Fermi National Accelerator Laboratory

QCD PHENOMENOLOGY OF PARTON DISTRIBUTION FUNCTIONS AT SMALL X

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The small x behavior of parton distributions is studied phenomenologically by examining in detail a series of QCD-evolved distribution sets obtained in a new global analysis of deep inelastic scattering and lepton-pair production experiments. The importance of 2-loop evolution is discussed. The main features and results of the global analysis are described. The range of small x behavior consistent with next-to-leading order QCD and current data is delineated. The extrapolated small x behavior is parametrized by effective Q-dependent power- and logarithmic-law parameters. Intriguing features of the evolution of these parameters with Q are presented. Alternative parametrizations based on the analytic solution for small x is also explored.

1. INTRODUCTION

This talk is concerned with the phenomenological study of parton distributions in the conventional perturbative QCD framework with emphasis on their small-x behavior. The investigation is based on a recent global analysis of parton distributions [1] which was motivated by the increasing need for more detailed knowledge on parton distributions for precision investigations of the Standard Model, for quantitative treatment of jet physics, as well as for information on the range of possible small-x behavior for the exploration of new physics at future accelerators.

In Sec. 2 we point out the importance of using 2-loop evolution of the parton distributions in the quantitative study of their small-x behavior. In Sec. 3 we describe the next-to-leading order (NLO) global analysis of current high statistics deep inelastic scattering and lepton-pair production data which forms the basis of our study. This analysis incorporates some new features designed to explore the small-x region more thoroughly than before. These allow us to investigate systematically possible extrapolations of the parton distributions into the small-x region consistent with NLO QCD and present data. The phenomenological consequences for the next generation of experiments are briefly summarized in Sec. 4. Finally, in Sec. 5, we study the parametrization of the allowed range of small-x behavior over the region of physical interest and show some quite interesting features which may provide some hints for theoretical analysis.

Although our investigation is based entirely on NLO leading twist perturbative QCD, there are reasons to believe that the numerical results are quite accurate within the range of physical interest, as we shall discuss in Sec. 2 and Sec. 5. This will be independently confirmed by other reports in this Workshop based on an all order Monte Carlo calculation [2] and on the comparative study of the leading log Q formulation vs. the leading log $1/x$ formulation of the small-x problem. [3]

2. SMALL-x BEHAVIOR OF 2-LOOP EVOLVED DISTRIBUTIONS

In perturbative QCD, the general expectation is that sub-leading terms in the perturbation series for a typical physical quantity introduce order α_s/π corrections to the leading expression. In reality, it has been known for some time, that NLO QCD hard scattering cross-sections for certain processes are numerically quite comparable to the LO expressions, usually for understandable reasons. [4] This fact, by itself, already requires the use of NLO parton distribution functions in the calculation of physical cross-sections so that consistent results can be obtained, especially if different physical processes are involved. We shall demonstrate that, independent of this consideration, it is important to employ NLO parton distributions in the quantitative study of their small-x behavior.

The question is: how much do the NLO parton distribution functions calculated with 2-loop evolution kernels [5] differ from the corresponding ones obtained with the familiar 1-loop evolution equation? [6] At first

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sight, it may look difficult to answer this question without elaborate numerical work since the 2-loop evolution kernels are extremely complicated. However, it is not hard to realize that significant modifications due to the next-order terms are most likely to be found in exceptional kinematic regions, in particular at small-x where any additional factors of $\log x$ or $1/x$ can easily overwhelm the extra power of α_s in the NLO term. With this in mind, it is useful to extract the dominant terms of the 2-loop evolution kernel at small-x and compare them with their counterpart in the LO kernel.

In general, the most singular term the n-loop kernel can have is of the form$(\alpha_s \log x)^n/x$. For the familiar 1-loop kernel,[6] only the P_{GF} and P_{GG} terms contain $1/x$ terms (without the log factor). It is also known that, in the $\overline{MS}$ scheme, the apparent $(\alpha_s \log x)^n/x$ ($n = 1,2$) terms in the 2-loop kernel elements[5] actually cancel out when implicit singularities in the Spence functions are taken into account, leaving behind only $1/x$ terms as the dominant ones at small-x. Shown in Table I is the comparison of the coefficients of the $1/x$ terms in the 1-loop and 2-loop kernel elements.

