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DARK MATTER IN THE UNIVERSE

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What is the quantity and composition of material in the Universe? This is one of the most fundamental questions we can ask about the Universe, and its answer bears on a number of important issues including the formation of structure in the Universe, and the ultimate fate and the earliest history of the Universe. Moreover, answering this question could lead to the discovery of new particles, as well as shedding light on the nature of the fundamental interactions. At present, only a partial answer is at hand: Most of the material in the Universe does not give off detectable radiation, i.e., is "dark;" the dark matter associated with bright galaxies contributes somewhere between 10% and 30% of the critical density (by comparison luminous matter contributes less than 1%); baryonic matter contributes between 1.1% and 12% of critical. The case for the spatially-flat, Einstein–de Sitter model is supported by three compelling theoretical arguments—structure formation, the temporal Copernican principle, and inflation—and by some observational data. If Ω is indeed unity—or even just significantly greater than 0.1—then there is a strong case for a Universe comprised of nonbaryonic matter. There are three well motivated particle dark-matter candidates: an axion of mass 10^{-6} eV to 10^{-4} eV; a neutralino of mass 10 GeV to about 3 TeV; or a neutrino of mass 20 eV to 90 eV. All three possibilities can be tested by experiments that are either being planned or are underway.

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I. Weighing the Universe: Dark Matter Dominates!

The Friedmann–Robertson–Walker cosmology, also known as the hot big bang model, provides a reliable and tested accounting of the Universe from about 10^{-2} sec after the bang until the present. It is so successful that it is known as the standard cosmology. In the context of this cosmology the critical density separates models that expand forever ($\rho < \rho_{\rm CRIT}$) from those that ultimately recollapse ($\rho > \rho_{\rm CRIT}$); $\rho_{\rm CRIT} \equiv 3H_0^2/8\pi G \simeq 1.88h^2 \times 10^{-29}\,{\rm g\,cm^{-3}} \simeq 1.05h^2 \times 10^4\,{\rm eV\,cm^{-3}}$, where the present value of the Hubble parameter $H_0 = 100h\,{\rm km\,sec^{-1}\,Mpc^{-1}} \simeq 1/3000h^{-1}\,{\rm Mpc}$. I will denote the ratio of the *total* energy density ρ (including a possible vacuum energy) to the critical density by $\Omega \equiv \rho/\rho_{\rm CRIT}$, and the fraction of critical density contributed by species i by, $\Omega_i \equiv \rho_i/\rho_{\rm CRIT}$. The flat Einstein–de Sitter model corresponds to $\Omega = 1$; the negatively curved model to $\Omega < 1$; and the positively curved model to $\Omega > 1$. The radius of curvature can be expressed

in terms of H_0 and Ω: $R_{\rm CURV} = H_0^{-1}/|\Omega - 1|^{1/2}$.

There are a variety of methods for determining Ω.[1] Broadly speaking they can be divided into two qualitatively different categories. First, there are the dynamical methods where the mass density is inferred by its gravitational effects; these include measuring the "rotation curves" of spiral galaxies, the virial masses of clusters of galaxies, and the local peculiar-velocity field. Second, there are the kinematic methods, which are sensitive to both the space-time geometry and the time evolution of the cosmic scale factor $R(t)$. They include the classic Hubble diagram (red shift–luminosity relation), the red shift–galaxy count relation, red shift–angular size relation, and others.[2]

Dynamical Methods

One can use Kepler's third law to determine the mass of a galaxy: $GM = v^2 r$, where v is the orbital velocity of a "test particle," r is its orbital radius, and M is the mass interior to the orbit (valid for a spherical mass distribution); or its statistical analogue, the virial theorem, to determine the mass of a gravitationally bound cluster: $GM = \langle v^2 \rangle r$ where M is the cluster mass, $\langle v^2 \rangle^{1/2}$ is the velocity dispersion of the galaxies, and r is the core radius of the cluster (orbits are assumed to be distributed isotropically).

For simplicity, one can imagine that one uses these methods to determine the "average mass per galaxy" and then multiplies it by the number density of galaxies to determine the average mass density ρ. In reality, astronomers use these methods to determine the mass-to-light ratio for spiral galaxies and for clusters of galaxies; from the mass-to-light ratio they infer the average mass density

$$\rho = \langle M/L \rangle \, \mathcal{L}, \tag{1}$$

where $\langle M/L \rangle$ is the mass-to-light ratio, and $\mathcal{L}$ is the luminosity density, whose value is about $2.4h \times 10^8 \, L_{B\odot} \, \mathrm{Mpc}^{-3}$ in the B_T system. The critical mass-to-light ratio is $(M/L)_{\rm CRIT} \simeq 1200h \, M_\odot/L_\odot$, where subscript $\odot$ refers to solar units.

"Rotation curves"—that is orbital velocity as a function of orbital distance—have been determined for numerous spiral galaxies. They are obtained by measuring the Doppler shifts of stellar spectral features and of the 21 cm radiation from neutral gas clouds (HI regions)—the stars and clouds act as gravitational test particles. Rotation curves are all qualitatively similar; they rise rapidly from the galactic center and remain flat ($v =$ const) out to the furthest distances that can be probed—eventually, one "runs out" of test particles, i.e., stars and gas clouds. Since $v =$ const implies $M(r) \propto r$, this means that one "runs out" of stellar light and 21 cm radiation before the mass of the galaxy has "converged." In some cases the 21 cm rotation curves have been determined to a distance that is three times that where the light has fallen to 1% of its value at the center of the galaxy.

By restricting oneself to the bright central regions of a galaxy one can use the rotation velocity to infer the amount of mass associated with the "luminous" part of the galaxy; doing so one finds that luminous matter contributes

$$\Omega_{\rm LUM} \lesssim 0.01, \tag{2}$$

which is far from the critical density. A similarly small value is obtained by using the mass-to-light ratio determined for the local solar neighborhood, $\langle M/L \rangle_{\rm local} \sim 2 - 3$.

Based upon the fact that many rotation curves stay flat out to distances far beyond where the surface luminosity of the galaxy is negligible, one can infer that there is much more matter associated with spiral galaxies that is dark (i.e., does not give off visible radiation) than is luminous. For our own galaxy the rotation velocity has been measured out to a distance of about 20 kpc, at which point the dark matter contributes about three times more mass than the luminous matter (for reference the solar system is about 8.5 kpc from the center of the galaxy). There is weaker evidence that this dark matter exists in a spherically-symmetric, extended halo with a density that varies as r^{-2} at large distances from the center of the galaxy.

Based upon the rotation curves, one can conclude that the dark halo material in spiral galaxies contributes *at least* three to ten times the mass density that luminous matter does,

$$\Omega_{\rm HALO} \gtrsim 0.03 - 0.10. \tag{3}$$

Since there is no convincing evidence for a rotation curve that "turns over" and decreases as $r^{-1/2}$ indicating that the halo mass has converged, it is possible that the halos of spiral galaxies extend a factor of order ten further and thereby provide the critical density.[a]

There is some evidence that individual elliptical galaxies contain significant amounts of dark matter, although the case is not as well established as that for spirals. Most cluster galaxies are ellipticals, and as I will now discuss there is strong evidence for dark matter in clusters.

Estimates of the mass density based upon the virial masses of clusters lead to

$$\Omega_{\rm CLUSTER} \simeq 0.1 - 0.3, \tag{4}$$

again indicating substantially more mass than that required to account for the light. Several points should be noted: (1) X-ray emission from hot intracluster gas indicates the presence of comparable or greater amounts of baryonic mass than that associated with the visible light (dark is a relative term!), but no where near enough to account for the cluster's virial mass. (2) Since only about one in ten galaxies resides in a large cluster, one can question whether or not the mass-to-light ratio—and value of Ω—deduced from clusters is indicative. However, there seems to be no question that clusters contain significant amounts of dark matter. (3) These determinations are based upon the assumption that the clusters are well virialized, single objects and that the galaxy orbits are distributed isotropically; moreover, the cluster core radius is inferred from the distribution of the visible galaxies. If galaxies have sunk deep into the cluster potential, e.g., due to dynamical friction, then the actual core radius of the cluster—and cluster mass—could be much larger (just as with galactic halos).[b]

[a] There are arguments to the contrary; e.g., mass estimates of the Milky Way and Andromeda based upon their velocity of approach seem to indicate that their halos could not be this large, although such arguments assume that the Milky Way and Andromeda are on a radial orbit and are approaching each other for the first time. Likewise, mass estimates of the Milky Way based upon the orbits of its satellite galaxies indicate the same, although it is assumed that the orbits are isotropically distributed.[3]

[b] It should be mentioned that in 1933 the astrophysicist Fritz Zwicky pointed out that the mass associated with the light in several clusters was much less than the mass required to bind the cluster—and thus was the first to identify the dark matter problem.

Peculiar Velocities

The velocity of a galaxy can be split into two pieces: the velocity due to the general expansion of the Universe (or Hubble velocity) which is radial and proportional to galaxy's distance from us; and the peculiar velocity, the velocity the galaxy has in addition to its Hubble velocity.[c] Any peculiar velocity that is not "supported" by a gravitational field will decay with time, inversely with the cosmic scale factor $R(t)$. Put another way, peculiar velocities arise due to the lumpy distribution of matter—and thereby offer a probe of the density field. In contrast, the distribution of bright galaxies only probes the distribution of light—and the two distributions need not be the same.

In the linear perturbation regime, i.e., $\delta\rho/\rho \lesssim 1$, the Fourier expansion of the velocity field, $\mathbf{v}_k$, is related to that of the density field, δ_k, $\mathbf{v}_k = -i\mathbf{k}R(t)\dot{\delta}_k(t)/|\mathbf{k}|^2$, and to a good approximation $|\mathbf{v}_k| \simeq \Omega^{0.6}H_0|\delta_k|/k$. Suppose the peculiar velocity of an object is primarily due to linear perturbations on the scale λ, then

$$\frac{\delta v}{c} \sim \Omega^{0.6}\left(\frac{\lambda}{H_0^{-1}}\right)\left(\frac{\delta\rho}{\rho}\right)_\lambda . \tag{5}$$

Even if the contribution from one Fourier component does not dominate, Eq. (5) still illustrates the correct dependence of the peculiar velocity upon Ω.[d] One can exploit this relationship in different ways: (i) input Ω and $\delta\mathbf{v}$ to infer $\delta\rho(\mathbf{r})/\rho$; (ii) input Ω and $\delta\rho(\mathbf{r})/\rho$ to infer $\delta\mathbf{v}$; or (iii) input $\delta\mathbf{v}$ and $\delta\rho(\mathbf{r})/\rho$ to infer Ω. The last of these alternatives is the one we are interested in here; however, what one can directly measure is $\delta n_G(\mathbf{r})/n_G$, and so one must relate $\delta n_G/n_G$ to $\delta\rho/\rho$ (n_G is the number density of "bright" galaxies). The simplest *ansatz* is to take them to be equal: "light traces mass." A slightly more general approach is to assume that "light is a biased tracer of mass:" $\delta\rho/\rho = b^{-1}(\delta n_G/n_G)$, where $1 \lesssim b \lesssim 3$ is the biasing factor.

