Fermi National Accelerator Laboratory

FERMILAB-Conf-90/219-T

B PHYSICS ON THE LATTICE*

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Theoretical developments within the $m_Q \to \infty$ limit of QCD and their applications to B physics are reviewed. Effective actions, symmetry relations and perturbative matchings are discussed for the continuum and lattice theories. An analysis of recent lattice results which emphasizes the situation for f_B is presented.

1. INTRODUCTION

In the last few years, B physics has become an exciting subject both because of the prospects for detailed experimental study[1] and recent theoretical developments. A partial list of the aspects of B physics that are interesting to theorists would include: (1) the spectrum of b quark mesons and baryons - e.g. the excitation spectrum for B_u, B_d, B_s, and B_c mesons; (2) the internal structure of B hadrons - e.g. the charge density and magnetic moment; (3) the electroweak decays of B mesons - e.g. the decay constant f_B, and semileptonic decays $b \to c$ which depend on $|V_{cb}|$ and $b \to u$ which depend on $|V_{ub}|$; (4) mixing in the B meson system - e.g. the $B_0 - \overline{B_0}$ mixing parameter which together with f_B, the KM mixing angles (including $|V_{td}|$ or $|V_{ts}|$) and the top quark mass determine the mixing strength; and (5) CP violation which can be studied in rare decay modes - e.g. $B \to \psi + K_S$ and $B \to \pi + \pi$. It is particularly exciting to us here at this conference as lattice gauge calculations will play an important role in understanding each of the items above.

Recent theoretical developments, which have generated such excitement about B physics, exploit the fact that the QCD interactions of a heavy quark takes a simple form in the limit that the mass of the heavy quark goes to infinity. Since the b quark mass is significantly heavier than the other mass scales (the QCD scale and the light quarks masses) which enter into the dynamics of B hadrons, it is likely a good approximation to treat the b quark in $m_Q \to \infty$ limit within B hadrons.

The most familiar systems in which quark masses are large compared to the QCD scale are the $(\bar{c}c)$ and $(\bar{b}b)$ family of resonances - the J/ψ and Υ resonances. These systems are nonrelativistic. In such systems, the typical momentum transfer (between the heavy quark, Q, and antiquark, $\overline{Q}$) $< p^2 >^{1/2}$ is small compared to the heavy quark mass, m_Q, but not bounded as $m_Q \to \infty$. Instead the relative three velocity $v = 2 < \vec{p}^2 >^{1/2} / m_Q$ is bounded below by $4\alpha_s/3$.

In contrast, for hadrons that contain only one heavy quark a different dynamics applies in the limit that $m_Q \to \infty$. In such systems the heavy quark becomes static. The momenta transferred are typically of the QCD scale and the scale of the light quark masses and remains bounded as the heavy quark mass goes to infinity. The B mesons and baryons are prime examples of such systems. It is also fruitful to consider the charm mesons and baryons as such systems. Hadronic systems involving the top quark would be ideal except for the fact that when top quarks are produced their weak decays occur so rapidly that hadronic states do not have time to fully form. This makes study of top hadrons extremely difficult.

There is also a very practical reason for studying B mesons on the lattice using an effective theory for the heavy quark. The continuum limit is approached

*Talk presented at the 1990 Symposium on Lattice Field Theory, Tallahassee, Florida, USA, October 8-12, 1990

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

as the lattice spacing becomes small compared to all the physical parameters in the problem. As the quark mass increases it requires larger lattices to satisfy both $m_Q a \to 0$ and a physical volume for the lattice sufficiently large to keep the finite size effects small. For a $24^3 \times 48$ lattice at $\beta = 6.0$, a physical volume of $(2.4\text{fm})^3$ implies that $m_c a \approx .9$ and $m_b a \approx 2.6$. Clearly direct simulation of B mesons using the usual lattice fermion action (Wilson or KS) for the b quark is impractical. Heavy-light methods[2, 3] were proposed to allow the direct calculation of the properties of hadronic systems with only one heavy quark - such as B mesons. The basic idea is simply to use an effective interaction which arises in the static[4, 6] or nonrelativistic[5] limit of the heavy quark motion. I will discuss this approach in detail.

There are two methods for studying B physics on the lattice:

1. Use the standard lattice action and simulate with heavy quark masses as large as possible while still keeping the systematic errors associated with the large value of $m_Q a$ manageable. Then use the scaling behaviour determined from the continuum effective theory in the $m_Q \to \infty$ limit to determine how to scale the results to the B system. This will be denoted the extrapolation method.

2. Use a lattice effective action for the heavy quark which will determine the physical quantities in the $m_Q \to \infty$ limit of QCD. Systematic inclusion of $1/m_Q$ corrections will then give the properties for the B systems to increasing accuracy. This will be denoted the static (or heavy-light) method.

Both methods rely on the properties of the heavy quark limit of QCD.

Before turning to the effective action approach on the lattice, I will discuss the various actions which have been used to study the $m_Q \to \infty$ limit in the continuum theory, illustrate how they lead to new symmetry relations and determine how physical quantities scale with m_Q in the heavy quark limit.

2. EFFECTIVE ACTIONS

If a heavy quark is bound within a physical meson or baryon in which the other masses and momenta are of the order of the QCD scale, then, in the rest frame of this physical system, the heavy quark is essentially at rest and on shell. In this limit, the heavy quark propagates only in time. Three effective actions have been used to study QCD in this limit:

(1) The static effective action[6, 7] is valid if the momentum transfers to the heavy quark are cutoff at a scale fixed relative to the QCD scale even as the heavy quark mass increases without bound[8]. The Lagrangian is given by[5, 6, 7, 9]:

$$\begin{aligned}\mathcal{L} &= i\psi^\dagger \mathcal{D}_0 \psi - \psi^\dagger m_Q \psi + \frac{1}{2m_Q}\psi^\dagger \vec{\mathcal{D}}^2 \psi \\ &\quad + \frac{1}{2m_Q}\psi^\dagger \vec{\sigma}\cdot\vec{B}\psi + O(\frac{1}{m_Q^2}) \qquad (2.1)\end{aligned}$$

where $\psi \equiv \frac{(1+\gamma_0)}{2}\psi_{\text{Dirac}}$ is a two component quark field and $B^i \equiv \frac{1}{2} g t^a \epsilon^{ijk} F^a_{jk}$ and $\mathcal{D}_\mu = \partial_\mu - i g t^a A^a_\mu$. The field ψ can be rescaled by $\tilde{\psi} = \exp(i m_Q t)\psi$ and $\tilde{\psi}^\dagger = \exp(-i m_Q t)\psi^\dagger$ to eliminate the order m_Q term in the action.