Table I

	1st Order	2nd Order
$P_{FF}(x)$	-	$N_f T_r C_F 40/9$
$P_{FG}(x)$	-	$N_f T_r C_G 40/9$
$P_{GF}(x)$	$C_F[1+(1-x)^2]$	$-N_f T_r C_F 40/9 + C_F C_G$
$P_{GG}(x)$	$2C_G$	$2N_f T_r\left[-\frac{23}{9}C_G + \frac{2}{3}C_F\right]$

where N_f is the number of quark flavors, $\{T_r, C_F, C_G\}$ are group factors and we have suppressed a common factor of $\alpha_s/2\pi$ for the 2nd order kernels. As noted earlier, the 1st order elements ($P^1_{FF}(x)$, $P^1_{FG}(x)$) are regular near $x = 0$, whereas their counterparts in 2nd order are proportional to $1/x$. Therefore, these kernel elements, hence the evolution of the quark distributions, will be *completely dominated by the 2-loop term* at small-x. The other two elements ($P^1_{GF}(x)$, $P^1_{GG}(x)$), have the same singularity as their 2-loop counterpart. Note, however, that the LO and NLO terms have opposite signs, and the 2nd order terms have rather large coefficients which help to overcome the extra α_s factor. Thus, we anticipate noticible influence of the 2-loop evolution kernel even in the evolution of the gluon

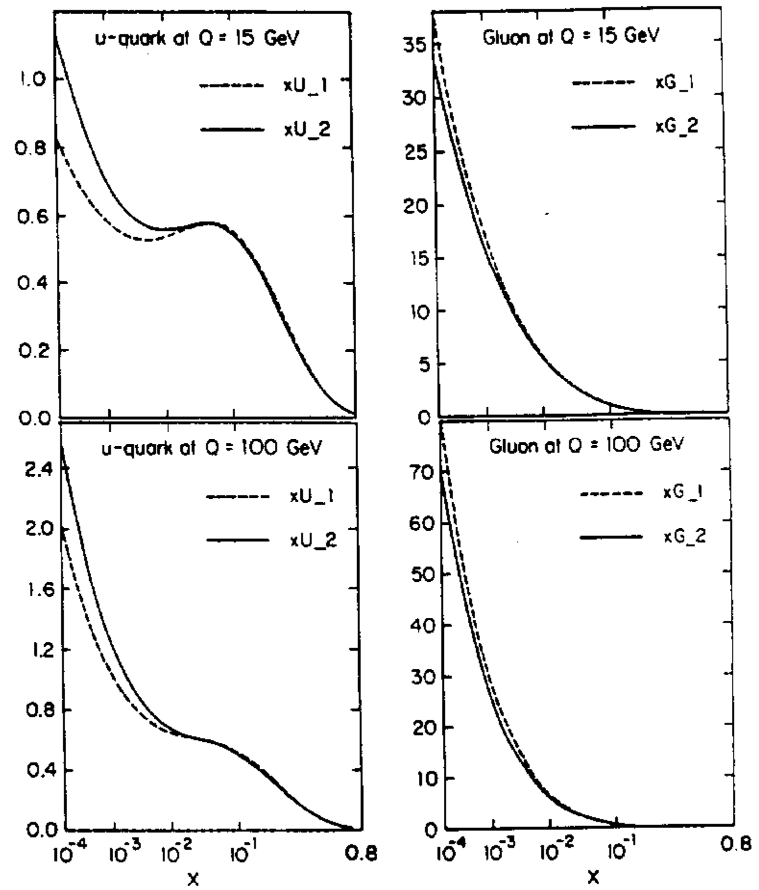

Figure 1: 2-loop *vs.* 1-loop evolved parton distributions: (a) & (b) are for the u-quark and gluon at $Q^2 = 225$; (c) & (d) are the same at $Q^2 = 10^4\ GeV^2$.

distributions at small-x, especially at moderate values of Q when α_s is not very small.[7]

These expectations are borne out by concrete numerical calculation using the *complete* expressions for the evolution kernel. In Fig. 1 we show the comparison between 1st and 2nd order evolved parton distributions at $Q^2 = 225$ and $10^4 GeV^2$ starting with the same initial distributions at $Q^2 = 15 GeV^2$ and using 1st and 2nd order running couplings which are comparable within this range. We see that the difference is quite substantial in the small-x region, particularly for the quarks; and it becomes of "normal" size, *i.e.* ~ 10 %, above $x \sim 10^{-2}$. This qualitative feature persists to rather large values of Q^2.[7]

We conclude from this example that in a quantitative study of small-x behavior of parton distributions, it is necessary to use 2-loop evolved distributions. It is natural to ask if it is sufficient to stop at the 2-loop level. Although there is no definitive answer to this question, it is worth noting that higher order kernels are no more singular than the 2-loop one, aside from log factors. Hence, even if resummation of higher order terms may lead to different asymptotic expressions, the difference between the complete asymptotic and the 2-loop evolved distributions may not be numerically significant within the range of x where the parton picture is valid or within the range of practical interest for foreseeable physics applications. We shall return to this point in the last section (Sec. 5) where specific results are presented. (See also Ref. [2] and Ref. [3] which confirm this.)