Using the IRAS catalogue of infrared-selected galaxies to determine the mass distribution (i.e., $\delta n_G/n_G$), several groups have used measurements of the local peculiar-velocity field[4] to infer $\Omega^{0.6}/b \simeq 1.$, with an estimated uncertainty of about 0.3 or so.[5] With some delight, I note that this technique seems to suggest that Ω is indeed close to unity, although I caution the reader that these results are still preliminary.[e]

Before going on to the kinematic methods, I mention that there are other dynamical methods for determining Ω, including the use of gravitational lens systems to measure cluster and galaxy masses, Virgo infall (which is similar to the peculiar-velocity method mentioned above), cosmic virial theorems, and pair-wise velocities of galaxies.[6]

In addition, there may or may not be another, more local dark matter problem. The mass density of the disk in our neighborhood can be determined by studying the motions of stars perpendicular to the plane of the disk, and by a "direct inventory" of the material

[c] Of course, we can only measure the component of the peculiar velocity that is parallel to the line of sight.

[d] More precisely, the peculiar velocity at position $\mathbf{r}$ is $\delta\mathbf{v}(\mathbf{r}) = -\Omega^{0.6}(H_0/4\pi)\int \delta\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')d^3r'/|\mathbf{r}-\mathbf{r}'|^3\rho$.

[e] The infrared bright galaxies tend to be spiral galaxies in the field, and so clusters are under-represented. The authors have tried to correct for this by including some important clusters, and find that their results do change significantly.

in the local neighborhood (stars, white dwarfs, gas, dust, etc.). In principle the two results should agree. The local mass density inferred from dynamics,[f] $1.3 \times 10^{-23}\,\mathrm{g\,cm^{-3}}$, is about a factor of two larger than can be accounted for by the local inventory.[7] This discrepancy of a factor of two may or may not be significant. In any case, it has little bearing on the "big" dark matter problem. Since the mass density of the local neighborhood is dominated by luminous matter, this additional dark matter—if it exists—makes a contribution to Ω that is at most comparable to that of luminous matter.

Moreover, this local dark matter cannot be due to halo material: Based upon the rotation curve of our galaxy and detailed models for the distribution of matter in our galaxy, the local halo density is estimated to be[8]

$$\rho_{\mathrm{HALO}} \simeq 5 \times 10^{-25}\,\mathrm{g\,cm^{-3}} \simeq 0.3\,\mathrm{GeV\,cm^{-3}}, \tag{6}$$

with an uncertainty of about a factor of two. The local halo density is about a factor of ten smaller than the local disk dark-matter density; put another way, if the halo material accounted for the disk dark-matter density, the local rotation velocity would be about a factor of three larger than its measured value!

Kinematic Determinations

There are a number of classic kinematic tests—luminosity–red shift (or Hubble diagram), angle–red shift, galaxy-number count–red shift—that can in principle be used to determine our cosmological model.[2] These tests depend upon the global space-time geometry and the time evolution of the scale factor. For example, the luminosity distance to a galaxy at red shift z, $d_L^2 \equiv \mathcal{L}/4\pi\mathcal{F}$, is related to the coordinate distance to the galaxy, $r(z)$, by

$$d_L^2 = r(z)^2(1+z)^2,$$
$$\int_0^{r(z)} \frac{dr}{\sqrt{1-kr^2}} = \int_{t(z)}^{t_0} \frac{dt}{R(t)}, \tag{7}$$

where the present value of the scale factor $R(t_0)$ is taken to be one and $(1+z) = R(t)^{-1}$. Since the evolution of the scale factor depends the equation of state, e.g., $p = 0$, matter-dominated, $R \propto t^{2/3}$; $p = \rho/3$, radiation-dominated, $R \propto t^{1/2}$; $p = -\rho$, vacuum-dominated, $R \propto \exp(Ht)$, the functional dependence of $r(z)$ does too. Thus, the red shift–luminosity distance relation depends upon both the curvature of space and the composition of the Universe. For a matter-dominated model

$$H_0 d_L = q_0^{-2}\left[zq_0 + (q_0-1)\left(\sqrt{2q_0 z + 1} - 1\right)\right] = z\left[1 + (1-q_0)z/2 + \cdots\right]. \tag{8}$$

where $q_0 \equiv -\ddot{R}_0/H_0^2 = \Omega(1+3p/\rho)/2 = \Omega/2$, and the second expression is an expansion in z.

The success or failure of this technique depends upon obtaining accurate luminosity distances for objects out to red shifts of order unity. Accurate luminosity distances requires the existence of objects of known luminosity (standard candles). Here lies the problem;

[f] This density is known as the Oort limit, in honor of the first astronomer to address this problem.

evolutionary effects are likely to be important, especially at high red shifts, and it is difficult to determine even the sign of the evolutionary effects let alone reliably estimate the magnitude! Nevertheless, there are some who believe that the K-band (2.2 μm) version of the Hubble diagram will prove useful,[9] as evolutionary effects are lessened.[g]

A kinematic test with great cosmological leverage and promise is the galaxy count–red shift relation. The number of galaxies seen in the red shift interval dz and solid angle $d\omega$ depends upon the number density of galaxies $n_G(z)$ and the spatial volume element, $dV = r^2 dr d\omega/\sqrt{1-kr^2}$. This relationship too depends upon both the spatial curvature and the time evolution of the scale factor. For a matter-dominated model,

$$\frac{dN_{\rm GAL}}{d\omega dz} = \frac{n_{\rm GAL}(z)[zq_0 + (q_0-1)(\sqrt{2q_0 z+1}-1)]^2}{H_0^3(1+z)^3 q_0^4[1-2q_0+2q_0(1+z)]^{1/2}},$$

$$\simeq z^2 n_{\rm GAL}(z)[1-2(q_0+1)z+\cdots]/H_0^3. \qquad (9)$$

For fixed (comoving) number density of galaxies, the galaxy count increases with decreasing Ω (or q_0) because of the increase in spatial volume. Loh and Spillar[10] have used the galaxy count–red shift test with a sample of about 1000 field galaxies—red shifts out to 0.75—to infer $\Omega = 0.9^{+0.7}_{-0.5}$ (95% confidence). Their result has drawn much criticism; in part because their red shifts are not spectroscopically determined (they are determined by six-band photometry) and because their results are sensitive to the assumptions made about galactic evolution.[11]

In principle, this test is less sensitive to evolution, provided that the number of galaxies remains constant and their luminosities do not evolve so drastically that they cannot be seen. Recent deep galaxies counts indicate an excess of galaxies at higher red shifts—indicative of a low value of Ω.[12] (If galaxy mergers are very important—as they may well be in cold dark matter scenarios—the number density of galaxies at higher red shifts would be expected to be larger.) At the moment, determinations of Ω based upon the galaxy number count test are not conclusive. However, many believe that this method has great potential because a large sample of objects can be used and it is less sensitive to evolution.

Primordial Nucleosynthesis and Ω_B

Primordial nucleosynthesis provides the most stringent and earliest test of the standard cosmology, probing it back to the epoch when $T \sim$ MeV and $t \sim$ sec. The primordial abundances of D, ^{3}He, ^{4}He, and ^{7}Li predicted in the standard (and simplest) model of primordial nucleosynthesis agree with the inferred primordial abundances of these light elements.[13] Moreover, this agreement can be used to constrain one cosmological parameter—the baryon-to-photon ratio η—and one parameter of the standard model—the number of light neutrino species N_ν.[14] Concordance between theory and observation requires:

$$3\times 10^{-10} \lesssim \eta \lesssim 5\times 10^{-10} \qquad \text{and} \qquad N_\nu \le 4.$$

[g] When one observes a galaxy of moderate red shift in the visible, the light one sees comes from the blue or UV part of the spectrum and is produced by massive stars that evolve rapidly. By contrast, observing in K-band, the light one sees was emitted in the red part of the spectrum and is produced by lower mass stars that evolve much more slowly.

The constraint to the number of light neutrino species has recently been confirmed by precise measurements of the properties of the Z^0 boson,[15] which imply $N_\nu = 3.0 \pm 0.1$. This is an impressive confirmation of the standard cosmology at this very early epoch.

Primordial nucleosynthesis provides the most precise determination of the baryon density. In converting the baryon-to-photon ratio to the fraction of critical density contributed by baryons two other parameters are needed: (i) the temperature of the cosmic microwave background (CMBR), which is now accurately determined to be $2.736 \pm 0.01\,\mathrm{K}$;[16] and (ii) the not so well known value of the present Hubble parameter, $0.4 \lesssim h \lesssim 1.0$.[h] The nucleosynthesis constraint can be written as

$$0.011 \lesssim 0.011h^{-2}\Omega_B \lesssim 0.019h^{-2} \lesssim 0.12. \tag{10}$$

Summary of Our Knowledge of Ω

What then is the present state of our knowledge concerning the mass density of the Universe? Let me try to summarize:

- Luminous matter contributes only a small fraction of the critical density: $\Omega_{\mathrm{LUM}} \lesssim 0.01$.
- Based upon primordial nucleosynthesis baryonic matter contributes: $0.011 \lesssim \Omega_B \lesssim 0.12$.[i]
- Based upon dynamical methods, the mass density associated with bright galaxies is $\Omega_{\mathrm{ABG}} \simeq 0.2 \pm 0.1$ (the ± 0.1 is not meant to be a formal uncertainty estimate).
- There is some evidence that Ω might be close to unity; e.g., analyses of the local peculiar-velocity field based upon the IRAS catalogue of galaxies, and the result of Loh and Spillar.

From this I would make the following inferences:

- ⋆ The dark component of the mass density dominates the luminous component by at least a factor of ten, and closer to a factor of 100 if $\Omega = 1$, and is more diffuse than the luminous component, e.g., the halos of spiral galaxies.
- ⋆ There is strong evidence for the existence of a dark component of baryons. This should not be too surprising since baryons can exist in a variety of low luminosity objects—white dwarfs, neutron stars, black holes, brown dwarfs, jupiters, etc.
- ⋆ At present there is no irrefutable case for a universal mass density that is larger than that permitted for baryons.
- ⋆ If Ω is significantly greater than 0.1—which is already suggested by mass-to-light ratios determined for clusters and the local peculiar velocity field—then there is a strong case for nonbaryonic dark matter. As I will discuss, there are three attractive particle dark-matter candidates whose relic abundance is expected to be close to critical: the axion, the neutralino, and a light neutrino.
- ⋆ If Ω is one, a discrepancy must be explained: why the estimates for the amount of material associated with bright galaxies is a factor of about five smaller. There are

[h] It also assumed that the only change in the baryon-to-photon ratio since the start of nucleosynthesis is the factor of 4/11 decrease caused by the transfer of the entropy in $e^\pm$ pairs to photons when $T \sim m_e/3$.