The terms in the action of order $1/m_Q$ are treated as perturbations to the static limit for the heavy quark. Thus, in leading order, the propagator for the quark field $\tilde{\psi}$ is given by:

$$S^0_F(x) = -i\delta(\vec{x})\mathcal{P}(x^0)\theta(x^0) \qquad (2.2)$$

where $\mathcal{P}(x^0)$ is the time ordered phase integral from 0 to x.

(2) The second form of the effective action - the nonrelativistic (NR) action[3, 5, 9] - is closely related to the first. The static action is a special case of this action, since the NR action is valid as long as the heavy quark is moving with low velocity (i.e. $|\vec{v}| \ll 1$). The form of the bare NR action is the same as the static action given in Eq. 2.1. However, in the nonrelativistic limit, the kinetic motion of the heavy quark cannot be ignored. Hence the expression for the quark propagator differs even in the free quark limit. For the static action, the free quark propagator is $1/(p_0 + i\epsilon)$; while for the nonrelativistic action, the propagator is $1/(p_0 + \vec{p}^2/2m_Q + i\epsilon)$. Therefore, the renormalization of operators and perturbative corrections to matrix elements determined

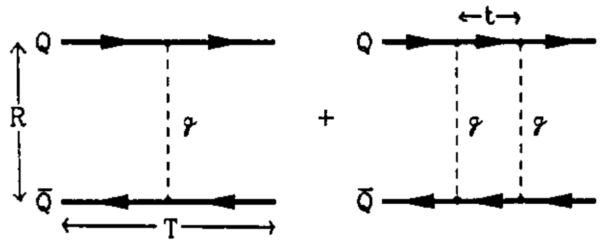

Figure 1: The first two terms in the QCD perturbative expansion for the scattering of a static quark and antiquark in Coulomb gauge. The static quark, Q and antiquark, $\overline{Q}$, are denoted by double solid lines and the Coulomb exchange, g, is denoted by a dashed line. In position space, the total time between initial and final states is denoted by T, while the relative time between interactions is denoted by t. The spatial $\overline{Q}Q$ separation is denoted by R.

from these two approaches can differ, even though the naive actions are the same. One might ask why, in the limit $m_Q \to \infty$, the kinetic energy contribution to the quark propagator can't always be treated perturbatively. The problem arises in certain physical situations in which the kinetic energy contribution is necessary to regulate the infrared behaviour. This cannot happen in a physical system with only one heavy quark[8]. However, in a bound state of a heavy quark-antiquark this does happen[9].

To illustrate the difference between the static and NR limit in a simple case, consider static quark-antiquark scattering. In Coulomb gauge the first two terms in the perturbative expansion for this process are shown in Fig. 1. The lowest order term is just a single Coulomb exchange. In position space this term is just given by $V(R) = -4\alpha_S/3R$. The second order term is given by $\frac{1}{2}[V(R)]^2$ integrated over the relative time, t, between the two interactions. The integration over relative time is only cutoff by the total time, T. This is the origin of the infrared divergence in this graph found in momentum space. In a physical heavy quark-antiquark system the large t behaviour is cutoff by the spatial propagation of the heavy quarks. No matter how small the kinetic energy term is, there is some time large enough that it cannot be neglected. On the other hand, integrating both terms over the total time, T, we see that these terms are simply the first and second order terms in the expansion of $\exp(-V(R)T)$ the static energy associated with the Wilson loop.

(3) The constant velocity effective action[10, 11] is a different generalization of the static action. The four velocity of the heavy quark (i.e. p^μ/m_Q) is fixed. All momentum transfers remain of the order of the QCD scale as for the static case. The Lagrangian for a heavy quark of four velocity v in lowest order in $1/m_Q$ is given by[11]:

$$\mathcal{L}_v = i\overline{\psi}_v \not{v} v^\mu \mathcal{D}_\mu \psi_v \qquad (2.3)$$

where $\psi_v = \exp(im_Q \not{v} v_\mu x^\mu)\psi$. Converting to two component notation we can define

$$\mathcal{L}_v = i\overline{\psi}_v^+ v^\mu \mathcal{D}_\mu \psi_v^+ - i\overline{\psi}_v^- v^\mu \mathcal{D}_\mu \psi_v^- \qquad (2.4)$$

where $\psi_v^\pm = \frac{1}{2}(1 \pm \not{v})\psi_v$. The total Lagrangian is given by:

$$\mathcal{L} = \int \frac{d^3v}{2v^0}\mathcal{L}_v \qquad (2.5)$$

There is a velocity superselection rule in this limit; fields associated with quarks moving with different velocities do not interact. The full Lagrangian (Eq. 2.5) is constructed to preserve a formal Lorentz invariance of the effective action. Note for $v^\mu = (\vec{0}, 1)$ the action Eq. 2.3 reduces to the static action. In the next section the symmetries of these effective actions will be explored.

3. SYMMETRY RELATIONS

To investigate the enhanced symmetries in the limit $m_Q \to \infty$ consider the static effective action given in Eq. 2.1. In leading order in $1/m_Q$ the Lagrangian has two types of symmetries. First, the Lagrangian is invariant under an arbitrary spin transformation of the quark field $\tilde{\psi} \to \exp(-i\vec{\alpha}\cdot\vec{\sigma})\tilde{\psi}$. Including both quarks and antiquarks this gives rise to an $SU(2) \otimes SU(2)$ symmetry. Second, if we allow for more than one flavor of heavy quarks then the Lagrangian, in leading order in $1/m_Q$, is just a sum of terms for each flavor of heavy quarks, hence for N flavors of heavy quarks there is a SU(N) flavor symmetry. Thus in the N flavor case the Lagrangian is symmetric under a full

$$SU(2N) \otimes SU(2N)$$

symmetry. This symmetry was first exploited by Isgur and Wise[10].

For the nonrelativistic effective action the kinetic energy term spoils the flavor symmetry. However the spin symmetry is maintained in leading order.

