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# W Boson plus Multijets at Hadron Colliders: HERWIG Parton Showers vs Exact Matrix Elements \*†

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## Abstract

Predictions of the parton shower simulation program HERWIG for the hadroproduction of a  $W$  boson plus  $n$  jets (for  $n \leq 4$ ) are compared with exact tree-level calculations. A variety of distributions relevant to new physics searches in  $p\bar{p}$  and  $pp$  collisions at 1.8 and 40 TeV respectively are compared. If one starts the parton shower simulation from the Drell-Yan subprocess  $q\bar{q} \rightarrow W$ , so that all  $n$  jets are generated by QCD bremsstrahlung, then the jet distributions generated are too low at low rapidities and at high transverse momenta. If one starts instead from the  $W$  plus 1 jet subprocesses, generating  $n - 1$  jets via bremsstrahlung, the shapes of the distributions agree much better, although an empirical factor is still required to bring the absolute cross sections into agreement.

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# 1 Introduction

The hadroproduction of a  $W$  or  $Z$  boson together with one or more jets constitutes a serious background in searches for almost any new physics at hadron colliders. A heavy standard-model Higgs boson or a non-standard  $Z'$ , for example, would be expected to decay copiously to  $WW$ . If one  $W$  decays hadronically then there is a severe background from non-resonant  $W + 2$  jet production. On the other hand, if both are required to decay leptonically the rate is suppressed and there is additional missing momentum. In top quark-antiquark pair production the signal has two  $W$ 's and two jets. If one  $W$  is allowed to decay hadronically the main background is  $W + 4$  jets. If one jet is missed, or if the process is  $t\bar{b}$  production, it is  $W + 3$  jets. Similarly, Higgs boson production by  $WW$  fusion can be tagged by picking up a fast forward and/or backward quark jet; the background is then  $W + 3$  or 4 jets. Thus in all these and many other cases, it is important to simulate in as much detail as possible the hadroproduction of a  $W$  plus  $n$  jets, where  $n \leq 4$ .

Two distinct approaches can be used to estimate the cross sections and distributions for  $W$  plus multijet final states. One is to use the exact tree-level matrix elements, which have now been calculated in full for  $W$  plus up to 4 jets [1-10]. This method provides the most reliable estimates for quantities that can be defined realistically at the parton level. It does not aim to give a complete description of the hadronic final state, although in principle the calculation could be combined with a model for jet hadronization. A disadvantage of this approach for some purposes is that it generates final state configurations with a range of weights, rather than unweighted events. Weights can become unphysically large for configurations near the singularities of the matrix element. Also, the amount of computer time per configuration increases factorially with the number of jets.

A second approach is to use a parton shower simulation to generate multijet final states starting from a simpler parton subprocess. Monte Carlo programs based on parton shower simulation were introduced ten years ago [11] and have greatly increased in sophistication since then [12-15]. Earlier programs [11, 12] were based on the leading collinear logarithmic approximation, which can only be strictly justified when the angles between pairs of jets are small. More recently, the use of 'coherent parton shower' algorithms has permitted the correct treatment of leading and next-to-leading infrared singularities as well [13-15]. For many purposes this appears to be sufficient to give rather accurate predictions even when one is not close to a singularity. In  $e^+e^-$  annihilation, simulations based on coherent parton showers have been found to give a good description of multijet cross sections [16] and of the detailed structure of 4-jet final states [17].

The main advantages of a parton shower simulation are that it generates unweighted parton configurations of arbitrary complexity and can readily be combined with a hadronization model to provide a complete event generator. Singularities of the matrix elements are regularized by the appropriate Sudakov form factors, and all multijet distributions are generated in proportion to their cross sections in a single Monte Carlo run. Thus, for example, cross sections for the production of up to 16 jets in  $e^+e^-$  annihilation at 2 TeV have been estimated [18] using the program of Ref. [13].

The disadvantage is that the parton shower approach cannot be justified rigorously when one is far from all singularities of the matrix element: specifically, when several jets are produced at large angles to each other with comparable energies. Such configurations correspond to multijet systems of high invariant mass. In these cases we need to check explicitly, in as many situations as possible,