3. A New Global Analysis of Parton Distributions

In this section, we will briefly describe a new global analysis of currently available high statistics data in the QCD framework which forms the basis of our study.[1] This global analysis is motivated by the need for reliable knowledge of quark and antiquark distributions within currently available energy range in precision tests of the Standard model, for better knowledge on the gluon distribution in the exploration of signals and background of new physics, as well as for more information on the small-x behavior which is essential for making projections for future high energies. In addition to the NLO QCD evolution, our phenomenological investigation emphasizes constraints imposed by existing data.

Although many parametrizations of parton distributions have been in wide use, most of the familiar leading-order (LO) distributions[8] are no longer suitable for current day applications for at least two reasons: (i) the LO evolution is insufficient for matching with the more accurate modern measurements on the one hand and the NLO calculations of hard cross-sections on the other; and (ii) the experimental data on which the original analyses were based have changed considerably, such that physical quantities calculated from these distributions now deviate significantly from up-to-date data.[9] Our study differs from other recent NLO parton distribution analyses[10] in several important aspects: (i) we emphasize the necessity of properly incorporating experimental systematic errors when fitting high statistics data from several independent experiments, and the importance of investigating the effect of different cuts on the data selection; (ii) we formulate the analysis in such a way that the functional form of the distributions can be easily adjusted to match the discriminating power of the data used; and (iii) our parametrization of the small-x behavior is particularly flexible so that the range of such behavior can be explored by comparison with data rather than being fixed by an assumption, as is usually done.

To be specific, we choose the following functional form for both the initial parton distributions at a given scale Q_0 as input to the evolution equation, and for parametrization of the resulting distributions at all (x, Q) after making the global fit to data and evolving over the entire range of Q for which applications are anticipated. For each parton flavor a (including the gluon), we have:

$$xf^a(x,Q) = e^{A_0^a} x^{A_1^a} (1-x)^{A_2^a} \ln^{A_3^a}\left(1+\frac{1}{x}\right) \qquad (3.1)$$

The incorporation of a logarithmic factor offers several advantages over the more conventional parametrizations using polynomials: (i) it is "QCD-motivated" in the sense that powers of logarithms arise naturally in perturbative calculations. (Even if this form is not an analytic solution to the evolution equation, the numerical solutions do tend to have log-like behavior over the x region of physical interest. See Sec. 5 for a detailed discussion of this point, including results on using a modified parametrization consistent with the *true* asymptotic solution); (ii) this functional form is positive-definite for all (x, Q) and it is smoothly varying as parton distributions should be, in contrast to other forms of parametrizations which often result in pathological behavior beyond certain range; and (iii) the presence of both a power and a logarithm factor offers a much more flexible scheme to explore the small-x behavior of the distribution functions as we shall discuss extensively below.

The deep inelastic scattering data sets used in this analysis are from the recent high-statistics neutrino experiment CDHSW[11], and from the muon experiments EMC[12] and/or BCDMS[13]. The continuum lepton-pair production (Drell-Yan process) data used are from the Fermilab experiments E288 and E605. A step-by-step procedure was used to determine the shape parameters of the valence quark, sea-quark and gluon distributions depending on the sensitivity of the measured quantities to these parameters. We obtained good fits to these data using EMC and BCDMS data separately or combined, provided reasonable Q^2 and W cuts are imposed in data selection (we used the values $10GeV^2$ and $4GeV$, respectively for the two cuts, after verifying that the results are insensitive to values above these); and provided an overall relative normalization constant for each data set is included as a fitting parameter. The set of normalization constants obtained this way agree well with those found independently by recent critical comparisons of reviews of these experiments.[14]

Although significant improvements are being made both on the experimental and theoretical fronts, our study shows that current data still do not uniquely determine the parton distributions. Specifically, the various sea-quark flavors are not yet well differentiated, and the gluon distribution is not yet well determined. Thus, certain assumptions concerning these distributions are necessarily incorporated in the global analysis. Our study yields a number of different sets of distribution functions depending on the data sets used, on the assumed small-x functional form, and on the assumed strange to non-strange sea-quark proportion. Details can be found in [1]. Specific results on the small-x behavior of parton distributions discussed in subsequent sections will be based on these fits.