The largest symmetry group is preserved in the fixed velocity effective action[10, 11, 12]. There, all the symmetries of the static action are preserved, and in addition the Lorentz invariance of the full Lagrangian, Eq. 2.5, can be exploited. For a meson, H, containing a heavy quark, Q, and a light antiquark, $\bar{l}$, the transformation properties under rotations and boosts associated with the Lorentz group can be described by a 4×4 matrix $\tilde{H}(v)$ which transforms as

$$\tilde{H}(v) \to D(\Lambda^{-1})\tilde{H}(\Lambda^{-1}v)D(\Lambda)$$

Here

$$D(\Lambda) = \exp\left(i\sigma^{\mu\nu}\epsilon_{\mu\nu}\right)$$

is the usual Lorentz representation. The appropriate transformation properties[12, 13] for a heavy-light pseudoscalar meson state is $\tilde{H}(v) = (1+\not{v})/2$ and for a vector meson state for polarization ε is $\tilde{H}^*(v) = \gamma_5\not{\varepsilon}(1+\not{v})/2$. Finally to eliminate any explicit mass dependence in the normalization of the matrix elements, define a noncovariant normalization

$$< H(p', s')|H(p, s) >= \delta_{s's}\delta(\vec{p'} - \vec{p})$$

where p, p' are momenta and s, s' are possible spins for the heavy-light state H.

Now these symmetries can be applied to obtain information about the mesons containing one heavy quark[10, 11, 12, 13].

- First consider the axial current $J_A^\mu(x) = \bar{l}(x)\gamma^\mu\gamma^5 Q(x)$. The matrix element of this current between a pseudoscalar heavy-light state, H, and the vacuum is given by

$$-i < 0|J_A^\mu(x)|H(p) > \equiv \frac{f_H p^\mu}{\sqrt{2E_H}} \exp\left(-ip\cdot x\right) \tag{3.1}$$

  where f_H is the meson decay constant. The covariant current J_A can be reexpressed in terms of the current in the effective theory $\tilde{J}_A$ as follows:

$$J_A^\mu(x) = \tilde{J}_A^\mu(x)\exp\left(-im_Q v^\mu x_\mu\right). \tag{3.2}$$

  Thus the matrix element for the effective current is given by

$$\begin{aligned} -i < 0|\tilde{J}_A^\mu(x)|H(p) >= \\ \frac{f_H m_H}{\sqrt{2E_H}} v^\mu \exp\left[-i(m_H - m_Q)v^\mu \cdot x_\mu\right]. \end{aligned} \tag{3.3}$$

  Since the current and matrix elements are defined to remove all explicit dependence on m_Q for large m_Q, the right hand side of Eq. 3.3 must also be independent of m_Q as $m_Q \to \infty$. The four momentum of the meson is replaced by the four velocity which is independent of m_Q and $E_H/m_H = v^0 = 1/\sqrt{1-\vec{v}^2}$ is independent of m_Q. Therefore, the binding energy $m_Q - m_H$ appearing in the exponential and the combination of the decay constant and m_H, $f_H\sqrt{m_H}$ which appears in the prefactor must also be independent of m_Q. These conclusions are easy to understand since the dynamics of the H meson would be expected to depend on the QCD scale and the reduced mass of the quark-antiquark system both of which are independent of m_Q in the limit $m_Q \to \infty$. In particular, in a potential model the combination $f_H\sqrt{m_H}$ is simply related to the magnitude of the wave-function at the origin of the H meson

$$f_H\sqrt{m_H} = \sqrt{12}|\psi(0)| \tag{3.4}$$

  and hence independent of m_Q.

  At this point I should comment that the scaling behaviour being discussed here use the relation (Eq. 3.2) between bare currents in the full and effective theory. These relations will be slightly modified when perturbative QCD corrections are included. This will result in additional logarithmic corrections to the scaling behaviour for f_H given above. I will discuss these corrections in the next section.

- Second, consider a general current $J(x)$ involving two heavy quarks, Q and Q', given by

$$\begin{aligned} \overline{Q}'(x)\Gamma Q(x) = \\ \exp\left[-i(m_Q v^\mu - m_{Q'} v'^\mu)x_\mu\right]\tilde{J}(x). \end{aligned} \tag{3.5}$$

  The matrix element of the current, $\tilde{J}$, between two heavy-light mesons, H and H' (containing the heavy quarks Q and Q' respectively) can be written in a form where the Lorentz structure of the matrix element is explicitly evaluated. Using the known transformation properties for the heavy mesons under Lorentz transformations on the heavy quark fields, the general form is given by

$$< H'(v')|\tilde{J}(0)|H(v) >= \frac{1}{2}\sqrt{\frac{m_{H'}m_H}{E_{H'}E_H}}\xi(v'\cdot v)Tr[\tilde{H}'(v')\Gamma\tilde{H}(v)]. \tag{3.6}$$

$\tilde{H}$ and $\tilde{H}'$ are the 4×4 matrices appropriate for the particular meson state as defined before.

The major result of this analysis is that there is a *universal* form factor ξ in leading order in $1/m_Q$. The form factor, ξ, is independent of the Lorentz structure of the current, $\tilde{J}$, and the spin and heavy flavor of either of the states H' and H. All spin and heavy flavor information is contained in the trace. This result was obtained by Isgur and Wise[10] and then in the fixed velocity formulation of Georgi[11] by Falk, Georgi, Grinstein and Wise[12].

As a simple application of this result consider the matrix element of the vector current between a single pseudoscalar heavy-light meson, H. Using $\tilde{H}=(1+\not{v})/2$, Eq. 3.6 becomes

$$< H(v')|\tilde{V}^{\mu}(0)|H(v) >= \frac{m_H}{\sqrt{E'_H E_H}}\xi(v'\cdot v)(\frac{v'^{\mu}+v^{\mu}}{2}). \tag{3.7}$$

The normalization of ξ at $v'\cdot v = 1$ can be determined by considering the special case of the meson at rest, where $v'\cdot v=1$. Since $\tilde{V}^0$ is the zero component of a conserved charge we conclude that $\xi(1)=1$.

The general result of Eq. 3.6 can be applied to many experimentally important processes. Prime examples are semileptonic decays of B mesons to D mesons. For the decays of a pseudoscalar meson $H=B$ to a final pseudoscalar $H'=D$ only the vector current contributes, and the matrix element is given by

$$< H'(v')|\tilde{V}^{\mu}(0)|H(v) >= \sqrt{\frac{m_{H'}m_H}{E_{H'}E_H}}\xi(v'\cdot v)(\frac{v'^{\mu}+v^{\mu}}{2}). \tag{3.8}$$

If the final state is a vector meson, $H'=D^*$ both vector and axial vector currents contribute. The matrix elements are:

$$< H'^*(v',\epsilon)|\tilde{V}^{\mu}(0)|H(v) >= -i\frac{1}{2}\sqrt{\frac{m_{H'}m_H}{E_{H'}E_H}}\xi(v'\cdot v)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^{\alpha}v^{\beta} \tag{3.9}$$

and

$$< H'^*(v',\epsilon)|\tilde{A}^{\mu}(0)|H(v) >= \frac{1}{2}\sqrt{\frac{m_{H'}m_H}{E_{H'}E_H}}\xi(v'\cdot v)[\epsilon^*_{\mu}(1+v'\cdot v)-v'_{\mu}v\cdot\epsilon^*] \tag{3.10}$$

There are many applications of these relations[10, 12]. For example, $\overline{B}\to D+e\nu$ and $\overline{B}\to D^*+e\nu$ decays are related in leading order in $1/m_b$ and $1/m_c$. Also since $\xi(1)=1$ the semileptonic decay $\overline{B}\to D+e\nu$ can be absolutely normalized. This allows a determination of $|V_{cb}|$ from this process.