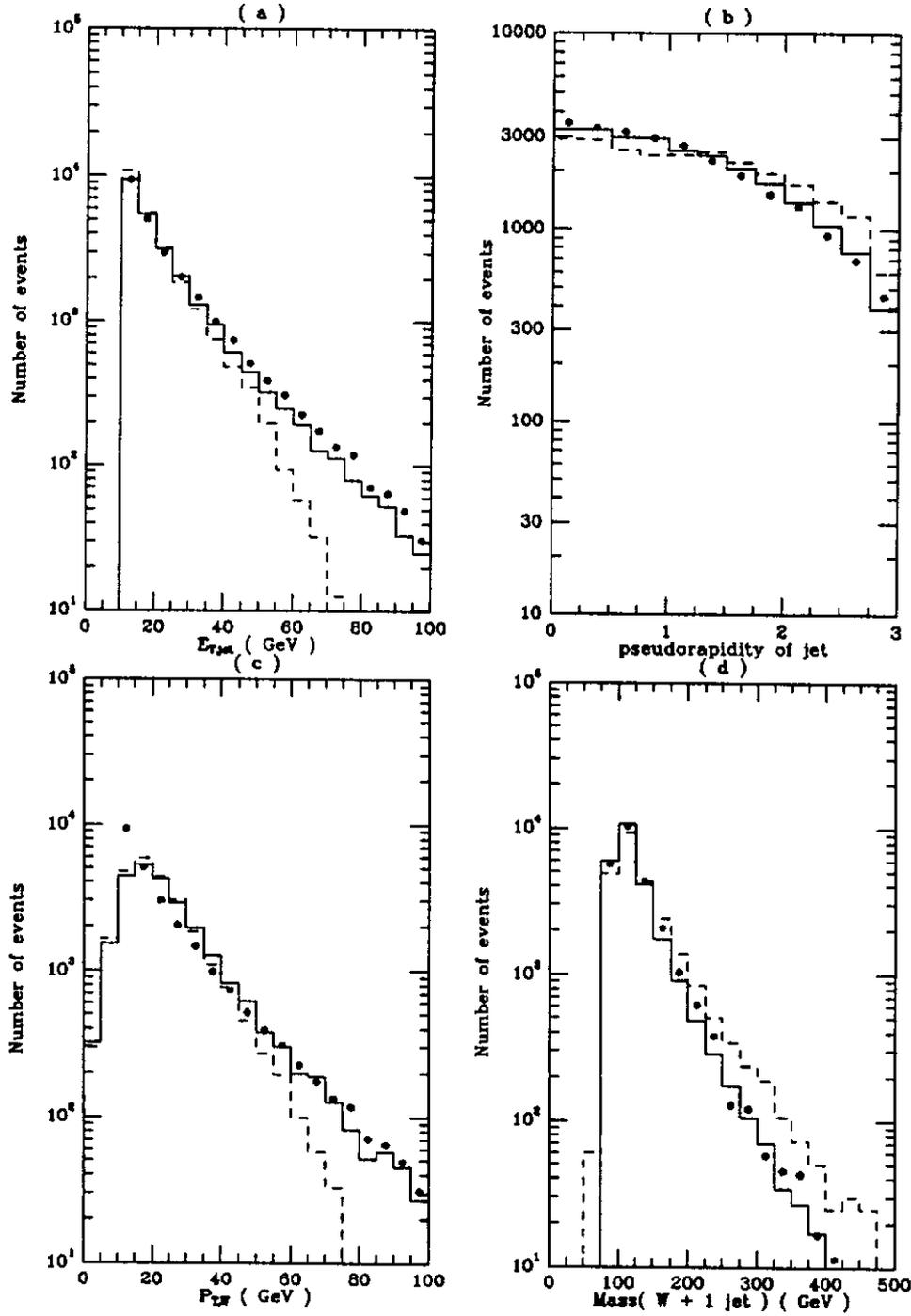


Figure 1:  $W + 1$  jet final states: 'W0' (dashed histograms) and 'W1' Monte Carlo (solid histograms) vs tree level matrix element (points) at  $\sqrt{s} = 1.8$  TeV, for  $E_{T, jet} > 10$  GeV. (a) Jet transverse energy; (b) Jet pseudorapidity; (c)  $W$  transverse momentum; (d)  $W + 1$  jet invariant mass.

Process	$n=0$	1	2	3	4	5
$V \rightarrow q\bar{q} + n g$	1	2	8	50	428	4670
$V \rightarrow q\bar{q} + q'\bar{q}' + (n-2) g$	-	-	4	24	196	2040

Table 1: Number of contributing Feynman diagrams.

whether distributions generated by the coherent shower algorithm are in reasonable agreement with the predictions of exact tree-level matrix elements.

Studies of  $W$  plus multijet backgrounds to new physics have been undertaken using a variety of parton-shower and matrix-element based approaches [19-24]. The new ingredients we shall exploit here are the coherent parton-shower program HERWIG [15, 25], which has been shown to give reliable results in the closely-related process  $e^+e^- \rightarrow Z^0 \rightarrow$  multijets [16, 17], and the recent matrix-element calculation for  $W + 4$  jets [9]. We also present results for the first time at supercollider energies ( $\sqrt{s} = 40$  TeV).

## 2 Matrix element calculations

Calculating multiparton matrix elements with a large number of partons is a far from trivial task, especially when a vector boson also participates in the process. While for  $n=0,1$  (which are the diagrams required to calculate the  $W + 0,1$  jet cross section) one can use standard techniques to calculate the squared matrix element, this becomes difficult for  $n=2$  (i.e.  $W + 2$  jets) [1] and impossible for  $n \geq 3$ . The reason is of course the rapid growth in the number of contributing Feynman diagrams, as demonstrated in Table 1 [8].

Several useful methods have been developed for calculating the matrix elements for these multijet events. The first is the helicity method as used by the CALKUL collaboration [2] which was later improved [3]. With this method the  $W + 2$  jet matrix element was easily calculated and short expressions were obtained [4].

To proceed beyond  $W + 2$  jets the helicity method becomes cumbersome, though it can be used to construct a program which calculates the  $W + 3$  jet cross section [5]. Any extension of this method to  $W + 4$  or more jets becomes a virtually impossible task.

The way to proceed is using recursive methods [6] combined with the Weyl van der Waerden spinor calculus [7] and the improved CALKUL helicity method. In this way compact expressions are obtained for the  $W + 3$  jet [8] and the  $W + 4$  jet matrix elements [9]. With the recursive method it is in principle possible to construct a computer program which calculates the  $W + n$  jet cross section. The recursive method only becomes efficient when at least 3 gluons are involved in the subprocess. Therefore a recursive formula was constructed for the subprocess  $W + q\bar{q} + n g$ . Once this routine is programmed it works for an arbitrary number of gluons. The only limitation is the available computer time. This is demonstrated in Table 2 [10], which gives the time per

