$. This affects the polarization of the K^* and leads to a discrepancy between model predictions and the experimental result for the ratio of the longitudinal to transverse polarization. However, the experimental situation is not yet completely clear; a recent result by Mark III[17] seems to indicate a different qualitative picture from that of E 691.

κ_i κ_f	p_f	q^2/m_i^2	$A_1(q^2)/Z_A$	$A_2(q^2)/Z_A$	$V(q^2)/Z_V$	$A_2/A_1(q^2)$
.118 .152	0.0	0.299	0.69 (11)			
	1.0	0.206	0.63 (11)	0.62 (27)	1.52 (30)	0.98 (37)
	$\sqrt{2}$	0.125	0.52 (12)	0.50 (53)	1.09 (35)	0.96 (79)
.118 .154	0.0	0.360	0.71 (12)			
	1.0	0.249	0.63 (15)	0.83 (65)	1.58 (50)	1.31 (86)
	$\sqrt{2}$	0.160	0.46 (20)	0.2 (12)	0.84 (61)	0.4 (21)
extr.	0.0	0.391	0.72 (13)			
$\kappa_s = .155$	1.0	0.271	0.63 (17)	0.93 (85)	1.61 (60)	1.5 (11)
SU(3) lim.	$\sqrt{2}$	0.177	0.42 (25)	.02 (150)	0.72 (75)	0.1 (28)
fit (I)		0.00	0.43 (09)	0.54 (66)	0.98 (50)	
fit (II)		0.00	0.43 (09)	0.55 (67)	0.98 (50)	

Table 3: The form factors for $D \to K^*$ on the $24^3 \times 40$ lattice at $\beta = 6.0$

On the theoretical side there are some recent computations by the ELC collaboration[3, 15] that are consistent with many of the E 691 measurements.

Our preliminary results for the decay $D \to K^*$ are shown in table 3 and figure 3. At this point these numbers have very large statistical uncertainties (especially the form factor A_2); the systematic errors are still to be included. As indicated in table 3 we have also not included the renormalization corrections; this, of course, does not affect A_2/A_1. Many aspects of the analysis are still under study. In particular, the hopping parameters used so far were chosen to match the physical meson masses in the pseudoscalar case[1], e.g. $\kappa_s = .154$. This does not appear to be a very good choice for the vector case, because the vector – pseudoscalar mass splitting is not reproduced well in our quenched calculation. In fact for the $K^*(892)$ meson $\kappa_s = .152$ rather than .154 seems to give the right mass in the chiral limit.

4. SUMMARY

We have completed our study of semi-leptonic decays to pseudoscalar final states based on existing lattices. Results on $D \to K^*$ presented here are preliminary.

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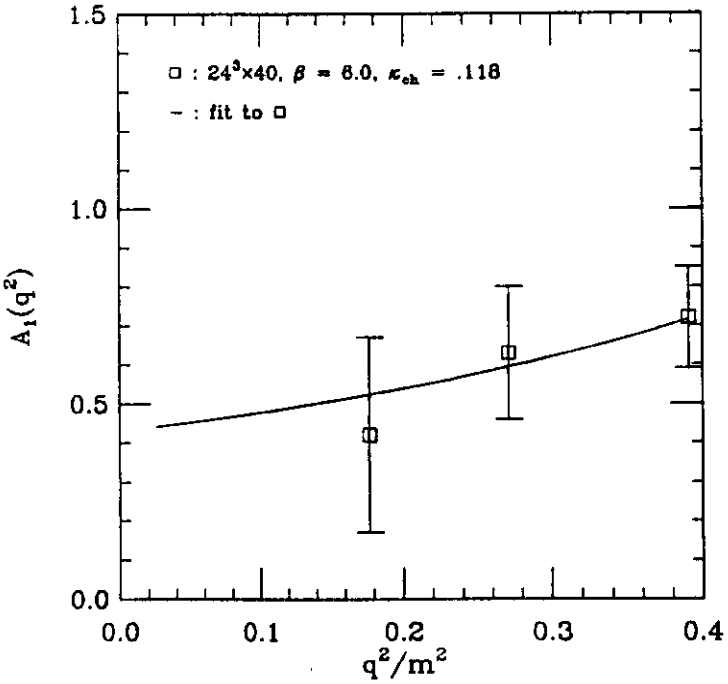

Figure 3: The form factor $A_1(q^2)$ vs. q^2/m_D^2

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