- Finally, consider the matrix elements of a general heavy-light current between a heavy meson and an ordinary light meson (eg. a pion, rho, or omega). Such matrix elements are needed if one is to use measurements of charmless exclusive semileptonic B decays to extract $|V_{ub}|$. Unfortunately, there is no reduction in the number of form factors[14, 15] in these processes even in the $m_Q\to\infty$ limit. On the other hand, when the momentum of the final light hadron is small, it is precisely these decays which can be computed using lattice methods[16]. The measurement of these form factors in that limit should be a important goal for lattice simulations in the coming years.

Now that some uses of symmetry relations have been examined[17], I will turn to the calculation of physical quantities in the heavy-quark limit. These calculations necessitate perturbative matching and lattice simulation.

4. PERTURBATIVE MATCHING

An appropriate lattice action for studying both light and heavy quarks in the $m_Q \to \infty$ limit of QCD is given by:

$$S_E = S_{gauge} + \sum_{flavors} S_{light} + \sum_{flavors} S_{heavy}. \quad (4.1)$$

Here S_{gauge} is the standard gauge action with coupling strength $\beta = 6/g^2$; S_{light} is the Wilson action for various flavors, f, of light quarks with mass determined by its associated hopping parameter, κ_f; and for each flavor of heavy quark, Q, the action S_{heavy} is given to order $1/m_Q$ by[18]

$$\begin{aligned}
S_{heavy} = & \qquad\qquad\qquad\qquad\qquad\qquad (4.2)\\
& (-a)^3 \sum_x \{Q^\dagger(x)[Q(x) - U_0^\dagger(x-\hat{0}a)Q(x-\hat{0}a)] \\
& + \frac{1}{2am_Q} \sum_{j=1}^{3} Q^\dagger(x)[U_j^\dagger(x-\hat{j}a)Q(x-\hat{j}a) \\
& \quad + U_j(x+\hat{j}a)Q(x+\hat{j}a) - 2Q(x)] \\
& + \frac{1}{2am_Q} \sum_{i,j,k=1}^{3} Q^\dagger(x)\sigma^k \epsilon_{ijk}(\frac{1}{8} \sum_{ni,nj=0}^{1} \frac{1}{2i} \\
& \quad [U_{P(ij)} - U_{P(ij)}^\dagger](x - n_i\hat{i}a - n_j\hat{j}a))Q(x)\}.
\end{aligned}$$

The first order $1/m_Q$ term in this action is just a lattice form of the kinetic energy term of Eq. 2.1. The second term is the spin interaction with the (spatially smeared) gauge plaquette, U_P. The gauge field dependence in the second term reduces to the chromomagnetic field in the continuum limit. Heavy antiquarks are included by adding a term to S_{heavy} with Q replaced with the charged conjugate field Q_c and U transformed by $U^\dagger(x-\hat{0}a) \to U^*(x)$.

Having defined the lattice action for the static limit, it remains to relate the matrix elements measured using this action on the lattice to the corresponding physical quantities in the continuum. This matching can be done analytically in perturbation theory. The general procedure is well known[19]; however, the application to the static effective theory is new.

The matching is done in two stages. The first step is to match the effective theory to the full theory and then the lattice effective theory is matched to the effective continuum theory.

The first quantity for which this procedure was applied is the matrix element of the axial current

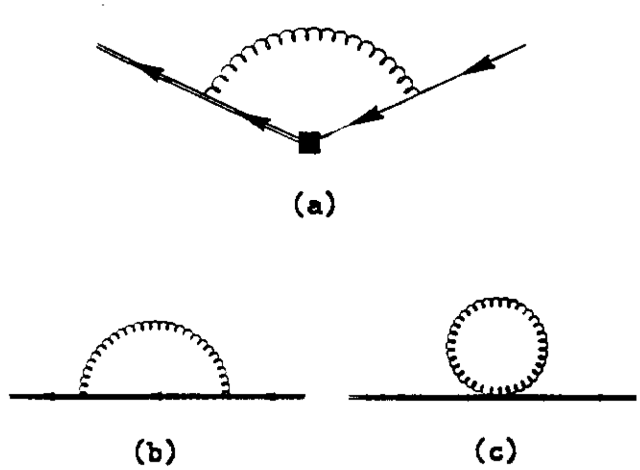

Figure 2: One loop corrections for the axial current between a heavy and light quark. The heavy quark is denoted by a double solid line, the light quark by a single solid line and the gluon by a curly line. The vertex correction is shown in (a). The usual self-energy correction for the heavy quark is shown in (b). For the lattice theory there is also a tadpole self-energy correction shown in (c). The standard light quark self-energy corrections are not shown.

between a heavy-light pseudoscalar meson and the vacuum[20, 21, 22]. This matching determines how to normalize the bare value of the decay constant measured on the lattice to its physical definition.

For example, the one loop vertex correction for the axial current is shown in Fig. 2(a). This vertex correction can be calculated by straightforward methods[6, 20] in the static effective field theories as well as in the full theory. The infrared divergences can be regulated using a gluon mass, λ. The ultraviolet divergences can be dimensionally regulated in the continuum full and effective theories using the $\overline{MS}$ scheme. The ultraviolet divergences are cutoff by the finite spacing, a, on the lattice. Finally, the matrix elements of the renormalized currents are defined at scale μ. It is also simple to compute the heavy quark wavefunction renormalizations shown in Fig. 2(b and c). with the three actions. The results for the vertex and heavy quark self-energy corrections in one loop are shown in Table 1. The standard light quark corrections[19] are also included in Table 1. Since the axial current is partially conserved, all

Correction	FULL	EFF	LAT
Z_V	$\ln(\frac{\mu^2}{\lambda^2})$ $+1$	$\ln(\frac{\mu^2}{\lambda^2})$ $+1$	$-\ln(\lambda^2 a^2)$ $+12.68$
Z_Q	$2\ln(\frac{m^2}{\lambda^2})+$ $\ln(\frac{m^2}{\mu^2})-4$	$2\ln(\frac{\mu^2}{\lambda^2})$	$-2\ln(\lambda^2 a^2)$ $+24.48$
Z_l	$-\ln(\frac{\mu^2}{\lambda^2})$ $+1/2$	$-\ln(\frac{\mu^2}{\lambda^2})$ $+1/2$	$\ln(\lambda^2 a^2)$ $+13.35$