One potentially useful source of information on the gluon distribution is direct photon production in hadronic collisions. At present, the experimental data still lack in range and precision compared to those on DIS and DY processes; and the theoretical interpretation of these data still suffer from unresolved uncertainties associated with isolation cuts and delicate choice of scale in next-to-leading order contributions. For these reasons, direct photon data has not been included in our global analysis so far. Improvements on both fronts will be most welcome, and they appear to be forthcoming.[15]

The traditional source of information on strange quark comes from dimuon events in deep inelastic neutrino scattering (assumed to be due to charm production). The proper interpretation of this process in the QCD formalism has been studies in much detail recently.[16] It was found that: (i) the contribution from gluons can be as significant as the strange quarks; (ii) the interpretation of this type of processes is inherently renormalization scheme dependent; hence (iii) the conventional leading-order analysis can lead to misleading results. Considerable clarification of this issue can be expected in the near future when existing large samples of dimuon data are examined in the light of this QCD analysis.[17]

4. Phenomenological Consequences of Small-x Extrapolations

Strictly speaking, global fits to existing experimental data can only determine the behavior of parton distributions in the x-range measurable at current time. Although theoretical efforts are being made to handle large $log(1/x)$ terms, there is as yet no widely accepted method for calculating the x-dependence of these distribution functions comparable to that for the Q-dependence by the QCD-evolution equation. In the context of our phenomenological study, we investigated the range of possible small-x behavior of parton distributions based on the functional form, Eq. (3.1), and the constraints imposed on the shape parameters by our global fits to existing data.

As mentioned previously, our functional form, Eq. (3.1), permits significantly more flexibility in studying the allowed range of small-x behavior of the parton distributions then the conventional parametrization based on a fixed power. We have used the x^{A_1} factor alone, the $log^{A_3}(1/x)$ factor alone, and both factors together in our phenomenological study. In all cases, we let at least one of the parameters A_1/A_3 of the initial gluon and sea-quark distributions be a variable parameter determined by fitting to data. This way, we do not prejudice the choice of these parameters; bearing in mind particularly that the value of these parameters does not have much real physical significance as commonly assumed because they change rather rapidly with Q in the lower Q range as the result of QCD evolution. (See next section for detailed results.)

Since available data in deep inelastic scattering and Drell-Yan processes only span a limited x range (0.05 - 0.75), we are able to obtain almost equally good fits for many combinations of the parameters A_1 and A_3. They lead to different extrapolations into the

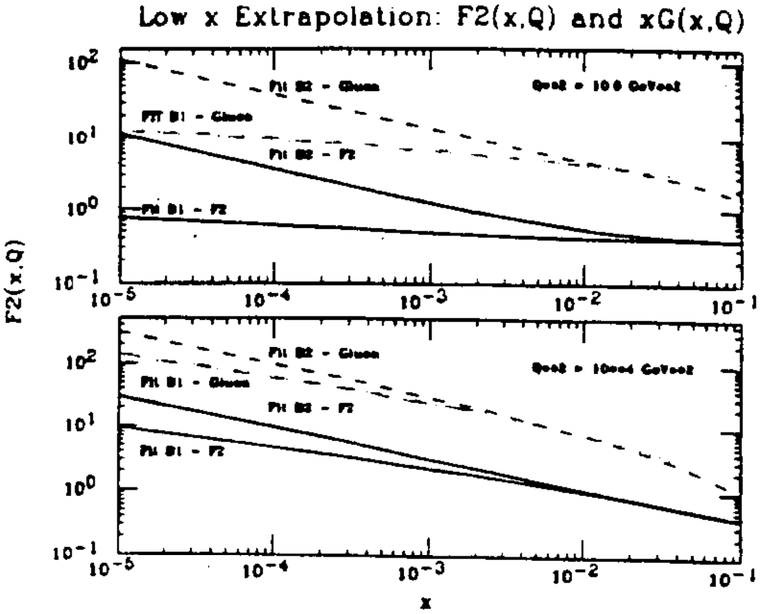

Figure 2: Extrapolated values of $F_2(x)$ and $xG(x)$ from two acceptable fits (B1 and B2) at low x for $Q^2 = 10\ GeV^2$ and $10^4 GeV^2$.