[i] If Ω is close to unity and the cosmological constant is zero, then h must be close to 0.5 to insure a sufficiently elderly Universe; in this case: $0.04 \lesssim \Omega_B \lesssim 0.12$.

two possibilities. The first, as previously mentioned, the halos of spiral galaxies could extend far enough to account for $\Omega = 1$ (and likewise for clusters). Second, there could be a component of the mass density that is more smoothly distributed, contributes $\Omega_{SM} \simeq 0.8$, and is not associated with bright galaxies; e.g., a population of low-luminosity galaxies that is more smoothly distributed than the bright galaxies—so-called biased galaxy formation—or a relic cosmological constant (more later).

⋆ There may be several dark matter problems—and with different solutions. While the most economical approach is to assume that all dark matter has the same composition, that need not be the case. As mentioned above there is already evidence that some of the baryonic matter is dark. Moreover, if there is indeed a local dark matter problem, its solution must involve "particles" that can dissipate energy and condense into the disk; it is very unlikely that axions, neutralinos, or neutrinos can do so. Taken at face value the observations seem to indicate that there is more dark matter in clusters (per galaxy) than in the halos of spiral galaxies—and if $\Omega = 1$—even more dark matter that is not associated with clusters.

To give a concrete example, consider an $\Omega = 1$, neutrino-dominated Universe ($m_\nu \simeq 92h^2\,\mathrm{eV}$). Because of their high speeds, neutrinos would be unlikely to find their way into potential wells as shallow as those of galaxies or perhaps even clusters. They would likely remain smooth on scales up to the neutrino free-streaming length, $\lambda_{FS} \simeq 40\,\mathrm{Mpc}/(m_\nu/30\,\mathrm{eV})$. The dark matter in galaxies would be baryons—perhaps white dwarfs that formed relatively recently in the local neighborhood and brown dwarfs that formed when the galaxy did in the halo—and the dark matter in clusters would be the neutrinos that eventually made their way into clusters.[j]

A Theoretical Prejudice

While the hard observational evidence for the flat, Einstein–de Sitter model is less than overwhelming, there are several compelling theoretical arguments: (i) the temporal Copernican principle—if $\Omega \neq 1$ the deviation of Ω from unity grows as a power of the scale factor, begging one to ask why Ω is just now beginning to differ from unity; (ii) structure formation—in $\Omega < 1$ models there is less time for the growth of density perturbations and larger initial perturbations are required; in fact, $\Omega < 0.3$ models with adiabatic density perturbations are inconsistent with the isotropy of the CMBR; and (iii) the flat, Einstein–de Sitter model is an inescapable prediction of inflation. To be sure, these arguments are not rooted in hard facts; however, the are sufficiently compelling to create a strong theoretical prejudice for $\Omega = 1$. From this point forward I will adopt this prejudice!

Dark Matter: New Physics or New Particles

Finally, there are some who have suggested another explanation for the dark matter problem: A deviation from Newtonian (Einsteinian) gravity at large distances.[18] Newtonian gravity (i.e., the weak field, slow velocity limit of general relativity) is well tested at distances from order 10^2 cm to the size of the solar system, order 10^{14} cm. However, the dark matter problem involves distance scales of order 10^{23} cm and greater. If gravity were

[j] In a neutrino-dominated Universe it is probably necessary for the dark matter in galaxies to be baryonic, as there seems to be évidence for dark matter in several dwarf galaxies in which there is not enough phase space to contain the necessary numbers of neutrinos.[17]

for some reason stronger on these scales there would perhaps be no need for additional "unseen" matter to explain flat rotation curves. For example, if G were a function of distance, say $G(r) \propto r$, then flat rotation curves would be consistent with constant mass interior to r—eliminating the need for unseen matter.

I opt for unseen matter. First, it seems unlikely that the same functional dependence for the strength of gravity could fit all the observations: While all spiral galaxies have flat rotation curves, the size of the luminous part of the galaxy can vary by almost a factor of ten, and clusters are even larger. Perhaps a more important reason is that of aesthetics: Not only is there no theoretical motivation for such a theory, but it seems difficult, if not impossible, to construct a relativistic theory of gravity in which G increases with distance. The one such theory I am aware is extremely complicated and leads to an unsatisfactory cosmology.[18] Were it the other way around—lack of compelling dark matter candidates and an attractive alternative theory of gravity—I would opt for new physics in the gravitational sector.

II. Why Not Baryons?

Given the existing observational evidence one has to be bold to insist that $\Omega = 1$. Moreover, this assumption seems to require one to go still further and postulate that most of the matter in the Universe is comprised of particles whose existence is still hypothetical! Before taking the big leap, I will comment on the possibility that baryons could contribute the critical density. There are two obstacles to this possibility: the nucleosynthesis constraint, $\Omega_B \lesssim 0.12$; and finding a place to hide the more than 99 invisible baryons for every visible baryon.

A number of different schemes have been suggested to evade the nucleosynthesis bound, for example, massive relic particles that decay into hadrons shortly after nucleosynthesis and initiate a second epoch of nucleosynthesis.[19] This scenario requires an unstable particle species with very special properties, and seems to lead to the overproduction of ^{6}Li and the underproduction of ^{7}Li. Perhaps the most clever idea is the scenario where the baryon-to-photon ratio is reduced after nucleosynthesis because photons suddenly come into thermal contact with "shadow particles" at a lower temperature, which leads to entropy transfer from the photons to the shadow world.[20]

Inhomogeneous Nucleosynthesis

The alternative to the standard scenario that has attracted the most attention is inhomogeneous nucleosynthesis.[21] If the quark/hadron transition is strongly first order and occurs at a relatively low temperature ($\lesssim 125\,\mathrm{MeV}$), baryon number can become concentrated in regions where the quark–gluon plasma persisted the longest. Moreover, due to the difference in the mean free paths of the proton and neutron around the time of nucleosynthesis, the high baryon density regions will become proton rich. Clearly, nucleosynthesis proceeds very differently, and two new parameters arise: the density contrast between the high and low baryon density regions and the separation of the high density regions.

While early calculations, done with two independent "zones" of differing baryon number density and proton fraction, suggested that $\Omega_B \sim 1$ could be made consistent with the observed light element abundances by an appropriate choice of these two parameters, more detailed calculations that allow for diffusion between the zones indicate that the predicted abundances for *all four* light elements conflict with observations if $\Omega_B \sim 1$—for all values

of the two parameters.[22] While this appears to be a sad end to an interesting idea, it does serve to emphasize the brilliant success of standard nucleosynthesis: The simplest model with no extra dials or knobs correctly predicts the primordial abundances of D, ^{3}He, ^{4}He, and ^{7}Li.

Where Is It?

Should one be able to evade the nucleosynthesis bound the next problem that one faces is where to put all those dark baryons. Ordinary stars, dust, and gas would all be "visible" in one way or another. Black holes and neutron stars do not necessarily provide an easy way out either. If, as seems likely, black holes and neutron stars evolve from massive stars, where are the heavy elements these stars produced? And remember, one is trying to hide 99 baryons for every baryon that is in a star. Perhaps massive black holes can form without overproducing heavy elements; however, there are other worries. If these black holes are too massive they will puff up the disk of the galaxy and disrupt binary stars by their gravitational effects, and lead to the (unobserved) lensing of distant QSOs. These considerations restrict the mass of black holes in the halo to be less than about $10^5 M_\odot$.[23]

White dwarfs, brown dwarfs (stars less massive than about $0.08 M_\odot$ which do not get hot enough for to burn hydrogen), or jupiters are better candidates.[24] All could have escaped detection thus far and *might* be detectable in planned experiments to look for microlensing of stars in the LMC by such objects in the halo of our galaxy. However, there is the issue of the large number of these objects needed. When one smoothly extrapolates the observed IMF (initial mass function of the most recent generation of stars) to these very small masses, one concludes that are far too few of these objects to account for the dark matter in the halo. It should be noted that the IMF is an empirical, rather than fundamental, relation, and some have suggested that when the galaxy formed most of its mass could have fragmented into small objects.

To summarize, it is not impossible to evade the nucleosynthesis bound, and there is no devastating argument to preclude astrophysical objects comprised of baryons from contributing critical density. However, the elegance of the nucleosynthesis argument and the difficulty of hiding so many baryons seem to suggest that nonbaryonic dark matter is a more promising option to pursue!

III. Particle Dark Matter

According to the standard cosmology,[25] at times earlier than the epoch of matter–radiation equality, $t \lesssim t_{\rm EQ} = 4.4 \times 10^{10} (\Omega h^2)^{-2}$ sec and $T \gtrsim T_{\rm EQ} = 5.5 (\Omega h^2)$ eV, the energy density of the Universe was dominated by a thermal bath of particles at temperature T. For reference, for $t \lesssim t_{\rm EQ}$, $T \sim {\rm GeV}/\sqrt{t/10^{-6}\,{\rm sec}}$.

While the extrapolation of the standard cosmology to very early times ($t \ll 1$ sec) is a bold step, there are several reasons to expect that such an extrapolation is at least self consistent, if not correct: (1) The splendid success of big bang nucleosynthesis, which tests the standard cosmology well into its radiation-dominated phase; (2) The fact that according to the standard model of particle physics the fundamental degrees of freedom are pointlike quarks and leptons, gauge bosons, and Higgs (scalar) bosons[k] whose interactions

[k] Of course, the existence of the Higgs sector has yet to be confirmed, and there could

are expected to remain perturbatively weak at very high energies; and (3) Quantum corrections to general relativity should be very small for times $t \gg 10^{-43}$ sec and temperatures $T \ll 10^{19}$ GeV.

The implications of this hot, early epoch for cosmology, and dark matter in particular, are manifold: At temperature T all particles of mass less than T should be present in numbers comparable to that of the photons; several phase transitions should take place (e.g., quark/hadron transition, chiral symmetry restoration, and electroweak symmetry restoration); and in the symmetry restored phase, the strength of all interactions—including "very weak" interactions that have yet to be discovered—should be comparable.

While the standard $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge theory of the strong and electroweak interactions does not offer any dark matter candidates—beyond the now dim hope that inhomogeneous nucleosynthesis could resurrect $\Omega_B \sim 1$—the speculations about fundamental physics beyond the standard model do. These well founded speculations include Peccei–Quinn (PQ) symmetry, technicolor, supersymmetry, grand unification, and superstrings. In the context of the hot big bang model these speculations lead to the prediction of various cosmological relics, including particles, topological defects (cosmic strings, domain walls, monopoles, and textures), and the baryon asymmetry of the Universe. The discovery—or nondiscovery—of an expected relic provides an important cosmological window on fundamental physics beyond the standard model. Since terrestrial experiments are hard pressed to probe the physics beyond the standard model, the Heavenly Laboratory has become an indispensable testing ground for fundamental physics.

In the context of the dark matter problem, the implications of theories that go beyond the standard model have great significance. Many of these theories predict particle relics whose contribution to the present mass density is comparable to the critical density! This is no mean feat, and for many of us is a strong hint that the idea of nonbaryonic particle relics as the dark matter is on the right track.

I have organized my discussion of particle dark-matter candidates into six broad categories: thermal relics; "skew" relics; axions; nonthermal relics; "significant-other" relics; and exotic relics. I have given the axion is own category not just because it is my favorite candidate, but also because the story of relic axions is a very rich one and spans three categories!