Table 1: Results for the one loop renormalizations of the axial current vertex, Z_V; the heavy quark wavefunction, Z_Q and the light quark wavefunction, Z_l. The results for the full theory are given in the column denoted FULL. The results for the effective continuum theory are given in the column denoted EFF[6, 20, 21]. The results for the lattice static theory are given in the column denoted LAT[20]. The renormalization scale in the continuum is μ and the gluon mass is λ. The lattice spacing is a. A common factor of $g^2/12\pi^2$ multiplies all the entries given here.

dependence on the renormalization scale μ drops out for combination $Z_V {Z_Q}^{1/2} {Z_l}^{1/2}$ corresponding to the renormalization of the matrix element of the axial current in the full theory. Futhermore, the dependence of the gluon mass is the same for each action since the infrared behaviour of perturbation theory is identical in all three theories.

In the final matching it is convenient to choose $\mu = 1/a$. Using the results in Table 1, the ratio of matrix elements between the lattice and continuum effective theories is

$$\frac{<0|J_A^0|B>_{LAT}}{<0|J_A^0|B>_{EFF}} = 1 + \frac{g^2}{12\pi^2}(30.35) \qquad (4.3)$$

and between the effective and full continuum theories is

$$\frac{<0|J_A^0|B>_{EFF}}{<0|J_A^0|B>_{FULL}} = \qquad (4.4)$$
$$1 + \frac{\alpha_S^{\overline{MS}}(\mu)}{3\pi}[\frac{3}{2}\ln(\frac{\mu^2}{m_b^2}) + 2].$$

Hence, the physical matrix element of the axial current between the B meson and the vacuum is related to the bare lattice quantity by

$$<0|J_A^0|B>_{FULL} = Z_A <0|J_A^0|B>_{LAT} \qquad (4.5)$$

where Z_A is given by

$$Z_A^{-1} = [1+\frac{\alpha_S^{\overline{MS}}(\mu)}{3\pi}(\frac{3}{2}\ln(\frac{\mu^2}{m_b^2})+2)][1+\frac{g_{lat}^2}{12\pi^2}(30.35)]. \qquad (4.6)$$

Choosing $\beta = 6.0$ so $g_{lat} = 1$, and $\alpha_S^{\overline{MS}}$ at $\mu = 2.0$ GeV for $\Lambda_{QCD} = 250$ MeV and four active quarks; $Z_{EFF}/Z_{FULL} = .97$, $Z_{LAT}/Z_{EFF} = 1.26$ and finally

$$Z_A = .82 \qquad (4.7)$$

Three comments on this result are in order:

(1)The previous numerical disagreement for some of the matching constants has been resolved. The method of Boucard, Lin and Pene[21] now agrees with the effective action method presented here[23].

(2)The renormalization group improved relation between the full and effective theory is given by

$$J_A^0(a^{-1}) = \qquad (4.8)$$
$$J_{LATA}^0(a^{-1})[\frac{\alpha_S(m_b)}{\alpha_S(a^{-1})}]^{\frac{\gamma_0}{2\beta_0}}(Z_{EFF}/Z_{LAT})$$
$$(Z_{FULL}/Z_{EFF})_{|\mu=m_b}[1+$$
$$\frac{\alpha_S(a^{-1})-\alpha_S(m_b)}{4\pi}(\frac{\beta_1\gamma_0}{2\beta_0^2}-\frac{\gamma_1}{2\beta_0})]$$

where here β is the QCD beta function and γ is the anomalous dimension for the axial current

$$\gamma = \gamma_0\frac{g^2}{16\pi^2} + \gamma_1(\frac{g^2}{16\pi^2})^2 + \cdots \qquad (4.9)$$

Both the first[24], $\gamma_0 = -4$ and the second[25], $\gamma_1 \approx -42$ for $N_f = 3$, coefficients have now been calculated.

(3)The size of Z_A is very sensitive to the choice of coupling constant for the lattice to continuum effective theory matching. The value chosen above corresponds to the bare lattice coupling. It has been argued by Lepage and Mackenzie[26] that a more appropriate coupling can be defined using the heavy quark potential measured on the lattice. With their choice, the value of Z_A is reduced to ≈ 0.65. Clearly, two loop calculations of these corrections would help in determining the proper lattice coupling for this matching.

5. LATTICE RESULTS

The focus of lattice simulations for B physics in the last year has been the extraction of a reliable signal for the ground state pseudoscalar heavy-light meson and the measurement of the value of the associated heavy-light meson decay constant. Since in the $m_Q \to \infty$ limit the dynamics is heavy flavor independent I will often use the B meson to mean any heavy-light meson. I will discuss the comparison of these results with the results from the extrapolation method at the end of this section.

All these measurements are made by considering the two point correlation between operators with appropriate quantum numbers to couple to the B meson ground state. The general form is

$$C^{(12)}(T) = <0|\mathcal{O}^{(1)}(T)\mathcal{O}^{(2)}(0)|0> \qquad (5.1)$$

and by assumption as $T \to \infty$

$$\begin{aligned} C^{(12)}(T) &\to <0|\mathcal{O}^{(1)}(T)|B><B|\mathcal{O}^{(2)}(0)|0> \\ &= Z_{(1)}Z_{(2)}\exp(-\mathcal{E}T) \end{aligned} \qquad (5.2)$$

where the mass in the static theory $\mathcal{E}$ is defined by $\mathcal{E} = M_B - m_b$ (i.e. minus the binding energy) and $Z_{(i)}$ is the coupling of the operator $\mathcal{O}^{(i)}$ to the B state.

In the static limit, the good quantum numbers are the total angular momentum (J), parity, and total angular momentum of the light quark (j_l). Different values of j_l are *not* degenerate even in the static limit. In the usual nonrelativistic terminology, the S-wave B mesons (the pseudoscalar and vector) are degenerate, but the P- wave mesons are split. There are two degenerate states with $(j_l, J) = (3/2, 2)$ and $(3/2, 1)$, and two with $(j_l, J) = (1/2, 1)$ and $(1/2, 0)$. The difference between the mass measured by this method for the various B meson states and that of the pseudoscalar B meson (the true ground state) is the excitation energy and is physical. In order $1/m_Q$, all remaining degeneracies are broken. It is to be expected that corrections of order $1/m_b$ to the spectrum with a given (j_l, J) should be of the order of ten percent of the splitting between ground and first excited state. A few initial measurements of excited states in the B meson system have been made[27, 28]; but much remains to be explored.