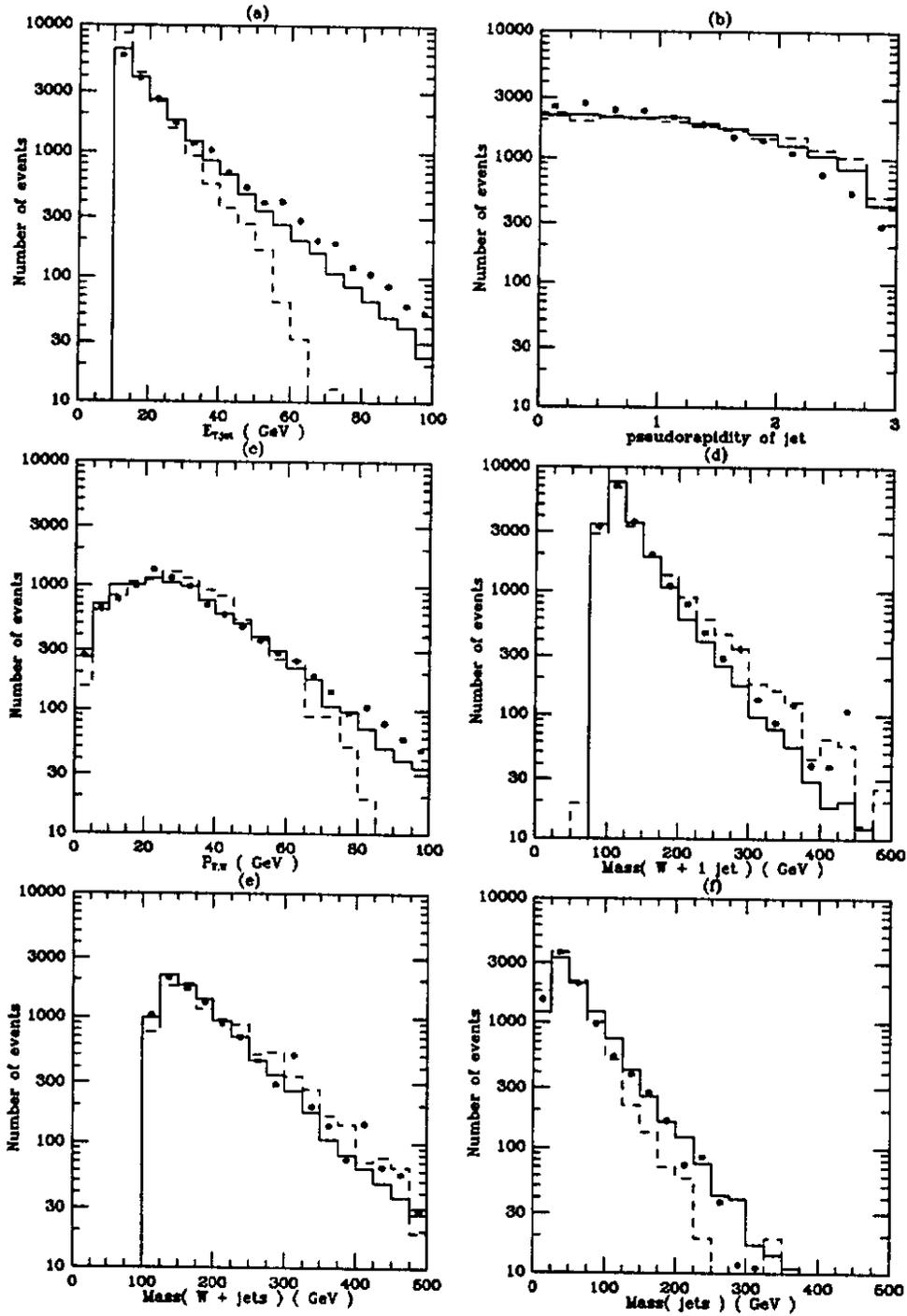


Figure 2:  $W + 2$  jet final states: (a)-(d) as in Figure 1; (e)  $W +$  multijet invariant mass; (f) Multijet invariant mass.

No. of gluons	Time in seconds
0	0.0022
1	0.0039
2	0.012
3	0.081
4	0.89
5	14.6

Table 2: Timing of a matrix element evaluation for the process  $V \rightarrow q\bar{q} + n g$  on a VAX 3500

accepted event required to evaluate the contributions of the  $W + q\bar{q} + n g$  subprocess to the  $W + n$  jet cross section.

The tree-level matrix element Monte Carlo programs give for each accepted event a set of final state momenta and its corresponding weight. The reconstructed jet momentum is identified with the parton momentum. Because of the jet defining cuts, which translate back on parton level to the same cuts on the partons, we stay well away from the singular regions of phase space (i.e. soft and/or collinear partons). This ensures that the weights of the events that survive the jet defining cuts do not fluctuate too much, and makes it possible to use this type of program for the calculation of exclusive jet cross sections in a reliable way.

For the present study, we defined the multijet cross sections by a minimum jet transverse energy of 10 GeV at  $\sqrt{s} = 1.8$  TeV and 50 GeV at  $\sqrt{s} = 40$  TeV. All jets were required to lie in the pseudorapidity interval  $|\eta| < 3$  and the minimum jet-jet separation in pseudorapidity and azimuth was chosen to be  $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.7$ , in line with the recommendations of Ref. [26]. For the QCD running coupling  $\alpha_S$ , the one-loop expression with scale  $Q^2 = m_W^2 + p_{TW}^2$  and  $\Lambda = 0.2$  GeV was used. The parametrizations of Eichten et al. (set 1) [27] were used for the parton densities in the proton.

### 3 Parton shower simulations

Two different types of parton shower simulations can be performed for  $W$  plus multijet production. The simplest [19, 21, 22] is to start from the bare Drell-Yan subprocess  $q\bar{q} \rightarrow W$  and generate all the jets via QCD bremsstrahlung from the incoming quark and antiquark. This method seems most suitable for estimating the  $W + 0$  jet cross section and the multijet distributions when jet transverse energies are small compared to the  $W$  mass. We call this a ‘ $W0$ ’ simulation.