Thermal Relics

Because the Universe was in thermal equilibrium at early times essentially all the known particles—and perhaps many particles that are yet to be discovered—were present in great abundance: When the temperature T was greater than the mass m of a species, a number comparable to that of the photons If thermal equilibrium were the whole story, it would be a very uninteresting one indeed: At low temperatures the equilibrium abundance of a species is exponentially negligible, a factor of order $(m/T)^{3/2}\exp(-m/T)$ less than that of the photons.

A massive particle species can only maintain its equilibrium abundance so long as the rate for interactions that regulate its abundance is greater than the expansion rate of the Universe: $\Gamma \gtrsim H$, where the expansion rate of the Universe $H = 1.67 g_*^{1/2} T^2/m_{\rm Pl}$ (g_* counts

well be some surprises at energies greater than $1/\sqrt{G_F} \sim 300$ GeV, corresponding to times earlier than 10^{-11} sec.

the total number of degrees of freedom of all relativistic species and $m_{\rm Pl} = 1.22\times10^{19}$ GeV). The expansion rate enters because it sets the rate at which the temperature is decreasing, $H = |\dot{T}|/T$, and therefore the rate at which phase-space distribution functions must change.

If we specialize to the case of interest for particle dark matter, a stable (or very long lived) particle, the reactions that control the abundance are pair production and annihilation, and their rates are related by detailed balance. The problem now reduces to a textbook example! The particle's number density n is governed by the Boltzmann equation, which takes the form[26]

$$\frac{dn}{dt} + 3Hn = -\langle\sigma|v|\rangle_{\rm ANN}\left(n^2 - n_{\rm EQ}^2\right), \tag{11}$$

where $\langle\sigma|v|\rangle_{\rm ANN}$ is the thermally averaged annihilation cross section times relative velocity and $n_{\rm EQ}$ is the equilibrium number density. It is more convenient to recast Eq. (11) in terms of the number of particles per comoving volume,[l] $Y = n/s$, where $s = 2\pi^2 g_* T^3/45$ is the entropy density, and the dimensionless evolution variable $x = m/T$:

$$\frac{dY}{dx} = -\frac{x\,s\,\langle\sigma|v|\rangle_{\rm ANN}}{H(T=m)}\left(Y^2 - Y_{\rm EQ}^2\right), \tag{12}$$

where $Y_{\rm EQ} = 0.278 g_{\rm eff}/g_*$ (for $x \ll 3$) and $0.145(g/g_*)x^{3/2}\exp(-x)$ (for $x \gg 3$), g is the species' number of internal degrees of freedom, and $g_{\rm eff} = g$ (for bosons) or $0.75g$ (for fermions). Eq. (12) is a particular form of the Ricatti equation that has no closed form solutions; it can be solved easily by approximation or numerical integration. I will highlight the evolution of a species' abundance.

Roughly speaking, the abundance tracks equilibrium until "freeze out," which occurs at temperature T_F, defined by $\Gamma = H$, where $\Gamma = n_{\rm EQ}\langle\sigma|v|\rangle_{\rm ANN}$ is the annihilation rate per particle. After that, annihilations cannot keep pace with the decreasing equilibrium abundance ("they freeze out"), and thereafter the number of particles per comoving volume remains roughly constant, at approximately its equilibrium value at freeze out: $Y_\infty \simeq Y(T_F)$. The mass density contributed by the relic particles today is

$$\rho = mY_\infty s_0 \qquad \text{or} \qquad \Omega h^2 = 0.28 Y_\infty (m/\,\text{eV}), \tag{13}$$

where $s_0 \simeq 7.1 n_\gamma \simeq 2970\,\text{cm}^{-3}$ is the present entropy density.

Hot and cold relics

There are two limiting cases: *hot* relics—species whose annihilations freeze out while they are still relativistic ($x_F \lesssim 3$); and *cold* relics—species whose annihilations freeze out while they are nonrelativistic ($x_F \gtrsim 3$). For a hot relic the present abundance is comparable to that of the photons, i.e., Y is of order unity. The weak interactions keep ordinary neutrinos in thermal equilibrium until a temperature of a few MeV; thus a neutrino species lighter than a few MeV is a hot relic, and

$$Y_\infty = \frac{0.278 g_{\rm eff}}{g_*(T_F)} \simeq 3.9\times10^{-2} \qquad \Omega_\nu = \frac{m_\nu}{92h^2\,\text{eV}}. \tag{14}$$

[l] In the absence of appreciable entropy production, the entropy per comoving volume $S \equiv R^3 s$ is conserved, implying that $s \propto R^{-3}$; thus the number of particles per comoving volume $N \equiv R^3 n \propto n/s$.

(There is an intermediate regime, referred to as *warm* relics; in this case the freeze out temperature is sufficiently high so that $g_*(T_F) \gg 1$ and Y_∞ is significantly less than order unity. For example, if $T_F \gtrsim 300\,\mathrm{GeV}$, g_* is at least 106.75, which is the total number of degrees of freedom in the standard model, and for a fermion with two degrees of freedom, e.g., a light axino or gravitino, $\Omega = m/910h^2\,\mathrm{eV}$.)

Freeze out for a cold relic occurs when the species is very nonrelativistic and the species' present abundance is significantly less than that of photons ($Y_\infty \ll 1$). In this very interesting case the relic abundance is inversely proportional to the annihilation cross section,

$$Y_\infty \sim \frac{4x_F/\sqrt{g_*}}{m m_{\mathrm{Pl}} \langle\sigma|v|\rangle_{\mathrm{ANN}}}, \tag{15}$$

where freeze out occurs for $x = x_F \simeq \ln[0.04 m_{\mathrm{Pl}} m \langle\sigma|v|\rangle_{\mathrm{ANN}} g/\sqrt{g_*}]$. In most cases of interest freeze occurs at $x_F \sim 20 - 30$, corresponding to $T_F \sim m/20 - m/30$ (in any case x_F only varies logarithmically).

This is a rather remarkable result: The relic abundance varies inversely with the strength of the species' interactions—implying that the weak shall prevail! Moreover, specifying that the species provides the critical density determines the annihilation cross section: $\langle\sigma|v|\rangle_{\mathrm{ANN}} \sim 10^{-37}\,\mathrm{cm}^2$—roughly that of the weak interactions!

Massive neutrinos

The simplest example of a cold relic is a "heavy" neutrino (mass greater than a few MeV). Provided its mass is less than that of the Z^0 boson, $\langle\sigma|v|\rangle_{\mathrm{ANN}} \sim G_F^2 m^2$ and

$$Y_\infty \simeq 6 \times 10^{-9} \left(\frac{m}{\mathrm{GeV}}\right)^{-3} \qquad \Omega_\nu h^2 \simeq 3 \left(\frac{m}{\mathrm{GeV}}\right)^{-2}. \tag{16}$$

That is, the relic abundance of stable neutrino whose mass is a few GeV would provide closure density. Since none of the three known neutrino species can be this massive and the SLC/LEP results rule out a fourth neutrino (unless it is heavier than about 45 GeV), this result, first discussed by Lee and Weinberg,[27] is only an interesting example.

For a neutrino more massive than about 100 GeV the annihilation cross section begins to decrease as m^{-2}, due to the momentum dependence of the Z^0 propagator. In this regime $Y_\infty \propto m$, and Ωh^2 varies as m^2, increasing to order unity for a mass of order a few TeV.[28]

Bringing everything together, the relic mass density of a stable neutrino species increases as m up to a mass of a few MeV; it then decreases as m^2 up to a mass of order 100 GeV; and finally it increases as m^2 for larger masses. A stable neutrino species can contribute critical density for three values of its mass: $\mathcal{O}(100\,\mathrm{eV})$; $\mathcal{O}(1\,\mathrm{GeV})$; and $\mathcal{O}(\,\mathrm{TeV})$. This behavior is generic for a particle whose annihilations proceed through a massive boson (here the Z^0).

Griest and Kamionkowski[29] have generalized this result. Unitarity provides a bound on the annihilation cross section of any pointlike species: $\langle\sigma|v|\rangle_{\mathrm{ANN}} \leq 8\pi/m^2$. This implies a lower bound to Ωh^2 that increases as m^2; requiring that Ωh^2 be no larger than one (based upon the age of the Universe[30]) results in an upper bound of 340 TeV to the mass of any stable, pointlike species.

Neutralinos

A more viable cold relic is the lightest supersymmetric partner or LSP. In supersymmetric extensions of the standard model a discrete symmetry, R-parity, is usually imposed (to ensure the longevity of the proton); it also guarantees the stability of the LSP. In most supersymmetric extensions of the standard model the LSP is the (lightest) neutralino (it could in principle be the sneutrino or gluino). The neutralino(s) are the four mass eigenstates that are linear combinations of the Bino, Wino, and two Higgsinos. In many models discussed early on, especially ones where the LSP was relatively light, the neutralino (by which I mean the lightest neutralino) was almost a pure photino state, and thus was referred to as the photino.

The minimal supersymmetric extension of the standard model has a number of parameters that must be specified: μ and M, two soft supersymmetry breaking mass parameters which are expected to be 100 GeV to few TeV; $\tan\beta = v_2/v_1$, the ratio of the two Higgs vacuum expectation values; the top quark mass; and the scalar quark and scalar lepton masses.[31] These parameters determine the composition of the neutralino, its mass, and its interactions. The parameter space of supersymmetric models is multidimensional and cumbersome to deal with.

To determine the relic neutralino abundance *all* one has to do is calculate the cross section for neutralino annihilation (the neutralino is a Majorana fermion). For a neutralino that is lighter than the $W^\pm$ boson, the final states are fermion–antifermion pairs and light Higgs bosons. For the most general neutralino this task has been done by Griest.[32] For neutralinos that are heavier than the $W^\pm$ boson, many additional final states open up: W^+W^-, Z^0Z^0, HH, HW, and HZ. This complicated cross section has been calculated by Kamionkowski and his collaborators.[33] Let me summarize the salient points.

- Because the scale of supersymmetry breaking is roughly of order the weak scale, "spartner" masses are of order the weak scale; since the interactions of the neutralino with ordinary matter involve the exchange of spartners, $W^\pm$ bosons, Higgs bosons, or Z^0 bosons, the neutralino's interactions are roughly weak in strength. Many of the qualitative features of the relic neutralino abundance are the same as for a neutrino.
- Over almost the entirety of the parameter space of the minimal supersymmetric extension of the standard model the relic neutralino abundance $\Omega_\chi h^2$ is greater than 10^{-3}; and in large regions of parameter space $\Omega_\chi h^2$ is of order unity. This of course traces to the fact that the neutralino's interactions with ordinary matter are roughly weak, and makes the neutralino a rather compelling dark matter candidate.
- Neutralinos can provide the critical density for masses from order 10 GeV to order 3 TeV (depending upon the model parameters). Fixing some of the parameters and examining $\Omega_\chi h^2$ as function of m_χ reveals a similar behavior as for neutrinos: $\Omega_\chi h^2 \sim 1$ for a mass in the GeV range and for a mass in the TeV range.
- Just as with a heavy neutrino, for large neutralino masses the annihilation cross section decreases as $1/m_\chi^2$; this results in a maximum neutralino mass that is cosmologically acceptable: 3.5 TeV. For $m_\chi \geq 3.5$ TeV, $\Omega_\chi h^2$ is greater unity for all models.
- Finally, the parameter space of models is constrained by unsuccessful accelerator-based searches for evidence of supersymmetry. Broadly speaking, the failure to find any evidence for supersymmetry has slowly pushed the expected mass of the neutralino upward.[34]

How accurately are relic abundances known?