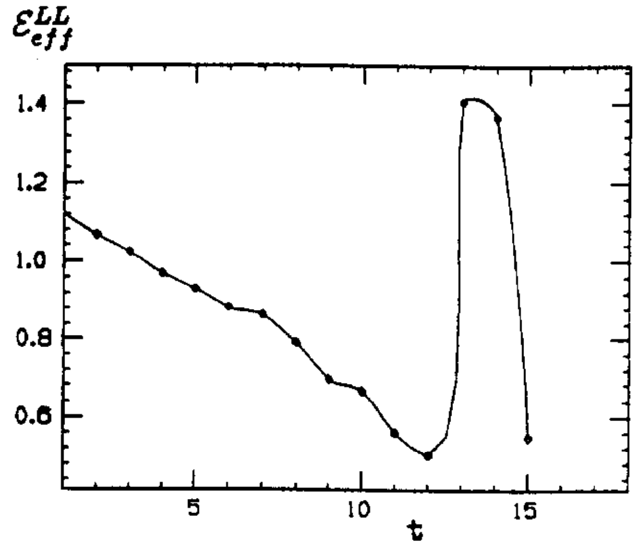

Figure 3: The effective mass associated with the axial current - axial current correlator measured from adjacent time slices. This effective mass is defined as $\mathcal{E}^{LL}_{eff}(t) = \ln(C^{LL}(t)/C^{LL}(t+1))$ and is shown as a function of the dimensionless time t. Results were obtained from 30 configurations on a $20 \times 10^2 \times 40$ lattice at $\beta = 6.0$ and $\kappa = .1515$ by Boucaud et. al.[29].

For the ground state pseudoscalar B meson, the original measurements of the correlator of Eq. 5.1 choose the time component of the axial current for the operators $\mathcal{O}^{(L)} = \mathcal{O}^{(1)} = \mathcal{O}^{(2)}$ and then

$$Z_L = \frac{f_B\sqrt{m_B}}{2m_B} Z_A^{-1} \qquad (5.3)$$

At the time of the last lattice conference: (1) It was clear from the results of Boucard, Pene, Hill, Sachrajda, and Martinelli[29] shown in Fig. 3 that using the local axial current for both operators in the correlator did not allow the extraction of the ground state. As can be seen in Fig. 3, the effective mass measured from adjacent time slices never reaches a plateau. (2) However, there was already preliminary evidence[16] showing that using a smeared operator could improve the situation. This year we have seen from a number of groups that this is indeed the case.

The basic idea of smearing is to enhance the coupling to the ground state by a suitable choice of a spatially non-local operator. This technique had already been applied successfully to the ordinary hadrons by the APE group[30].

A smeared operator can be used on the source side ($t = 0$) or the sink side ($t = T$) or both. The corresponding correlators will be denoted by C^{SL}, C^{LS}, and C^{SS} respectively,

For the correlator with two smeared operators, there should be a large contribution from the ground state with mass $\mathcal{E}$ and smeared operator coupling Z_S as well as remaining contributions from excited states with mass $\mathcal{E}' > \mathcal{E}$ and couplings Z'_S. Therefore, for large times the C^{SS} correlator is given by

$$C^{SS}(T) \to \sum_{excited} ({Z'_S}^2 \exp(-\mathcal{E}'T)) + Z_S^2 \exp(-\mathcal{E}T). \tag{5.4}$$

Similarly, the correlator of the local axial current with one smeared operator should behave like

$$C^{LS}(T) \to \sum_{excited} (Z'_L Z'_S \exp(-\mathcal{E}'T)) + Z_L Z_S \exp(-\mathcal{E}T). \tag{5.5}$$

Hence the ratio

$$C^{LS}(T)/C^{SS}(T) \to Z_L/Z_S[\frac{1 + \sum Z'_L/Z_L Z'_S/Z_S \exp[-(\mathcal{E}' - \mathcal{E})T]}{1 + \sum(Z'_S/Z_S)^2 \exp[-(\mathcal{E}' - \mathcal{E})T]}] \tag{5.6}$$

will approach Z_L/Z_S at sufficiently large time. Thus, by measuring these two correlators, the value Z_L and hence f_B can be extracted.

In the last year, three groups have produced results for f_B using this method:

1. Allton, Sachrajda, Lubicz, Maiani, and Martinelli[31] have results for $\beta = 6.0$ on a $10^2 \times 20 \times 40$ lattice. I will denote this group RS.

2. Alexandrou, Jegerlehner, Gusken, Schilling, and Sommer[27] have results for $\beta = 6.0$ on $8^3 \times 36$ and $12^3 \times 36$ lattices, for $\beta = 5.74$ on a $8^3 \times 24$ lattice, and for $\beta = 5.82$ on a $6^3 \times 28$ lattice. I will denote this group W.

3. Bernard, Labrenz, and Soni[32] have results for $\beta = 6.0$ on a $16^3 \times 40$ lattice. I will denote this group UCLA.

I will focus on the data at $\beta = 6.0$ for which all three groups have results.

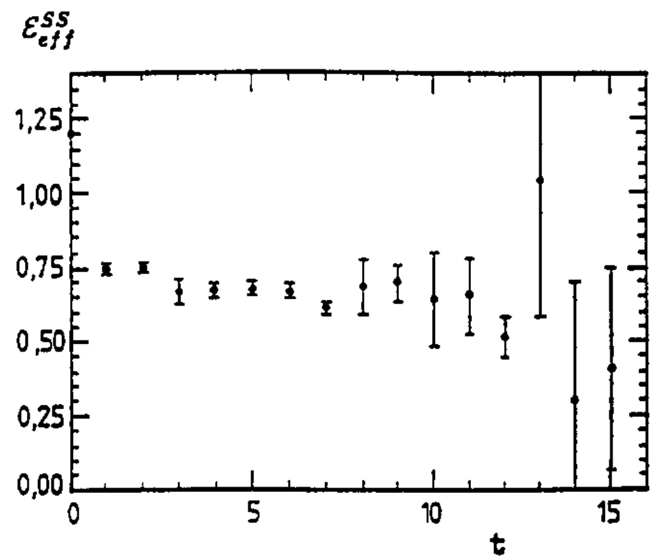

Figure 4: The effective mass, $\mathcal{E}^{SS}_{eff}$, for the smeared correlator measured using adjacent time slices. The lattice parameters are the same as in Fig. 3. Unlike the situation for $\mathcal{E}^{LL}_{eff}$, a true plateau is seen in this data from the RS group[31].