The ‘ $W0$ ’ type of shower simulation cannot be adequate for jet transverse energies greater than or of the order of the  $W$  mass. For if a gluon in one of the incoming parton showers is emitted with  $E_T > m_W$ , then the hardest vertex in the diagram is that emission vertex and not the Drell-Yan subprocess. Such an event is more correctly described as a QCD hard subprocess in which a  $W$  is radiated from an incoming line. Thus to avoid double counting one must always restrict the

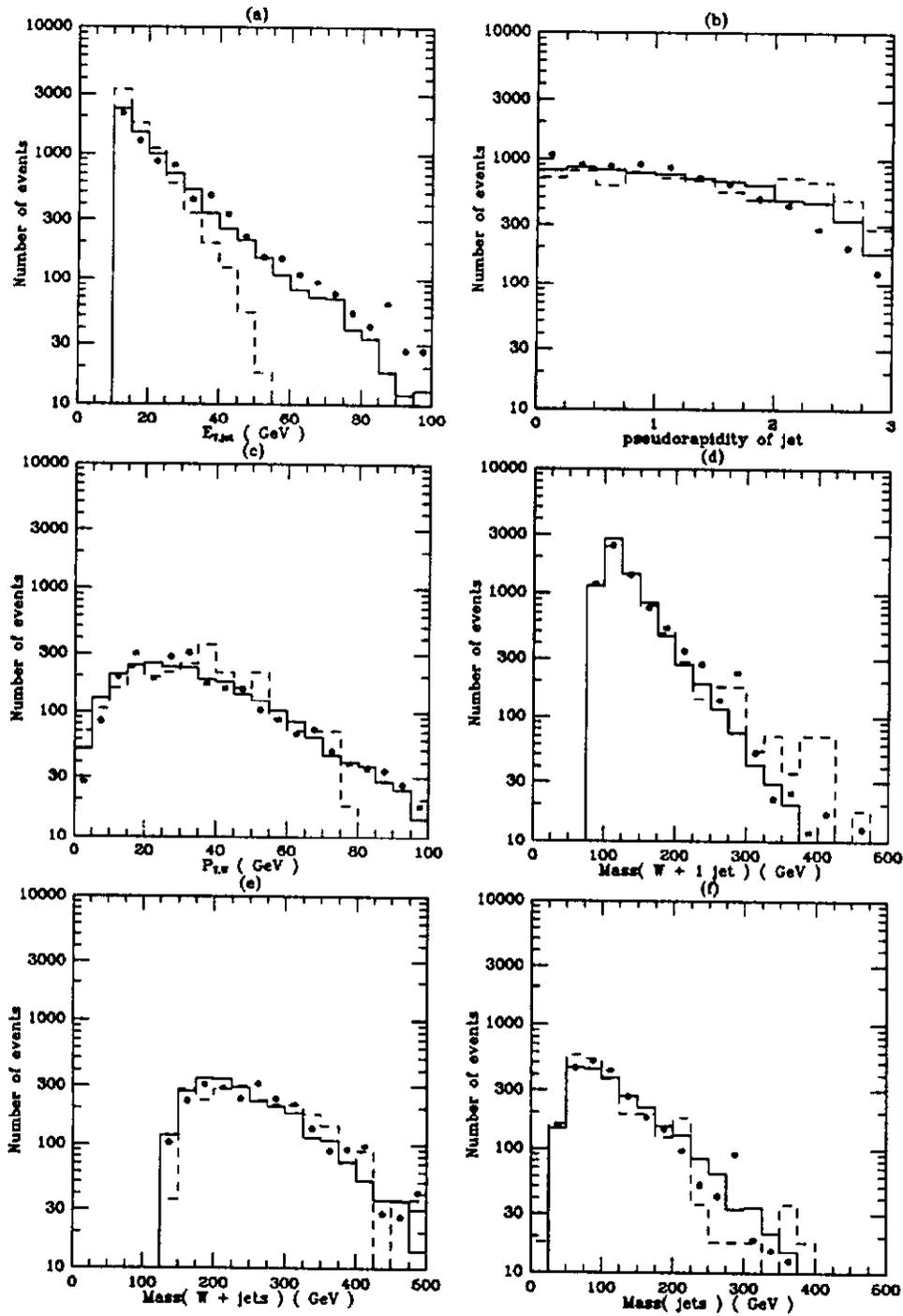


Figure 3: As Figure 2, but for  $W + 3$  jet final states.

transverse energies of objects radiated in the parton showers to be less than the scale of the hard subprocess.

The production of multijets with arbitrarily high transverse energies can be simulated consistently provided one starts instead from the  $W + 1$  parton subprocesses  $q\bar{q} \rightarrow Wg$ ,  $gq \rightarrow Wq$  and  $g\bar{q} \rightarrow W\bar{q}$ . Then the jet with highest  $E_T$  is generated in the hard subprocess and the others come from QCD bremsstrahlung. We shall call this a ‘ $W1$ ’ simulation. To our knowledge, this approach has been considered before only in Ref. [24], to study  $W + 2$  jet background in searches for heavy vector bosons.

For both types of simulation, we used the same jet definitions as in the matrix element calculations described above. To find jets we used the jet finder GETJET [28] from the ISAJET simulation package, with a jet cone  $\Delta R = 0.7$  and cell size  $(\Delta\eta, \Delta\phi) = (0.25, 15^\circ)$ . The HERWIG (version 4.5) parameters were left at their default values, except that the QCD scale  $QCDLAM$  was increased to 0.2 GeV, the maximum cluster mass parameter was correspondingly increased to 4.0 GeV, and the EHLQ 1 structure functions were used, for consistency with the matrix element calculations. For the ‘ $W1$ ’ option the hard subprocess required a lower limit on the outgoing parton transverse momentum, which we took to be the same as the minimum jet transverse energy.