Calculating the relic abundance of a species that was once in thermal equilibrium has become a routine chore for the particle cosmologist. Because of the importance of this calculation, it is prudent to consider the inherent uncertainties. They are easy to identify.[35] Recall that freeze out involves the competition between the expansion rate and the annihilation rate. The annihilation rate as a function of temperature is determined by the properties of the species—and is thus a given. In calculating the expansion rate we have assumed that the Universe was radiation dominated at freeze out; further we assumed that there was no entropy production since freeze out, so that Y_∞ remains constant.

- If there the entropy per comoving volume increased by a factor of γ after freeze out, then the relic abundance Y_∞ is *decreased* by the same factor γ. Entropy release could occur in a first-order phase transition, or through the out-of-equilibrium decay of a massive particle species.
- Additional forms of energy density in the early Universe (e.g., scalar fields, or shear) serve to increase the expansion rate at fixed temperature. This in turn leads to an earlier freeze out, at a larger abundance. Increasing $H(T)$ then can *increase* Y_∞. While we can be confident that the Universe was radiation dominated by the epoch of nucleosynthesis, freeze out for most dark matter candidates occurs earlier, at a time when we cannot exclude the possibility that there were additional contributions to the energy density.

Skew Relics[36]

In discussing thermal relics I tacitly assumed that the abundance of the particle and its antiparticle were equal. For a Majorana fermion (like the neutralino) this is necessarily so; a Dirac fermion (or a scalar species) can carry a conserved (or at least approximately conserved) quantum number, and if the net particle number is sufficiently large it will determine the relic abundance of the species. Baryon number provides a simple example; if there were no net baryon number, baryons and antibaryons would annihilate down to a relic abundance $n_b/s = n_{\bar{b}}/s \simeq 10^{-19}$, which is significantly smaller than that observed, $n_b/s \simeq \eta/7 \sim 10^{-10}$. As is well appreciated the relic baryon abundance is determined by the net baryon number: $n_b/s = n_B/s$ (the net baryon number density $n_B = n_b - n_{\bar{b}}$).

The same can occur for any species whose net particle number is conserved, e.g., a heavy Dirac neutrino whose net particle number is conserved because of conservation of family lepton number. Denote the net particle number per comoving volume by n_L/s (L for lepton number). Since the relic abundance cannot be less than the net particle number, it follows roughly that: If the net particle number is greater than the would-be freeze out abundance, the relic abundance is determined by it, $Y_\infty = n_L/s$; on the other hand, if the net particle number is smaller than the would-be freeze out abundance, the net particle number plays no important role and the relic abundance is given by the usual freeze out abundance, $Y_\infty \simeq Y(x_F)$.

In the case that the relic abundance is determined by the net particle number

$$\Omega h^2 = \left[\frac{n_L/s}{10^{-10}}\right]\left(\frac{m}{35\,\mathrm{GeV}}\right); \tag{17}$$

that is, a particle species of mass 35 GeV with a net particle number comparable to the baryon asymmetry would contribute the critical density.

Axions

Peccei-Quinn (PQ) symmetry with its attendant pseudo-Nambu-Goldstone boson—the axion—remains the most attractive and promising solution to the strong-CP problem.[37] Moreover, the axion arises naturally in supersymmetric and superstring models. One might call PQ symmetry and the axion the simplest and most compelling extension to the standard model!

The axion mass and PQ symmetry breaking scale are related by

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{f_\pi m_\pi}{(f_a/N)} \simeq \frac{0.62\,\mathrm{eV}}{(f_a/N)/10^7\,\mathrm{GeV}}, \tag{18}$$

where f_a is the PQ symmetry breaking scale, $z \simeq 0.56$ is the ratio of the up to down quark masses, f_π and m_π are pion decay constant and mass, and N is the color anomaly of PQ symmetry. At present there is little theoretical guidance as to the key parameter: the axion mass, although a variety of astrophysical and cosmological arguments leave open only two "windows" for the axion mass:[38] 10^{-6} eV to 10^{-3} eV and 3 eV to 8 eV (hadronic axions only).

Relic axions arise due to three distinct mechanisms: thermal production[39]—for an axion of mass greater than about 10^{-4} eV axions thermalize shortly after the QCD transition and, today, like neutrinos, should have a relic abundance of order $30\,\mathrm{cm}^{-3}$; and two coherent processes, the "misalignment" mechanism[40] (see below) and axionic string decay[41]—since PQ symmetry breaking involves the spontaneous breakdown of a global $U(1)$ symmetry, strings are produced; they decay by radiating (among other things) axions. While the thermal population of axions dominates for axion masses greater than about 10^{-2} eV, there are strong astrophysical constraints in this mass range which preclude an axion more massive than about 8 eV. Thus, thermal axions can contribute at most 10% of critical density (more later on thermal axions).

For axion masses greater than about 10^{-2} eV misalignment and axionic string decay are the dominant production processes, and sufficient numbers of axions can be produced to provide closure density. The importance of axionic string decay is still a matter of intense debate. It seems to be agreed that axion production through this mechanism is somewhere between being comparable to and about 100 times more important than the misalignment mechanism,[41] further that if the Universe inflated either before or during PQ symmetry breaking, the number of axions produced by axionic strings is negligible. In the "no inflation" case, if axionic string decay is as potent as is claimed by some authors, axions provide the critical density for an axion mass of about 10^{-3} eV.

Let me briefly describe the misalignment mechanism. The free energy of the vacuum depends upon the axion field because this field modulates the phase of the instanton amplitude. At low temperatures the free energy has a maximum value of about Λ_{QCD}^4, is periodic in the "axion angle" $\theta \equiv a/(f_a/N)$, and is minimized at a value of $\theta = 0$. The mass of the axion is determined by the curvature of the free energy at $\theta = 0$ and is given approximately by Eq. (18). At high temperatures instanton effects are strongly suppressed, and for $T \gg \Lambda_{\mathrm{QCD}}$ the free energy is essentially independent of the axion field. Thus, when PQ symmetry breaking occurs ($T \sim f_a$), no value of the axion angle is singled out dynamically, and one expects that the value of the axion angle in different causally distinct regions will be randomly distributed between $-\pi$ and π. Thus the primeval energy density

associated with the misalignment of the axion field should be of order Λ_{QCD}^4. Around a temperature of order Λ_{QCD} instanton effects become potent, and the axion mass starts to "turn on." When the axion mass exceeds $3H$ the axion field will begin to relax toward $\theta = 0$. Because it has no efficient way to shed energy, the field is left oscillating. The energy density in oscillations of the axion field behaves as nonrelativistic matter during the subsequent evolution of the Universe, and may be interpreted in particle language as a gas of zero-momentum axions.

The contribution of these axions to the present mass density of the Universe is estimated to be[40]

$$\Omega_a h^2 \simeq 0.13 \times 10^{\pm 0.4} \Lambda_{200}^{-0.7} f(\theta_1^2)\theta_1^2 (m_a/10^{-5}\,\mathrm{eV})^{-1.18}. \tag{19}$$

where $\Lambda_{QCD} = \Lambda_{200} 200\,\mathrm{MeV}$, and θ_1 is the initial misalignment angle. The function $f(\theta_1^2)$ accounts for anharmonic effects, and is of order unity (and specifically $f \to 1$ for $\theta_1 \ll 1$). The $10^{\pm 0.4}$ factor is an estimate of theoretical uncertainties—e.g., in the temperature dependence of the axion mass. Provided that $\theta_1 \sim \mathcal{O}(1)$ closure density in axions is achieved for a mass somewhere between $10^{-6}\,\mathrm{eV}$ and $10^{-4}\,\mathrm{eV}$, and for a mass less than about $10^{-6}\,\mathrm{eV}$ axions "overclose" the Universe.[m]

The unusual dependence of the axion energy density upon the axion mass is easily understood. Regardless of the value of the axion mass, the energy density associated with the initial misalignment of the axion field is of order Λ_{QCD}^4; once the axion field starts to oscillate that energy density red shifts as R^{-3}. The axion field begins to oscillate when the axion mass $m_a(T) \simeq 3H$: For smaller masses the axion oscillations begin later, and the energy density trapped in the misalignment of the axion field is diminished less.

Since the initial misalignment angle θ_1 is a random variable, at the time of PQ symmetry breaking the value of θ_1 will be different and uncorrelated in different causally distinct regions of the Universe. In the absence of inflation, these different regions are very small, and today the Universe is comprised of a very large number of regions that each had a different value of θ_1. To obtain the average axion energy density, one uses the *rms* average of θ_1, which is just $\pi/3$, in Eq. (19). In this circumstance axions provide closure density for a mass in the range of $10^{-6}\,\mathrm{eV}$ to $10^{-4}\,\mathrm{eV}$.

If the Universe inflated before or during PQ symmetry breaking the fluctuations in the axion field take an entirely different form. While the average of θ_1^2 over many causally-separate volumes is still $\pi/3$, the practical relevance of this fact is nil, because the entire presently observable Universe lies within one causal region where θ_1 is constant. A number of authors[42] have pointed out that an axion of mass smaller than $10^{-6}\,\mathrm{eV}$ could lead to $\Omega_a \sim 1$, provided that θ_1 was sufficiently small:

$$\theta_1 \simeq h(m_a/10^{-6}\,\mathrm{eV})^{0.59}. \tag{20}$$

In this case, then, we would be living in a rare, axion-poor region of the Universe. If the Universe did indeed undergo inflation, the fundamental laws of physics do not determine

[m] Overclose is not completely accurate; if the Universe is open, the production of axions—or any other particle—cannot change the geometry and close it. More precisely, a larger value of Ωh^2 leads to an earlier epoch of matter–radiation equality and ultimately to a more youthful Universe. Requiring that the Universe be at least 10 Gyr old and $h \gtrsim 0.4$ constrains $\Omega h^2 \lesssim 1$.[30]

θ_1. Despite its cosmic import the local value of this parameter is an "historical accident," and can only be determined through direct measurement of $\Omega_a h^2$ and m_a.[n]

Nonthermal Relics

The axion provides *two* examples of how a relic can be produced coherently rather than thermally: the misalignment mechanism and axionic string decay. For both of these processes the number of axions produced is highly superthermal, as is clear since theses productions mechanism dominate thermal production for $m_a \lesssim 10^{-2}$.