Before turning to the results, a few more details about constructing the spatially smeared operator discussed above are in order. A number of different operators were chosen by the various groups. The RS group worked in Coulomb gauge and used nonlocal operators of the general form

$$\mathcal{O}^S(t) = Q^\dagger(\vec{x}, t)\gamma^5 l(\vec{0}, t) \tag{5.7}$$

They constructed smearing operators from the operator $\mathcal{O}^S$ by averaging the position of the heavy quark relative to the light quark over spatial cubes of length 3, 5, and 7 on a side. They also employed operators where this procedure was done twice. The UCLA group worked in Landau gauge and used operators smeared over cubes of length 3 and 5 on a side. In contrast, the W group smeared in a gauge invariant manner. They averaged the relative position of the heavy quark with a weighting function with one of two forms: (1) a simple Gaussian and (2) an exponential from the spatial propagator of a scalar field with adjustable mass.

All these methods succeeded in producing a dramatic improvement in the ability to extract the effective mass of the ground state. The results for the RS group with smearing are shown in Fig. 4.

Various choices for the detailed smearing opera-

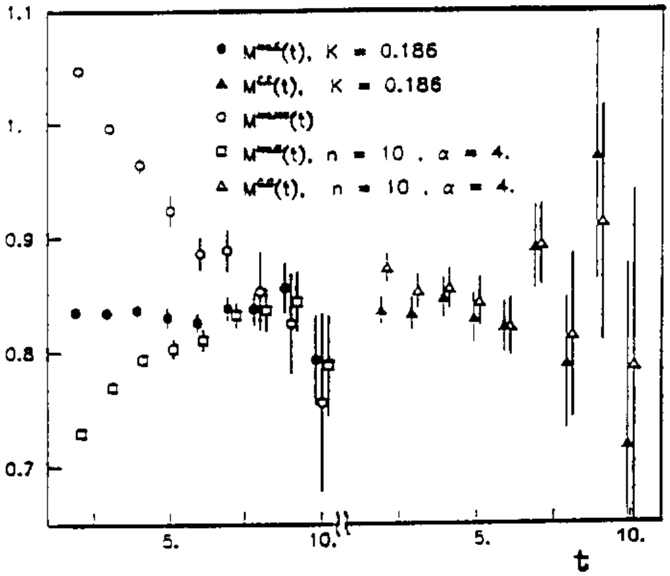

Figure 5: The time dependence of the effective mass for various smeared correlators. Results were obtained from 100 configurations on a $8^3 \times 24$ lattice at $\beta = 5.7$ and $\kappa = .166$ by the W group[27]. They used the local axial current (L) and two types of gauge invariant smearings: Exponential (E) and Gaussian (G). Here the correlators are denoted as follows: open circles (LL), solid circles (LE), open squares (LG), solid triangles (EE), and open triangles (GG). (For details see the original work[27]).

tor do affect the short time behaviour of the correlator as can be clearly seen in Fig. 5. This is expected as different smearing operators will have different couplings to the excited states. For the C^{LS} correlator (with a number of choices for the smearing), the effective mass approaches its asymptotic value from below. This, at first, surprising result can be seen from Eq. 5.5 to happen whenever $(Z_L' Z_S' / Z_L Z_S)$ is negative. Nonrelativistically, the first excited state which can contribute will have a radial wavefunction which changes sign; hence, this behaviour would be explained if the smearing weight in the region outside the node in the wavefunction is sufficiently strong.

Now consider the results at $\beta = 6.0$ for the RS and W groups. The comparison of mass values extracted at various κ values for the light quark is given in Table 2. The agreement between the groups is excellent. However, the mass measured here

$$M_B - m_b(\text{bare}) = M_B - m_b(\text{renorm}) - \delta m \quad (5.8)$$

kappa	RS	kappa	W
.1515	$.663 \pm .014$	.1525	$.687 \pm .012$
.1530	$.643 \pm .018$	.1540	$.666 \pm .013$
.1545	$.618 \pm .017$	.1550	$.653 \pm .014$

Table 2: Results for the mass $m_B - m_b$ in lattice units for the RS and W groups. The various values of κ for the light quark mass are indicated with the resulting value.

Group	f_B (GeV)
RS	$.310 \pm .025$
W	$.300 \pm .020$
UCLA	$.390 \pm .028$

Table 3: Results for f_B from the RS, W, and UCLA groups. The results presented here are all for $\beta = 6.0$ and have been scaled to a common value for the lattice spacing $1/a = 2.0$ GeV. No systematic errors are shown in this table.

is unphysical since δm is linearly divergent[20] as $a \to 0$.

For the comparison of f_B I have adjusted their results so the all are reported for the same value of $1/a$. The RS group used $1/a = 2.0$, while the W group used $1/a = 2.3$ and the UCLA group used $1/a = 1.75$. I will use $1/a = 2.0$ GeV and the renormalization $Z_A = .8$. The results for f_B are shown in Table 3. The agreement is excellent and the result surprisingly large[34].

The RS and W groups reports the value of f_B for light quark mass extrapolated to zero; i.e. $\kappa = \kappa_{critical}$; while the UCLA value is for $\kappa = .156$. There is a slow variation of the decay constant with the light quark mass as shown in Fig. 6.

We are now in a position to compare the static method with the extrapolation method discussed in the introduction. This comparison is made for $f_H\sqrt{m_H}$ in Fig. 7.

In the scaling limit $f_H\sqrt{m_H}$ should be approximately constant (up to the logarithmic scale violation given in Eq. 4.8). It is clear that the two

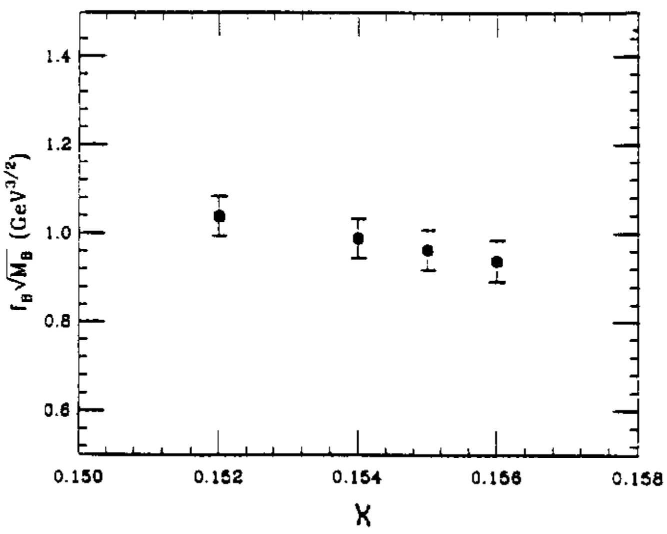

Figure 6: The dependence of $f_B\sqrt{m_B}$ on κ. Results from the UCLA group[32] at $\beta = 6.0$. $\kappa_{critical} \approx .157$.