To reduce the computing time for event generation and jet finding, no soft underlying event was generated, i.e. only particle production associated with the hard subprocess was simulated. We checked at  $\sqrt{s} = 1.8$  TeV that this made little difference to the distributions

## 4 Results

Results of the matrix element calculations are shown by the points in Figs. 1–6, and those of the HERWIG ‘ $W0$ ’ and ‘ $W1$ ’ simulations by the dashed and solid histograms respectively.

We see that as expected the jet transverse energy and  $W$  transverse momentum distributions from the ‘ $W0$ ’ option always fall below the matrix element calculations when transverse momenta become of the order of the  $W$  mass. In addition the jet pseudorapidity distribution is too low at low rapidities and too broad, showing that the QCD bremsstrahlung contribution alone is too forward-peaked. On the other hand, the  $W + \text{jet}$ ,  $W + \text{multijet}$  and multijet invariant mass distributions reproduce the matrix element results fairly well.

Our conclusion on the results of the ‘ $W0$ ’ simulation option is that while it may be adequate for certain distributions at moderate transverse momenta, it necessarily underestimates the cross section at large  $p_T$ . It also gives a jet pseudorapidity distribution that is too forward-peaked.

The ‘ $W1$ ’ type of simulation is seen to give better results than the ‘ $W0$ ’ option for all the distributions we investigated. There is still a shortfall in the jet transverse energy distributions at the highest values of  $E_T$ , which becomes more serious as the number of jets increases. Part of this could be a real dynamical effect: jets with high  $E_T$  are more likely to be resolved into sub-jets with lower  $E_T$ . Therefore as  $E_T$  increases a growing fraction of the cross section feeds into higher jet multiplicities at lower transverse energies. Such a higher-order effect would not be seen in the tree-level matrix element calculations.

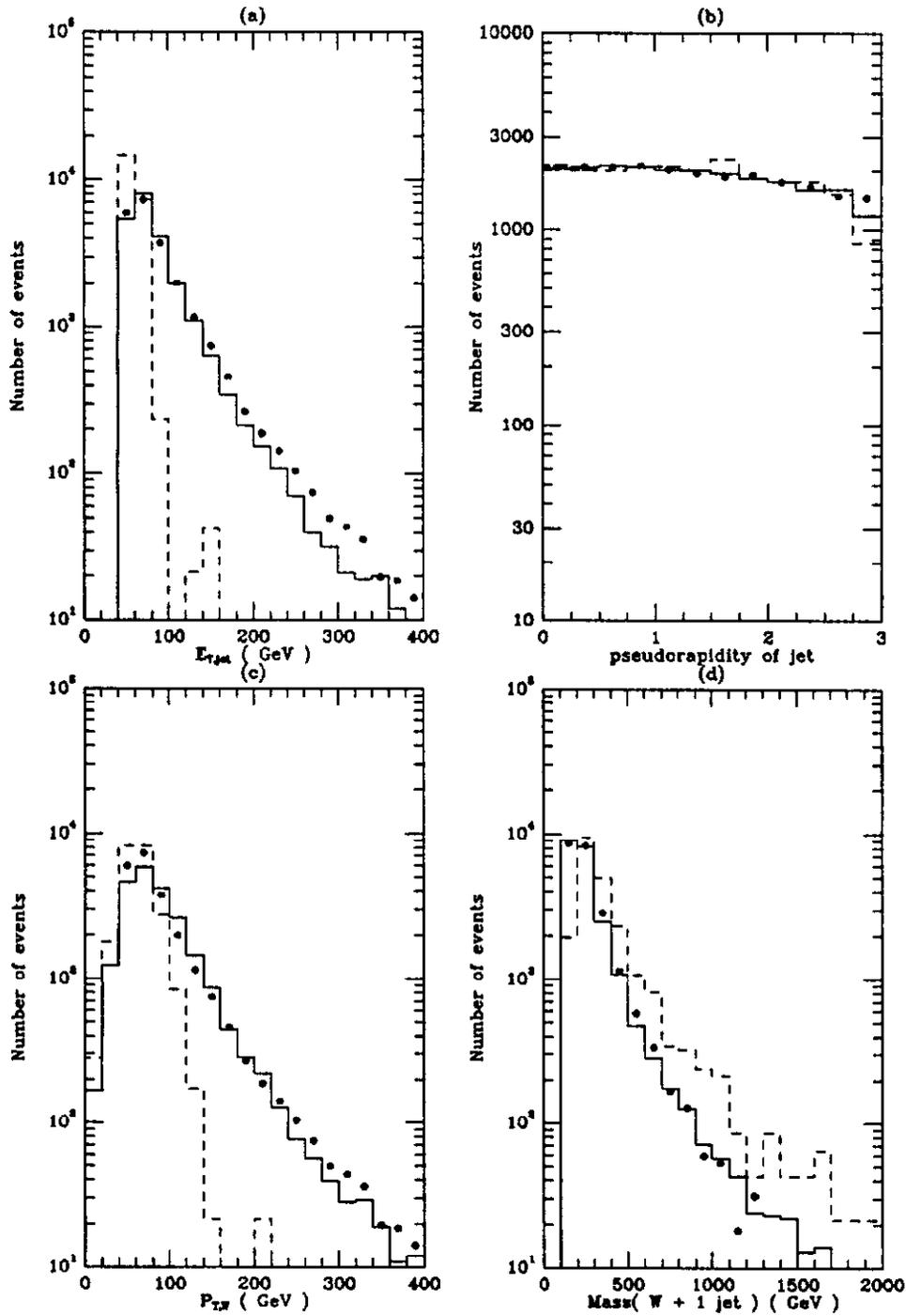


Figure 4: As in Figure 1, but for  $\sqrt{s} = 40$  TeV and  $E_{T,jet} > 50$  GeV.