There are other examples of nonthermal relics. The most familiar is the superheavy magnetic monopole. The monopole is a topologically nontrivial configuration of gauge and Higgs fields. Monopoles are produced as topological defects in a symmetry breaking phase transition where a semi-simple group $\mathcal{G}$ is broken down to a smaller group $\mathcal{H}$ that contains a $U(1)$ factor; e.g., $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$. Because of the finite size of the particle horizon in the standard cosmology, after symmetry breaking the Higgs field can only be correlated on distance scales less than $H^{-1} \sim ct$—and thus must be uncorrelated on larger scales. Because of this fact of order one monopole per horizon volume will be produced. Monopole annihilation is ineffective, the monopoles produced should be with us today. This production process, which relieves on the fact that the Higgs field cannot be correlated on scales larger than the horizon, implies quite generally that order one topological defect per horizon volume should arise in a phase transition. It is known as the "Kibble mechanism."

For the simplest symmetry breaking patterns GUT monopoles are so copiously produced by the Kibble mechanism that they overclose the Universe by a factor of about 10^{10}! Moreover, there are other stringent astrophysical bounds to their relic abundance. Inflation solves the monopole problem by expanding the horizon to a size that is larger than our present Hubble volume, and thus predicts less than one monopole in the Universe due to the Kibble mechanism.

More complicated symmetry breaking schemes can reduce the relic monopole abundance to an acceptable level; and it is possible that significant numbers of monopoles can be produced as thermal pairs after inflation. It is very difficult to make a sensible prediction for the relic abundances of monopoles; however, magnetic monopoles of mass 10^{19} GeV could provide closure density and have a flux that is consistent with all the astrophysical constraints.[44]

There are other examples of nonthermal relics, including soliton stars.[45] Soliton stars are regions of false vacuum that are stabilized by dynamics rather than topology. (By contrast, magnetic monopoles, domain walls, and cosmic string are regions of false vacuum that are stable for topological reasons.) For example, imagine a closed region of false vacuum associated with a scalar field ϕ. Such a region is unstable and should collapse. However, if there are particles inside this region whose mass when they are in the false vacuum is *less* than when they are in the true vacuum, they can exert pressure and stabilize the region. Whether soliton stars are an interesting dark matter candidate remains to be

[n] One might then be left with the impression that if the Universe underwent inflation, any axion mass can provide closure density provided that θ_1 is appropriately small. Additional, very important constraints emerge when fluctuations in the axion field that arise during inflation are taken into account.[43]

seen.

"Significant-Other" Relics

Up to this point I have focused on relics that contribute the critical density. A relic from the early Universe can be interesting and significant even if it contributes only a fraction of the critical density; e.g., most cosmologists consider baryons ($\Omega_B \sim 0.1$) and microwave photons ($\Omega_\gamma \sim 10^{-4}$) to be interesting relics, in spite of their small contributions to Ω! I will use the term "significant-other" for such relics.

I will mention two possible significant-other relics: a neutralino and an axion of mass 3 eV to 8 eV. While it is possible that the neutralino contributes the critical density, it need not be the case. However, in the minimal supersymmetric extension of the standard model, the neutralino contributes at least 0.1% of the critical density; thus, if Nature exhibits low-energy supersymmetry, the neutralino is at the very least a significant-other relic! Moreover, efforts to directly detect relic neutralinos could still be successful even if they are only a significant-other relic. Needless to say the implications of their discovery for cosmology and particle physics would be almost as profound.

Axions of mass 3 eV to 8 eV arise as thermal relics and would contribute only about 1% of the critical density. Such an abundance is sufficient to permit their detection through their decay to two photons.[46] The axion mean lifetime

$$\tau(a \to 2\gamma) \simeq 6.8 \times 10^{24}\, \xi^{-2}\, (m_a/\,\mathrm{eV})^{-5}\,\mathrm{sec}, \tag{21}$$

where $\xi \equiv [E/N - 2(z+4)/3(z+1)]/0.72 \simeq (E/N - 1.95)/0.72$ and E is the electromagnetic anomaly of PQ symmetry. In the simplest axion models, $E/N = 8/3$ and $\xi = 1$.

Relic thermal axions will fall into the various potential wells that develop in the Universe as structure formation proceeds. Today they will be found in extended structures such as the halos of galaxies and clusters of galaxies, as they cannot dissipate energy and collapse further. They will decay and produce photons of wavelength $\lambda_a \simeq 24800\,\text{Å}/(m_a/\,\mathrm{eV})$. This radiation will be Doppler-broadened due to the velocities that axions have in these objects—for galaxies $\Delta\lambda/\lambda \simeq v/c \sim 10^{-3}$ and for clusters $\Delta\lambda/\lambda \simeq v/c \sim 10^{-2}$—and for distant objects the line will also be red shifted. The most favorable case for their detection is to search for the radiation from decaying axions in clusters. The intensity of the axion line is approximately[47]

$$I_{\mathrm{cluster}} \sim 10^{-17}\, \xi^2\, (m_a/3\,\mathrm{eV})^7\, \mathrm{erg\,cm^{-2}\,arcsec^{-2}\,\text{Å}^{-1}\,s^{-1}}/(1+z_c)^4,$$

where z_c is the red shift of the cluster.

The background against with which this line must compete is the "night sky," which at a ground-based observatory is dominated by the glow of the atmosphere and includes many strong lines. The baseline intensity of the night sky is $10^{-17}\,\mathrm{erg\,cm^{-2}\,arcsec^{-2}\,\text{Å}^{-1}\,s^{-1}}$. By subtracting "off-cluster" measurements from "on-cluster" measurements one can eliminate the night-sky background. This past May, two students, M. Ted Ressell and Matthew Bershady, and I used the 2.1 m telescope at Kitt Peak to search for axion radiation in three clusters using this technique. The spectra we took span 3600 Å to 8600 Å with 10 Å resolution. Our "on-off" subtractions allow us to search for such a line with a sensitivity of less than 3% of the night sky for the mass range from 3.1 eV to 7.9 eV. Our results should be ready for publication this December.[48]

Exotic Relics

Thus far I have focused on particle relics that today would behave like ordinary nonrelativistic matter. There are more exotic possibilities. Since the amount of matter associated with bright galaxies seems to contribute only 20% of critical, and a strong theoretical prejudice for $\Omega = 1$ exists, several relics have been suggested that today would contribute an almost uniform energy density of 80% of the critical density. A uniform contribution to the mass density would not show up in the dynamical measurements, thereby solving the "Ω problem." The exotic candidates include a relic cosmological constant,[49] very light cosmic strings that are either fast moving or exist in a tangled network,[50] or relativistic particles produced by the recent decays of a massive relic.[51] Whether or not we have to resort to such exotics to savage our strong prejudice remains to be seen.

IV. Implications for Structure Formation in the Universe

According to the standard cosmology, structure formation proceeds via the Jeans (or gravitational) instability: Small primeval density perturbations begin to grow once the Universe becomes matter dominated, and then develop into the structure that we observe today. The structure-formation problem is essentially an initial data problem: Specify the primeval density perturbations and the quantity and composition of matter, and let it go!

We now have well motivated suggestions for both pieces of initial data.[52] For the density perturbations, there are several choices: inflation-produced, constant-curvature (Harrison–Zel'dovich) perturbations; inflation-produced, isocurvature perturbations; and topological relics, such as cosmic strings or texture, as the seed perturbations. For the matter content, there are the following suggestions: $\Omega = 1$, $\Omega_B \sim 0.1$, and $\Omega_X \sim 0.9$, where generically X is hot dark matter (a light neutrino species), cold dark matter (axions, neutralinos, magnetic monopoles, ...), or perhaps warm dark matter (a keV mass particle, such as an axino,[53] righthanded neutrino, or gravitino).[o]

The suggestion that weakly interacting relic particles comprise the bulk of the mass density of the Universe and contribute $\Omega \sim 1$ has been a particularly important one, and virtually all scenarios of structure formation now include nonbaryonic dark matter. For good reason; in a "particle dark-matter" Universe density perturbations can begin growing as soon as the Universe becomes matter dominated, while in a baryon-dominated Universe density perturbations cannot begin to grow until decoupling. Further, linear perturbations in a low-Ω model cease growing at a red shift $z \sim \Omega^{-1}$. Thus, in a low-Ω model larger amplitude perturbations are required. Low-Ω models with curvature-perturbations conflict with the observed isotropy of the CMBR if $\Omega \lesssim 0.3$.

Two "stories" of structure formation have been studied in some detail: hot dark matter and cold dark matter (both with inflation-produced, constant-curvature perturbations). Hot dark matter seems to be ruled out, as galaxies form too late.[54] Cold dark matter is the most successful paradigm for structure formation yet proposed.[55] Other scenarios

[o] In the context of structure formation, hot, warm, and cold refer to the velocity dispersion of the relic particles around the time of matter–radiation equality; hot corresponds to relativistic and cold to very nonrelativistic. For thermal relics this matches the previous nomenclature; in general nonthermal relics have very small velocity dispersions and behave like cold dark matter.

involving cosmic strings and texture are presently less well developed. In any case, the "hints from the early Universe" as to the initial data for structure formation have served well to bring this problem into sharper focus. Next, I will digress briefly to discuss my candidate for the "best-fit model" of the Universe.

The Best-fit Universe

Cold dark matter does a remarkably good job of describing the Universe on scales less than about $20h^{-1}$ Mpc. However, it appears to have a number of shortcomings: deficient large-scale structure, deficient galaxy counts, the age problems, and the Ω problem. No one of these problems is sufficiently troublesome to falsify the cold dark matter paradigm—yet—but taken together they are worrisome. As we shall see, the addition of a cosmological constant simultaneously addresses all of these problems.

As a reference point, the conventional cold dark matter scenario is: a flat Universe whose composition is $\Omega_B \sim 0.1 \ll \Omega_{CDM} \sim 0.9$, with $h \sim 0.5$ (to have a sufficiently old Universe) and inflation-produced Harrison–Zel'dovich curvature perturbations whose spectrum after the epoch of matter–radiation equality is[55]

$$|\delta_k|^2 = \frac{A\,k}{(1+\beta k+\omega k^{1.5}+\gamma k^2)^2}. \tag{22}$$

Here δ_k is the amplitude of the Fourier component of comoving wavenumber k ($\equiv 2\pi/\lambda$), A is an overall normalization constant, $\beta = 1.7(\Omega h^2)^{-1}$ Mpc, $\omega = 9.0(\Omega h^2)^{-1.5}\,\mathrm{Mpc}^{1.5}$, and $\gamma = 1.0(\Omega h^2)^{-2}\,\mathrm{Mpc}^2$.

The basic idea of the best-fit model is simple; retain the flatness, but add a cosmological constant.[49,56] The model I discuss here is: (i) Hubble constant of around $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ ($h = 0.7$)—a nice compromise value; (ii) $\Omega_B \sim 0.03$—near the central value implied by nucleosynthesis; (iii) $\Omega_{CDM} \sim 0.17$—sufficiently greater than the baryonic component so that the mass density is dominated by that of the cold dark matter; (iv) Ω_Λ—cosmological constant corresponding to an energy density $\rho_\Lambda \equiv \Omega_\Lambda \rho_{CRIT} \simeq 3.2\times 10^{-47}\,\mathrm{GeV}^4 = (2.4\times 10^{-3}\,\mathrm{eV})^4$. I am not wed to these particular values and I simply use this set for definiteness. (If the ratio of the mass densities of CDM and baryons is somewhat smaller, then the decoupling of matter and radiation can have an effect on the spectrum of density perturbations, which is to boost power on large scales.[57] If the "best-fit model" is still deficient in large-scale power, this effect could improve the situation.)