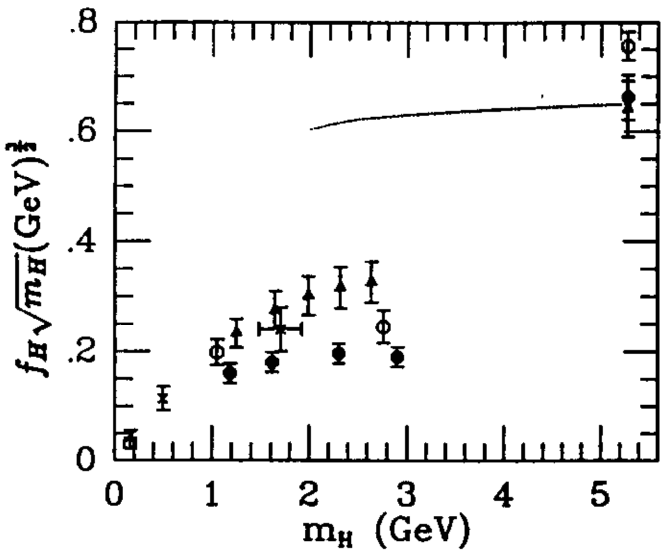

Figure 7: Comparison of values for the scaling form for the meson decay constant, $f_H\sqrt{M_H}$, from extrapolation and static methods. Results for a Wilson heavy quarks are denoted as follows: C. Bernard et. al.[33] solid circles; T. A. DeGrand et.al.[35] open circles; and M. B. Gavela et. al.[36] crosses; C. R. Allton et. al. [37] open triangles. Results extent up to pseudoscalar meson masses, M_H of $\approx$ 3 GeV. Results for a static heavy quark are denoted as follows: RS group[31] solid triangle; W group[27] open circle; and UCLA group[32] solid circle. The static results shown at $M_H = M_B$ are valid in the limit of an infinitely massive heavy quark. The scaling behaviour for the static case (Eq. 4.8) leads to a variation with M_H depicted by the dotted line.

methods have not converged. If the scaling region is already setting in at the mass of the D meson mass, then there is a significant systematic difference between the two methods.

The systematic effects which could account for all or part of the disagreement between the static and extrapolation methods of determining f_B are:

- Important finite size effects (FSE) or finite a scale violation effects. The W group[27] have found that there is no significant volume dependence changing the spatial extent from 0.7 to 1.05 fm. Also, the W group[27] only found 15% scale violation from $\beta = 5.7$ to 6.0. Goity has analyzed the FSE in the continuum for large volumes.[38]

- Large $m_Q a$ corrections particularly for the extrapolation method. Both the UCLA group[32] and the RS group[31] have presented preliminary data at larger β. At present, the situation remains uncertain. An improved action for Wilson fermions has been developed[39] which will help in understanding the first order $m_Q a$ corrections.

- Large perturbative correction Z_A for f_B in the static effective theory.

- Large $1/m_Q$ corrections to the $m_Q \to \infty$ limit.

One general consistency test, suggested by the differences seen here, is to measure wavefunction of the B meson. Since the difference between the two methods remains large even for a relatively heavy light quark, one might use the relation between the decay constant and the wavefunction at the origin

$$f_B\sqrt{m_B} = \sqrt{12}|\psi(0)| \qquad (5.9)$$

to extract a value of f_B.

Preliminary studies of this method were reported at this conference[28, 32]. Labrenz[32] reports that indeed the wavefunction of a heavy-light meson is more compact than that of a light hadron, and that the value of f_B measured from this method is roughly consistent with the large value reported using the static effective action. The small size of the B meson is consistent with the lack of volume dependence of the results for f_B found by the W group[27]. It would also help explain the intermediate time behaviour of the various C^{LS} correlators and this may aid in finding a better smearing operator for the B meson. Clearly much remains to be understood, but this approach looks promising.

We must eventually compute the $1/m_Q$ corrections to the static limit. To date the only progress has been in the continuum effective theory. In particular the one loop matching has been done for all the operators which contribute to the $1/m_Q$ corrections to f_B[40]. The matching between the lattice and continuum effective theories has not been done yet and none of the $1/m_Q$ corrections has yet been measured on the lattice.

6. SUMMARY AND OUTLOOK

There have been major theoretical developments in B physics in the last few years:

- Effective actions for QCD in the $m_Q \to \infty$ limit have been developed. Three forms which are particular useful are the static, nonrelativistic, and fixed velocity effective actions.

- Applications of the symmetries in the $m_Q \to \infty$ limit have lead many useful relations for B and D physics.

- For these effective actions, perturbative matching and renormalization has been done, to one loop, for a number of physical quantities.

- The $1/m_Q$ corrections to the heavy quark limit have come under intense study. In particular, the perturbative renormalizations for all the $1/m_Q$ contributions to f_B have been done for the continuum static and fixed velocity actions.

Furthermore significant progress has been made through lattice simulations in the last year. The focus of much of the effort has been on the extraction of f_B. The status is as follows:

- By employing smearing techniques, numerically reliable estimates of f_B have been obtained by a number of groups in the static effective theory approach.

- At present there is a sizable gap between the value of f_B obtained by the static quark and extrapolation approach.

The coming year should bring a number of improvements in this situation for f_B:

- One expects the first lattice measurements of the $1/m_Q$ corrections.

- More study of the wavefunction and other detailed properties of heavy-light mesons should aid in understanding the origin of the disagreement between these two methods.

- For the extrapolation method, studies at larger β should allow a clear determination as to whether the large mass scaling limit has actually set in by the mass of the D meson.

Of course there is a rich and varied set of other applications of heavy-light methods to the B meson system. We have only begun the study of B physics on the lattice.

ACKNOWLEDGEMENTS

I thank C. Allton, C. Bernard, A. Duncan, B. Hill, J. Labrenz, P. Mackenzie, C. Sachrajda, S. Sharpe, A. Soni, and H. Thacker for useful discussions. I also am grateful for the hospitality of the Institute of Theoretical Physics in Santa Barbara where some of this review was prepared.

This research supported in part by the National Science Foundation under Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration.

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