small x region, hence map out for us the range of possible small-x behavior consistent with current data. In Fig. 2 we show the range of extrapolated gluon distribution function $G(x,Q)$ as well as the $F_2(x,Q)$ deep inelastic scattering structure function bracketed by two typical fits (labelled B1 and B2) near the opposite ends of the allowed spectrum. Two values of Q^2, $10GeV^2$ and 10^4GeV^2, are shown. At the lower Q value, the two $F_2(x,Q)$ curves differ by as much as a factor of 15 at the extreme small-x end of 10^{-5}; while the gluon distributions differ by a factor of 9. As one would expect, the difference diminishes as Q increases due to the QCD evolution. However, even at Q^2 of $10^4\ GeV^2$ there is a factor of 3 difference in F_2 and a factor of 2 difference in G at $x = 10^{-5}$. These facts underscore the important role HERA measurements can play in determining the correct behavior of the parton distributions.

Hadron collider processes can also help to shed light on the small x behavior of parton distributions. In Fig. 3 we show next-to-leading order calculation of the y-distribution of lepton-pairs (Drell-Yan process) at the Tevatron energy for dimuon mass $Q = 20$ GeV using the same two set of parton distributions

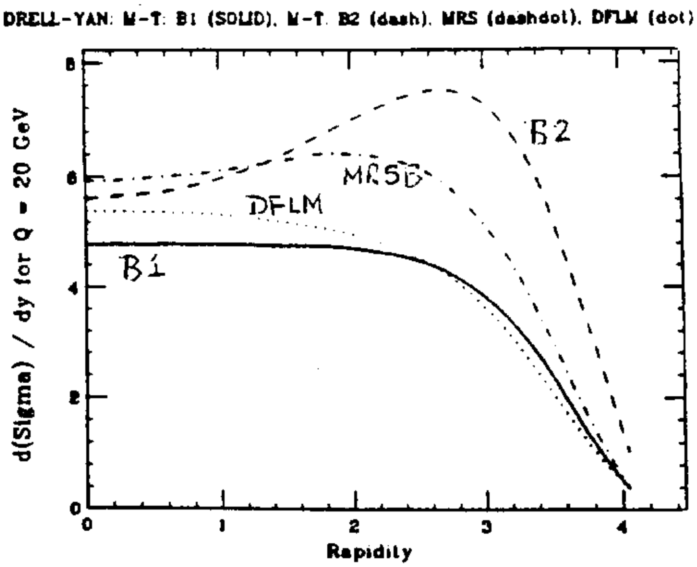

Figure 3: Predictions for low mass ($Q = 20GeV$) Drell-Yan pair production at the Tevatron Collider for selected parton distribution functions.

B1 and B2 along with the MRSB and DFLM distributions. Here we see a dramatic difference in the anticipated shape of the y-distribution, especially between the B2 curve and the others. This sensitivity is due to the contribution of the small-x parton distributions to the Drell-Yan cross-section – especially in the forward-backward directions. This striking effect has been known for some time, based on crude estimates[18]. The current calculation, using parton distributions known to be consistent with all current experiments, confirms the importance of the collider lepton-pair measurements in probing parton distributions at small x.

5. Exploring the Small-x Behavior of Parton Distributions

In this section we use a series of 2-loop evolved parton distributions, all of which fit current data as described above, as the basis for the phenomenological exploration of their small-x behavior. As we shall see, the results are quite interesting; and they may provide hints for complementary theoretical studies. For the two ways of parametrizing parton distributions we shall be discussing, we first use the initial distributions of each fit to generate parton distributions over the extended region ($10^{-5} < x < 1$ and

$10 < Q^2 < 10^8 GeV^2$) by the NLO evolution equation, then try to fit the results to the assumed functional forms with Q-dependent parameters, requiring (if possible) an accuracy comparable to that of the original fit to data. In this way, we can examine the *effective* shape parameter, such as $(A_0 - A_3)$ in Eq. (3.1), for arbitrary Q.