For this model the total matter contribution $\Omega_{NR} = 0.2$, and today the vacuum energy density dominates the matter energy density by a factor of four. In general the ratio $\rho_{NR}/\rho_\Lambda = 0.25(1+z)^3$. At red shifts greater than about $z_\Lambda \simeq 0.59$ the matter energy density dominates, and the model behaves just a flat, CDM model. To determine when this model becomes matter dominated one simply sets $\Omega h^2 = \Omega_{NR} h^2 \simeq 0.098$: $T_{EQ} = 0.54\,\mathrm{eV}$; $t_{EQ} \simeq 4.5\times 10^{12}$ sec; and $z_{EQ} \simeq 2300$. Once the radiation energy density is negligible ($z \ll z_{EQ}$), the scale factor evolves as

$$R(t) = \left(\frac{\Omega_{NR}}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3}\left(3\sqrt{\Omega_\Lambda}H_0 t/2\right), \tag{23}$$

where the value of the scale factor today is taken to be one.

The Ω problem

A cosmological constant behaves just like a uniform mass density (with equation of state $p = -\rho$). As such, it would not affect determinations of Ω based upon dynamics (galactic halos and cluster virial masses). These measurements of the masses of tightly bound systems are insensitive to the contribution of a uniform background energy density because the average density in these objects is much greater than the average density of the Universe. Likewise, determinations of Ω based upon the peculiar velocities induced by the clumpy matter distribution would only reveal the clumpy, matter component. Thus, all current dynamical determinations that indicate $\Omega \simeq 0.1 - 0.3$, would be consistent with a flat Universe ($\Omega = 1$) with $\Omega_{\rm NR} = 0.2$.

The age problems

As is well appreciated the addition of a cosmological constant *increases* the age of a flat Universe. The age of a Λ model is

$$t(z) = \frac{2H_0^{-1}}{3\sqrt{\Omega_\Lambda}} \sinh^{-1}\left[\sqrt{\Omega_\Lambda/\Omega_{\rm NR}}/(1+z)^{3/2}\right]; \tag{24a}$$

$$t_0 \equiv t(z=0) = \frac{2H_0^{-1}}{3\sqrt{\Omega_\Lambda}} \sinh^{-1}\left[\sqrt{\Omega_\Lambda/\Omega_{\rm NR}}\right] = \frac{2H_0^{-1}}{3\sqrt{\Omega_\Lambda}} \ln\left[\frac{1+\sqrt{\Omega_\Lambda}}{\sqrt{\Omega_{\rm NR}}}\right]. \tag{24b}$$

The present age of a Λ-model is always greater than $2H_0^{-1}/3$ and for $\Omega_\Lambda = 0.8$, $t_0 = 1.1H_0^{-1} \simeq 15.5\,$Gyr, an age which is comfortably consistent with the age as determined from the radioactive elements, from the oldest globular clusters, and from white dwarf cooling (e.g., see Ch. 1 of Ref. 52 and references therein). Moreover, a Λ model is older than its matter-dominated counterpart at any given epoch, so that objects at a given red shift have had more time to evolve. For $z \gg z_\Lambda$, $t(z) \to 2H_0^{-1}/3\sqrt{\Omega_{\rm NR}}(1+z)^{3/2}$, which is a factor of $\Omega_{\rm NR}^{-1/2}$ older than a flat, matter-dominated model; at these early epochs the "best-fit model" is a factor of 1.6 older than the conventional CDM model.

Large-scale structure

The spectrum of density perturbations at matter-radiation equality, $(\delta M/M) \propto k^{3/2}|\delta_k|$, decreases monotonically with λ and its wavelength scale is determined by the value of Ωh^2. The spectrum "shifts" to larger length scales as Ωh^2 is decreased. Supposing that the spectrum is normalized on the scale $\lambda = 8h^{-1}\,$Mpc (a common normalization is: $\delta M/M \simeq 1$ for $\lambda \simeq 8h^{-1}\,$Mpc), decreasing Ωh^2 increases the power on all scales greater than the normalization scale. Put another way, the ratio of the characteristic scale in the spectrum, $\lambda_{\rm EQ} = 13(\Omega h^2)^{-1}\,$Mpc, to the scale of nonlinearity in the Universe, $\lambda_{\rm NL} \simeq 8h^{-1}\,$Mpc, is $\lambda_{\rm EQ}/\lambda_{\rm NL} \simeq 1.6/\Omega h$; in the "best-fit model" this ratio is a factor of 3.5 greater than in a model with $\Omega = 1$ and $h = 0.5$ (conventional cold dark matter, or the "most well motivated model"), implying more power on large scales. Needless to say, this can only help with the problem of deficient large-scale structure.

To be specific, if the spectrum of perturbations is normalized by $(\delta M/M)_{\lambda=8h^{-1}\,\rm Mpc} = 1$,[p] I find that: $A = 4.4 \times 10^6\,{\rm Mpc}^4$ for $\Omega = 1$ and $h = 0.5$ (conventional CDM) and $A = 2.5 \times 10^7\,{\rm Mpc}^4$ for $\Omega_{\rm NR} = 0.2$ and $h = 0.7$ ("best-fit model"). On large scales

[p] I have used the "top hat" window function $[W(r) = 1$ for $r \le r_0$ and $= 0$ for $r \ge r_0]$ to define M, so that $(\delta M/M)^2 = (9/2\pi^2)\int_0^\infty k^2|\delta_k|^2[\sin(kr_0)/k^3r_0^3 - \cos(kr_0)/k^2r_0^2]^2\,dk$, where $r_0 = 8h^{-1}\,$Mpc.

($\lambda \gg \lambda_{\rm EQ}$) $\delta M/M \propto \sqrt{A}/\lambda^2$; it follows that $\delta M/M$ for the "best-fit model" is a factor of 4.7 bigger on large scales.

Growth of density perturbations

Subhorizon-sized, linear density perturbations grow as the scale factor during the matter-dominated regime ($z \lesssim z_{\rm EQ} \simeq 23000\Omega h^2$), and remain roughly constant in amplitude when the Universe is radiation dominated, curvature dominated ($z \lesssim z_{\rm CURV} \simeq \Omega^{-1} - 2$; $z_{\rm CURV} \simeq 3$ for $\Omega = 0.2$), or vacuum-energy dominated ($z_\Lambda \simeq [\Omega_\Lambda^{-1} - 1]^{1/3} - 1 \simeq 0.59$). For a nonflat, $\Omega = 0.2$ model the reduction in the growth of perturbations relative to a flat model is very significant: about a factor of 20. By contrast, in flat-Λ models perturbations grow almost unhindered until the present (see Refs. 49 and 58). In the "best-fit model" the growth factor is only a factor of 0.8 less than $z_{\rm EQ}$, or about 1800. For comparison, in the conventional CDM model the growth factor $z_{\rm EQ} \simeq 5800$, only about a factor of three more growth.

Microwave anisotropies

For conventional CDM the predicted CMBR temperature anisotropies are about a factor of three or so below the current level of observed isotropy (depending upon the angular scale and biasing factor b).[59] One might worry that because the "best-fit model" has more power on large scales and the growth factor for perturbations is smaller the predicted CMBR anisotropies might violate current bounds. That is not the case. The reason involves the angular size on the sky θ of a given scale λ at epoch z:

$$\theta(\lambda, z) = \lambda/r(z); \tag{25a}$$

$$r(z) = \int_{t(z)}^{t_0} \frac{dt}{R(t)} = \frac{2H_0^{-1}}{3\Omega_\Lambda^{1/6}\Omega_{\rm NR}^{1/3}} \int_{\sinh^{-1}\left[\sqrt{\Omega_\Lambda/(1+z)^3\Omega_{\rm NR}}\right]}^{\sinh^{-1}\left[\sqrt{\Omega_\Lambda/\Omega_{\rm NR}}\right]} \frac{du}{\sinh^{2/3} u}, \tag{25b}$$

where $r(z)$ is the coordinate distance to an object at red shift z. In a flat, matter-dominated model $r(z) = 2H_0^{-1}\left[1 - 1/\sqrt{1+z}\right] \to 2H_0^{-1}$ for $z \gg 1$, and $\theta(\lambda, z \gg 1) \simeq 34.4''\,(\lambda/h^{-1}\,{\rm Mpc})$. For the "best-fit model" $r(z \gg 1) \simeq 3.9H_0^{-1}$ and $\theta(\lambda, z \gg 1) \simeq 17.7''\,(\lambda/h^{-1}\,{\rm Mpc})$.

In a flat Λ-model the horizon is further away and a given length scale has a smaller angular size. Since the temperature fluctuations on a given angular scale are related to the density perturbations on the length scale that subtends that angle at decoupling, in the "best-fit model" temperature fluctuations on a given angular scale are related to density perturbations on a *larger* scale λ. While the "best-fit model" has more power on a *fixed* (large) length scale, a fixed angle θ corresponds to a *larger* length scale, where the amplitude of perturbations is smaller because $\delta M/M$ decreases with λ.

Consider the temperature fluctuations on large-angular scales ($\theta \gg 1°$); they arise due to the Sachs–Wolfe effect and $(\delta T/T)_\theta \simeq (\delta\rho/\rho)_{\rm HOR}/2$ on the scale $\lambda(\theta)$ when that scale crossed inside the horizon. For the Harrison–Zel'dovich spectrum the horizon-crossing amplitude is constant, so that $\delta T/T$ is independent of angular scale (for $\theta \gg 1°$). The CMBR quadrupole anisotropy is related to the amplitude of the perturbation that is just now crossing inside the horizon: $\lambda_{\rm HOR} \sim 2H_0^{-1} \sim 12000\,{\rm Mpc}$ (conventional CDM) and $\lambda_{\rm HOR} \sim 3.9H_0^{-1} \sim 16700\,{\rm Mpc}$ ("best-fit model"). Evaluating the normalized spectra on these scales it follows that the large-angle temperature fluctuations in the "best-fit model"

are only a factor of 1.2 larger than for conventional CDM, in spite of the fact that the "best-fit model" has significantly more power on large scales.

The amplitude of the temperature fluctuations on small angular scales ($\theta \ll 1°$) is proportional to the amplitude of the density perturbations at the time of decoupling ($z_{\rm DEC} \sim 1000$), on the scale $\lambda(\theta)$. In the "best-fit model" perturbations have grow by a factor of about $0.8 z_{\rm DEC}$ since decoupling, while those in the "most well motivated model" have grown by a factor of $z_{\rm EQ}$. On the other hand the length scale corresponding to the angular scale θ is larger for the "best-fit model." The net result is that the temperature fluctuations on an angular scale of 1° are also only about a factor of 1.2 larger.