$n$	Matr. el.	MC( $W0$ )	MC( $W1$ )	MC( $W^*$ )
0	1637(3)	1140(6)		
1	534(3)	295(3)	263(2)	421
2	160(3)	45(1)	103(1)	170
3	42(2)	4(1)	26.4(5)	45
4	8(1)	1(1)	5.4(2)	9
5			0.8(1)	1.3
6			0.2(1)	0.3

Table 3:  $W + n$  jet cross sections (pb) at  $\sqrt{s} = 1.8$  TeV

All the distributions shown in Figs. 1-6 are normalized to the same number of entries and therefore we have still to check the absolute cross sections. These are shown for  $\sqrt{s} = 1.8$  TeV in Table 3. (All cross sections include a branching fraction of  $1/9$  for  $W \rightarrow e\nu$ .) We see that the ‘ $W0$ ’ cross sections fall progressively further below the matrix element values as the number of jets  $n$  increases. On the other hand, at this energy the ‘ $W1$ ’ predictions remain at a roughly constant fraction (about 60%) of the matrix element values for all  $n = 1, \dots, 4$ . The column headed ‘ $W^*$ ’ shows the ‘ $W1$ ’ Monte Carlo predictions multiplied by an empirical factor of 1.6: they are then in fair agreement with the matrix element results.

The predicted cross sections at  $\sqrt{s} = 40$  TeV are given in Table 4. Here the ‘ $W0$ ’ predictions fall drastically short, as was already clear from Figs. 4-6. In this case the ‘ $W1$ ’ results also fall further below the matrix element values as  $n$  increases. We have found that this is primarily a consequence of the increase in the jet transverse energy cut from 10 to 50 GeV rather than the increase in  $\sqrt{s}$ : similar ratios between the ‘ $W1$ ’ and matrix element results are obtained at  $\sqrt{s} = 1.8$  TeV if we require  $E_{T,jet} > 50$  GeV. Empirically, the following enhancement factor gives improved agreement:

$$\sigma_n(W^*) = 1.6 \left( \frac{m_W^2 + E_{T,min}^2}{m_W^2} \right)^{2(n-1)} \sigma_n(W1),$$

as shown in the last column of Table 4. This factor also reproduces the results given in the last

$n$	Matr. el.	MC( $W0$ )	MC( $W1$ )	MC( $W^*$ )
0	23.73(3)	21.4(1)		
1	2.89(2)	0.45(1)	2.15(2)	3.4
2	1.30(8)	0.004(2)	0.48(1)	1.5
3	0.44(4)		0.07(1)	0.4
4	0.20(4)		0.008(2)	0.1
5			0.0012(4)	0.03

Table 4:  $W + n$  jet cross sections (nb) at  $\sqrt{s} = 40$  TeV

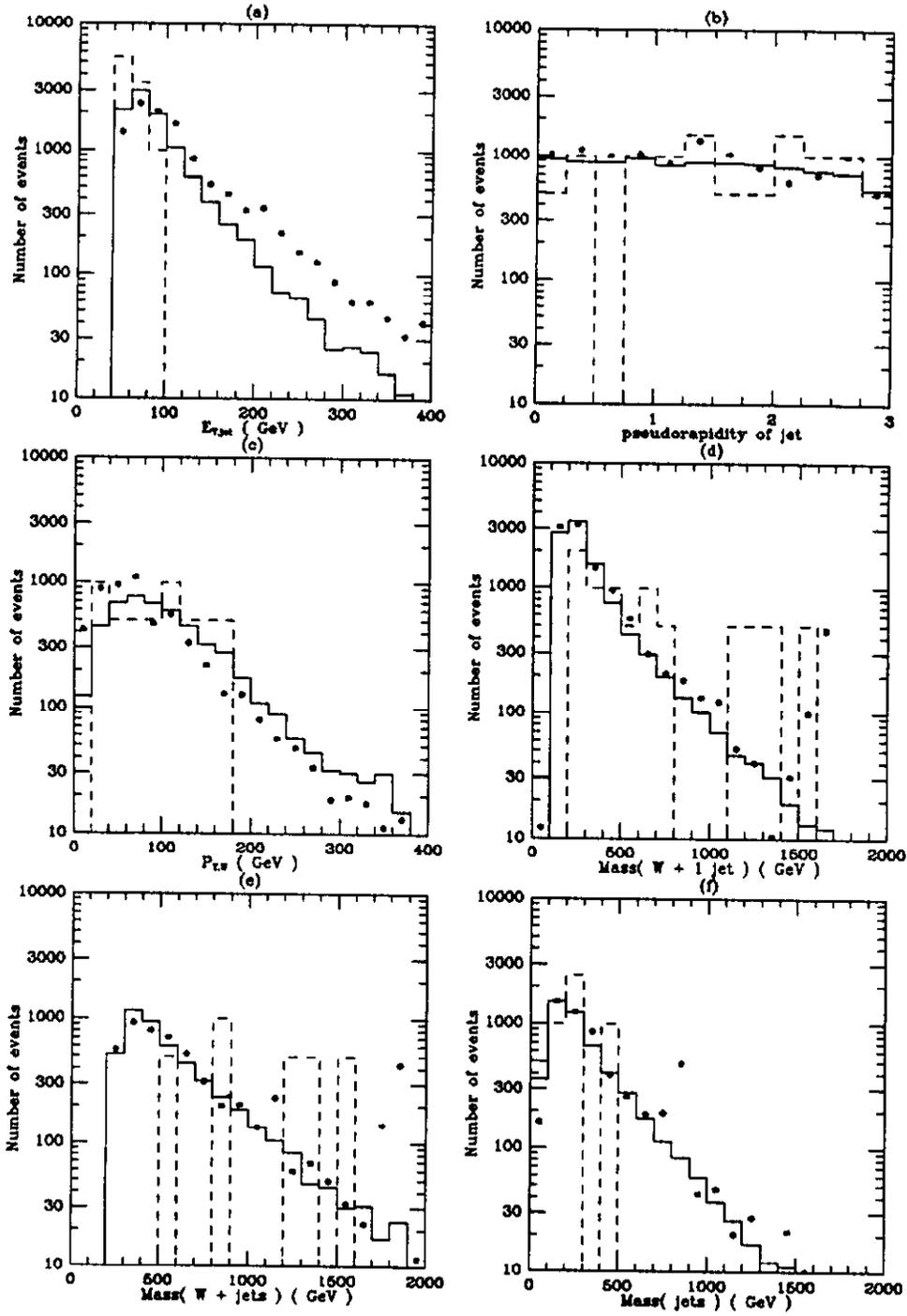


Figure 5:  $W + 2$  jet final states: (a)-(d) as in Figure 4; (e)  $W +$  multijet invariant mass; (f) Multijet invariant mass.

column of Table 3, since  $E_{T,min}$  is negligible compared with  $m_W$  in that case.

## 5 Conclusions

We conclude that the use of the HERWIG Monte Carlo program starting from the  $2 \rightarrow 2$  subprocesses of  $W + 1$  parton production gives distributions of a wide range of multijet observables that agree quite well with those generated using exact tree-level matrix elements. In the case of the absolute  $W +$  multijet cross sections, an empirical enhancement of the HERWIG values, by a factor depending mainly on the minimum jet  $E_T$  and the number of jets, is required for agreement with the matrix element results. It is not clear how much of this factor is due to the inherent inaccuracies of the matrix elements used in HERWIG, and how much is due to real fragmentation effects and the jet finding algorithm. In the meantime, it would seem prudent to include the enhancement factor when using HERWIG to predict absolute cross sections for  $W +$  multijet production.

## References

- [1] R.K. Ellis and R.J. Gonsalves, Proceedings of the Workshop on High Energy Physics, Eugene, 1985 (ed. D.E. Soper), pg. 287.
- [2] D. Danckaert et al., Phys. Lett. 114B (19203) 82; F.A. Berends et al., Nucl. Phys. B206 (1953) 82; Nucl. Phys. B206 (1982) 61; P. de Causmaecker, Ph.D. thesis, Leuven University, 1983; F.A. Berends et al., Nucl. Phys. B239 (19382) 84; Nucl. Phys. B239 (1984) 395; Nucl. Phys. B264 (19243) 86; Nucl. Phys. B264 (1986) 265.
- [3] R. Kleiss, Nucl. Phys. B241 (1984) 235; Z. Xu, Da-Hua Zhang and L. Chang, Tsinghua University Preprints, Beijing, TUTP-84/4, TUTP-84/5, TUTP-84/6 and Nucl. Phys. B291 (1987) 392; F.A. Berends, P.H. Daverveldt, R. Kleiss, Nucl. Phys. B253 (1985) 441; R. Kleiss and W.J. Stirling, Nucl. Phys. B262 (1985) 235; J.F. Gunion and Z. Kunszt, Phys. Lett. 161B (1985) 333.
- [4] R. Kleiss and W.J. Stirling, Zeit. Phys. C40 (1988) 419.
- [5] K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B313 (1989) 560.
- [6] F.A. Berends and W.T. Giele, Nucl. Phys. B306 (1988) 759; W.T. Giele, Ph.D thesis, University of Leiden (1989).
- [7] F.A. Berends and W.T. Giele, Nucl. Phys. B294 (1987) 700.
- [8] F.A. Berends, W.T. Giele and H. Kuijf, Nucl. Phys. B321 (1989) 39.
- [9] F.A. Berends, W.T. Giele, H. Kuijf and B. Tausk, FERMILAB-PUB 90-213-T/Leiden Preprint.
- [10] H. Kuijf, private communications.