Galaxy counts

Because the coordinate distance to an object of given red shift is greater in a flat Λ model, there is greater volume per red shift interval per solid angle, which increases the number of galaxies in $dz d\omega$. To see roughly how this goes, consider the deceleration parameter

$$q_0 = \Omega(1 + 3p/\rho)/2 = (1 - 3\Omega_\Lambda)/2 \simeq -1.2, \tag{26}$$

where Ω is the total energy density ρ divided by the critical energy density and p is the total pressure. From Eq. (2) one can see that the galaxy-number count is significantly increased by the addition of a cosmological constant, $dN_{\rm GAL}/dz = z^2 n_{\rm GAL}[1 - 3z + \cdots]$ compared to $z^2 n_{\rm GAL}[1 + 0.4z + \cdots]$.

Large-scale motions

The *rms* peculiar velocity of a volume defined by the "window function" $W(r)$, averaged over all such volumes in the Universe, is

$$\langle v^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty k^2 |\mathbf{v}_k|^2 |W(k)|^2 dk, \tag{27}$$

Using a gaussian window function $[W_{r_0}(r) = \exp(-r^2/2r_0^2)]$ and normalizing the spectrum as above, the *rms* peculiar velocity expected on the scale $r_0 = 50h^{-1}$ Mpc is[60]

$$v_{50} \simeq 83h^{-0.9}\,{\rm km\,s^{-1}} \simeq 160\,{\rm km\,s^{-1}} \qquad (\Omega = 1,\ h = 0.5);$$

$$v_{50} \simeq 83\Omega_{\rm NR}^{-0.33} h^{-0.9}\,{\rm km\,s^{-1}} \simeq 200\,{\rm km\,s^{-1}} \qquad (\Omega_{\rm NR},\ h = 0.7).$$

While the *rms* peculiar velocity on the scale of 50 Mpc is still far short of $700\,{\rm km\,s^{-1}}$, it is larger, owing to fact that there is more power on large scales.[f]

Motivation

As its name suggests, it is a model motivated by observations and not aesthetics: Conventional cold dark matter is clearly better motivated. While the conventional CDM model has one question to answer—why the ratio of the baryon density to that of cold dark matter is of order unity (see below)—in the "best-fit model" one must also address "why now?"—why is the cosmological constant just now becoming dynamical important? (This problem is similar to the flatness problem, where the question is, why is the curvature

[f] The comparison of theoretical expectations to the peculiar-velocity data is far more complicated than just computing $\langle v^2 \rangle$ for a gaussian window function.[61] The point I wish to make here is that adding a cosmological constant increases peculiar velocities.

radius just now becoming comparable to the Hubble radius?) Moreover, there is the issue of the cosmological constant itself: At present there is every reason to expect a cosmological constant $\rho_\Lambda = \Lambda/8\pi G \sim m_{\rm Pl}{}^4$ that is some 122 orders of magnitude larger than observations permit[r] (Supersymmetry *might* be able to help in this regard, reducing the estimate to $\rho_\Lambda \sim G_F^{-2}$, which is only 56 orders of magnitude too large!) The strongest statement that one can make in defense of a relic cosmological constant of the desired size is that no good argument exists for *excluding* it!

V. A New Dimensionless Cosmic Ratio[63]

Dimensionless numbers play a crucial role in physics and in cosmology, and attempts to understand their origin often lead to important insights. There are a number of dimensionless ratios in cosmology: the baryon-to-photon ratio, the fractional abundances of the light elements, the amplitude of the primeval density perturbations, and the ratio of the neutrino and photon temperatures. If there is a significant amount of nonbaryonic matter in the Universe, we have a new dimensionless ratio to understand

$$r \equiv \frac{\Omega_{\rm B}}{\Omega_{\rm X}} \sim 0.1. \tag{28}$$

In particular we can ask why r is order unity, and not say 10^{-20} or 10^{20}?

We can try to express r in terms of fundamental quantities. To begin, write

$$r = \frac{m_B}{m_X}\frac{n_B/s}{n_X/s}. \tag{28'}$$

One of the great successes of particle cosmology is the dynamical explanation of the baryon asymmetry, or baryogenesis.[64] While the specific details of baryogenesis are still lacking, generally one expects that $n_B/s \sim \epsilon/g_*$, where $g_* \sim 100 - 1000$ counts the number of degrees of freedom at the epoch of baryogenesis (10^{14} GeV?) and $\epsilon \sim 10^{-8} - 10^{-7}$ is a measure of the C, CP violation in the baryon number violating sector and—on general grounds—is order $(\alpha/\pi)^N$ ($\alpha = g^2/4\pi$, g is a Higgs coupling). The quantity n_X/s is the relic abundance of X particles per comoving volume, a quantity that can be calculated as we seen above. Now consider the implications of $r \sim 0.1$ for the various relics previously discussed.

Heavy neutrino/neutralino

For a thermal relic like a heavy neutrino or neutralino whose interactions are weak, $n_X/s \sim 1/m_X^3 m_{\rm Pl} G_F^2$ (see *Thermal relics* above). The condition that r be of order unity implies

$$\frac{G_N}{G_F} \sim \sqrt{\frac{\epsilon m_B m_X^2}{g_* m_{\rm Pl}{}^3}} \ll 1, \tag{29}$$

and thus is related to the fact that the weak scale is much smaller than the Planck scale.

[r] There is one interesting explanation of why the cosmological constant is "probably" zero: Coleman and others[62] have argued that due to wormhole effects the wavefunction of the Universe is very sharply peaked at zero cosmological constant.

Axion

The relic abundance of axions can be expressed as $n_a/s \sim f_a^2/\Lambda_{\rm QCD} m_{\rm Pl}$. The condition that r be of order unity implies

$$\frac{f_a}{m_{\rm Pl}} \sim \epsilon \left(\frac{m_B}{\Lambda_{\rm QCD}}\right)^2 \sim \epsilon, \tag{30}$$

and thus is related to the fact that the PQ symmetry breaking scale is somewhat less than the Planck scale.

Light neutrino

The relic abundance of a light neutrino species, n_ν/s, is of order unity. If we assume that light neutrino masses arise through the see-saw mechanism, then $m_\nu \sim m_f^2/M$, where m_f is a typical fermion mass and M is the large energy that characterizes lepton number violation. The condition that r be of order unity implies

$$m_f^2/m_B \sim \epsilon M/g_*, \tag{31}$$

and thus is related to the fact that fermion masses are much smaller than the scale of lepton number violation.

Skew relic

Consider a skew relic whose net particle number per comoving volume is comparable to that of baryon number (perhaps its net particle number was produced at the same time as the baryon number, e.g., a heavy neutrino). In this case the fact that r is of order unity is related to the fact that the mass of the skew relic is comparable to that of a nucleon.

In a sense, all of these relations only tell us what we already knew and put in. However, this exercise does illustrate the fact that r can be related to fundamental quantities in physics, and raises the hope that this very important dimensionless cosmological ratio may some day have a more fundamental explanation. Apparently, that explanation will have to wait until we have a better understanding of the various energy scales that arise in particle physics.

VI. Summary

What do we know about the quantity and composition of the matter in the Universe? Most of the matter in the Universe is dark, with luminous matter contributing less than 1% of the critical density. The best estimates of the amount of matter associated with bright galaxies is $\Omega_{\rm ABG} \simeq 0.1 - 0.3$; however, there are some observations that suggest that Ω might be larger, perhaps even equal to one. Based upon primordial nucleosynthesis, we can be confident that baryons contribute between 1.1% and 12% of the critical density—more than that of luminous matter, but far less than the critical density. It is no means impossible that baryons account for the entire mass density of the Universe.

While there may already be evidence for nonbaryonic matter—if Ω is indeed 0.2—if our strong theoretical prejudice for $\Omega = 1$ is correct, nonbaryonic matter *must* account for the bulk of the mass density in the Universe. In any case, it is certainly a hypothesis worthy of careful consideration.

Theories of fundamental physics that go beyond the standard model have profound implications for the earliest moments of the Universe; indeed, many of us believe that the

"blueprint" for the Universe traces to events that took place during that epoch. Theories that unify the particles and interactions predict the existence of new, stable particles (or additional properties for known particles, e.g., neutrino masses), and remarkably enough, the relic abundances calculated for a number of these new particles is comparable to that required to close the Universe. For many, this is what makes the particle dark-matter hypothesis so compelling. Needless to say, the discovery of such a relic would not only solve a cosmological puzzle, but would also shed light on the theory that unifies the forces and particles.

By now there is a virtual zoo of particle dark-matter candidates.[65] However, three candidates are particularly well motivated and attractive. They are an axion of mass 10^{-6} eV to 10^{-4} eV, a neutrino of mass $92h^2$ eV, and a neutralino of mass 10 GeV to 3 TeV. Peccei–Quinn symmetry and its axion resolve a nagging and serious difficulty of the standard model: the strong-CP problem. The neutralino is a very robust prediction of theories that incorporate low-energy supersymmetry. Low-energy supersymmetry provides some understanding of the hierarchy problem (the large disparity between the weak scale and the Planck scale), and is further motivated by superstring theories. Neutrinos actually exist—and come in three flavors!—and in many extensions of the standard model small neutrino masses are predicted. Moreover, the first results of the SAGE experiment,[66] together with the results of the Homestake and Kamiokande II solar neutrino experiments, suggest that nonadiabatic MSW neutrino oscillations may be *the solution* to the solar neutrino problem.[67] If this is so, it implies a mass for the μ or τ neutrino in the range 10^{-4} eV to 10^{-2} eV. Speculating (upon supposition to be sure) that this is the mass for the μ neutrino, a simple see-saw scaling estimate for the τ neutrino mass might just put it in the cosmologically interesting range. While cold dark matter provides a far more promising paradigm for structure formation than does hot dark matter, I am certain that cosmology could learn how to live with a neutrino-dominated Universe.

Particle dark matter is an attractive and compelling hypothesis, and the next step is to test it. A variety of experiments are underway, and more are planned.[68] The experimental efforts encompass a diversity of approaches, involving conventional laboratory and accelerator experiments, large-underground detectors, and experiments built expressly to detect the dark matter particles in our local neighborhood. The search for evidence of supersymmetry is going on at accelerator laboratories all over the world. Indirect evidence for the existence of particle dark matter in our own halo could come from the annihilation products of particle dark matter in the halo or from particle dark matter that has accumulated in the sun or earth. The GALLEX and SAGE experiments may well provide information about neutrino masses, and a nearby supernova or a long-baseline neutrino oscillation experiment could provide definite evidence for neutrino masses. The MACRO experiment in the Gran Sasso Laboratory is operating and can search for both relic magnetic monopoles and high-energy neutrinos from particle dark-matter annihilations in the sun or earth. First-generation Sikivie-type detectors to search for cosmic axions have been built and successfully operated;[69] a second generation detector with sufficient sensitivity to detect halo axions in the our neighborhood has been proposed.[70] Low-background, cryogenic detectors designed to detect the keV energies deposited by halo neutralinos that elastically scatter within the detector are under development in laboratories all over the world, and low-background ionization detectors have already been used to search for heavy

neutrinos and cosmions.[71]

The answer to the simple question—What is the Universe made of?—may well be answered soon. If the bulk of the matter in the Universe is nonbaryonic, this discovery will rank as one of the most important of the century, and will have profound implications for both cosmology and particle physics.

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