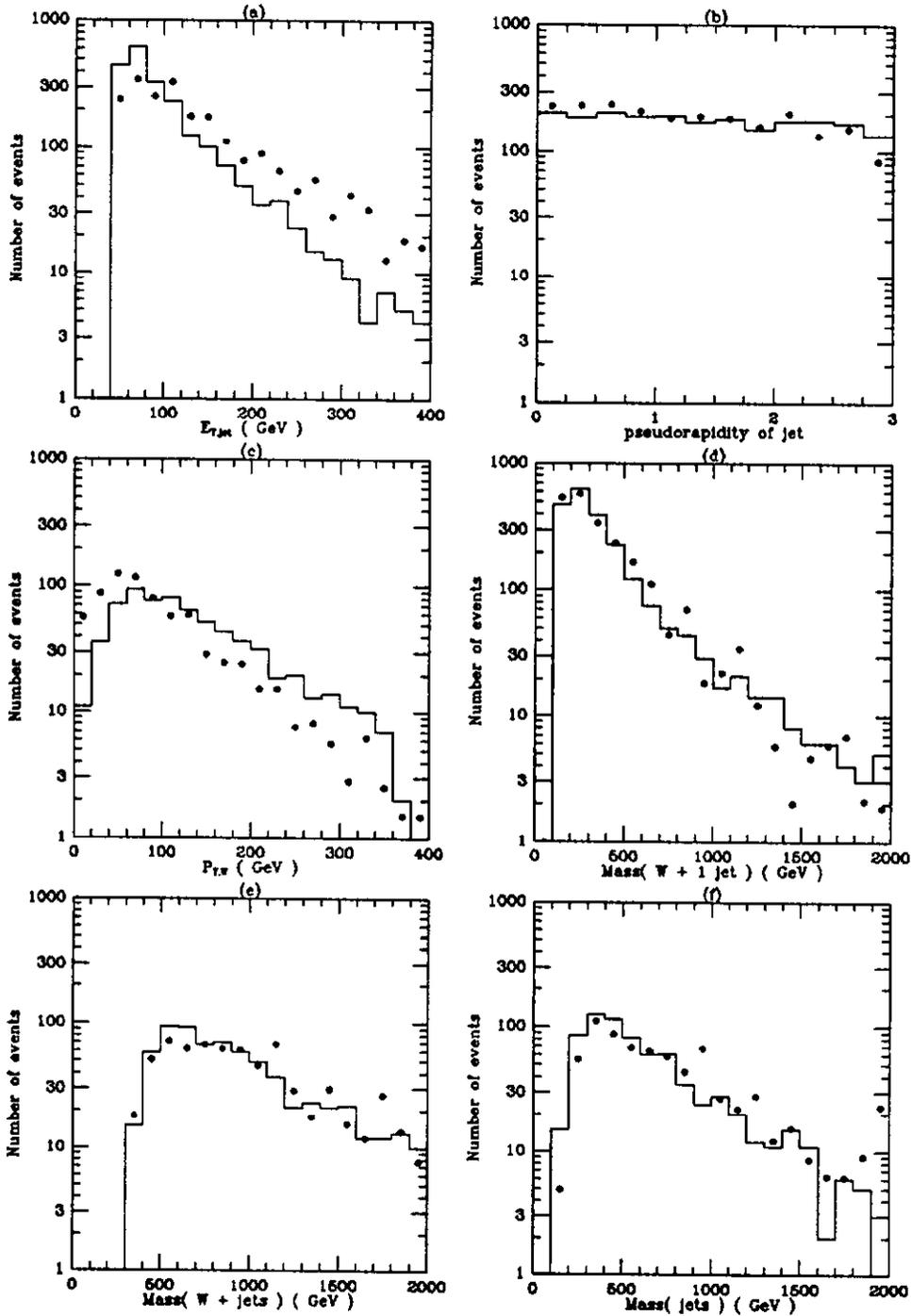


Figure 6: As in Figure 5, but for  $W + 3$  jet final states.

- [11] G.C. Fox and S. Wolfram, Nucl. Phys. B168 (1980) 285; R. Odorico, Nucl. Phys. B172 (1980) 157.
- [12] COJETS, R. Odorico, Comput. Phys. Commun. 32 (1984) 139;  
 PYTHIA, H. Bengtsson and G. Ingelman, Comput. Phys. Commun. 34 (1985) 251;  
 ISAJET, F. Paige and S. Protopopescu, in 'Supercollider Physics', Proc. Oregon Workshop on Super High Energy Physics (World Scientific, Singapore, 1986) ed. D.E. Soper, p.41; FIELD-AJET, R.D. Field, Nucl. Phys. B264 (1986) 687.
- [13] G. Marchesini and B.R. Webber, Nucl. Phys. B238 (1984) 1; B.R. Webber, Nucl. Phys. B238 (1984) 492.
- [14] M. Bengtsson and T. Sjöstrand, Phys. Lett. 185B (1987) 435.
- [15] HERWIG, G. Marchesini and B.R. Webber, Nucl. Phys. B310 (1988) 461;  
 S. Catani, G. Marchesini and B.R. Webber, Cambridge preprint Cavendish-HEP-90/11.
- [16] OPAL Collaboration, M.Z. Akrawy et al., Phys. Lett. 235B (1990) 389;  
 DELPHI Collaboration, P. Aarnio et al., CERN preprint EP/90-89;  
 L3 Collaboration, B. Adeva et al., L3 preprint 011 (July 1990).
- [17] OPAL Collaboration, M.Z. Akrawy et al., CERN preprint PPE/90-97 (July 1990).
- [18] G. Marchesini and B.R. Webber, in Proc. Workshop on Physics at Future Accelerators, La Thuile, Italy, 1987 (CERN, Geneva, 1987), vol.II, p.364.
- [19] V. Barger, T. Gottschalk, J. Ohnemus and R.J.N. Phillips, Phys. Rev. D 32 (1985) 2950.
- [20] V. Barger, T. Han, J. Ohnemus and D. Zeppenfeld, Phys. Rev. D 40 (1988) 2888.
- [21] H. Baer, V. Barger and R.J.N. Phillips, Phys. Lett. 221B (1989) 398.
- [22] R.D. Field and T. Gottschalk, Phys. Rev. D 35 (1987) 875.
- [23] F. del Aguila, L. Ametller, R.D. Field and L. Garrido, Phys. Lett. 201B (1988) 375;
- [24] F. del Aguila, L. Ametller, R.D. Field and L. Garrido, Phys. Lett. 221B (1989) 408.
- [25] HERWIG version 4.6, I.G. Knowles, G. Marchesini, M.H. Seymour and B.R. Webber, available from the authors.
- [26] S.D. Ellis and J. Huth, contribution to these Proceedings.
- [27] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. 56 (1984) 579; *ibid.* 58 (1986) 1065.
- [28] F.E. Paige, private communication. See also B. Flaughner, K. Meier and K. O'Shaughnessy, contribution to these Proceedings.