5.1. "Conventional Parametrization" of Small-x Behavior

In our parametrization of the parton distributions, Eq. (3.1), the two parameters $\{A_1, A_3\}$ jointly control the small-x behavior of the distributions. By contrast, in a "conventional parametrization" such as:

$$xf(x,Q) = e^{B_0} x^{B_1} (1-x)^{B_2} (1 + B_3 x) \qquad (5.1)$$

the small-x behavior is effectively controlled by the single parameter B_1 for any given Q. Thus it is attractive to consider this alternative parametrization when investigating the small-x region. One may also be tempted to use this functional form to parametrize the parton distributions for all Q as we did with the function form Eq. (3.1). We found this to be impossible, however, because the parametrization Eq. (5.1) cannot accurately represent the QCD evolved distributions at large values of Q, especially for the gluon distribution, as Eq. (3.1) can. In this subsection we shall use this convenient parametrization to discuss the "initial distribution" at low Q, where it is good, and then briefly look at the Q-dependence of the B_1 parameter to get a qualitative feel of the Q-evolution of the overall small x behavior.

In most published parton distribution sets, one adopts some parametrization similar to Eq. (5.1) for the parton distributions at an initial $Q = Q_0$, and fix the parameter B_1 at some specific value determined by certain "theoretical" considerations, *e.g.* $B_1 = 0$ from conventional Regge behavior or $B_1 = -\frac{1}{2}$ from resummation of large $log(1/x)$ terms.[19] These choices are highly subjective since there is no theoretical guidance on what the appropriate value of Q is for these behavior to be applicable; and since the value of the effective B_1 varies quite rapidly with Q in the low Q range. This is illustrated in Fig. 4 where the parameter B_1 for the gluon is plotted against Q for a series of distributions with differing small-x behavior as described before. (The curve marked $\overline{u}$ represents th corresponding parameter for the sea-quark $\overline{u}$.) We se the rapid variation of B_1 in the $Q = 2 - 5GeV$ rang except for the set with the lowest value of B_1 close t $-\frac{1}{2}$ where it is relatively stable. It should be obviou from examining this plot that one cannot meaningfull compare the values assigned to B_1 in the various par ton parametrization sets unless they refer to a singl standard starting Q_0 value. In reality, different group use rather different Q_0 values ranging from 1.5 to 5. GeV or higher.

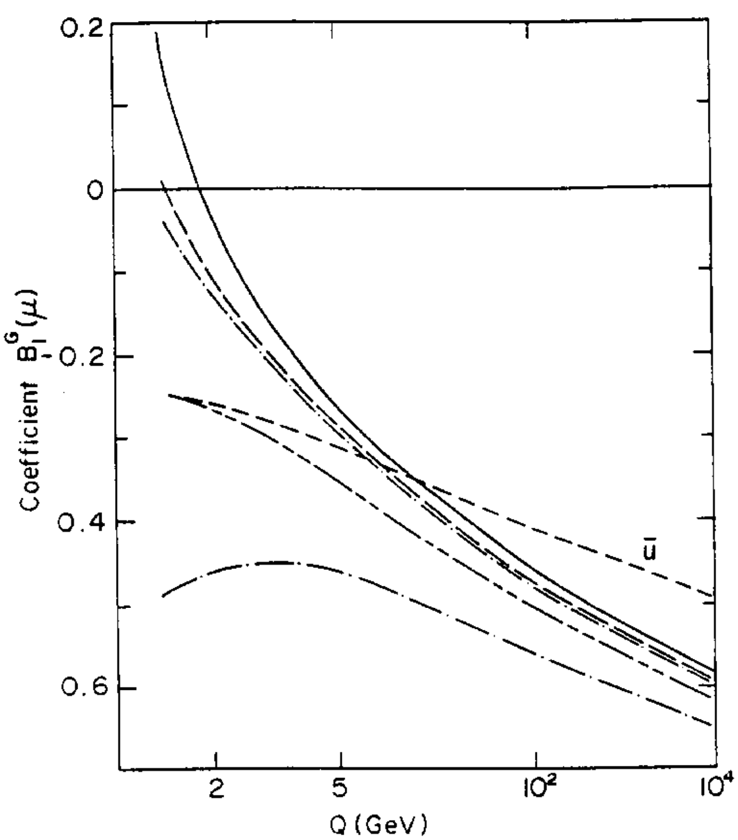

Figure 4: The parameter B_1, Eq. (5.1), for th gluon from the various parton distribution sets plotte against Q.

Over the wider range of Q shown in Fig. 4, th effective power B_1 becomes increasingly negative fo all sets of distributions as Q increases, reflecting th softening of the parton distributions at short distance as dictated by the evolution equation. Interestingl the curves in Fig. 4, representing a wide spectrum o small-x behavior at low Q, show a tendency to con verge toward a common asymptote at large Q. Th fact that for sufficiently large Q the effective power B becomes less than -0.5 in all cases, resulting in singula

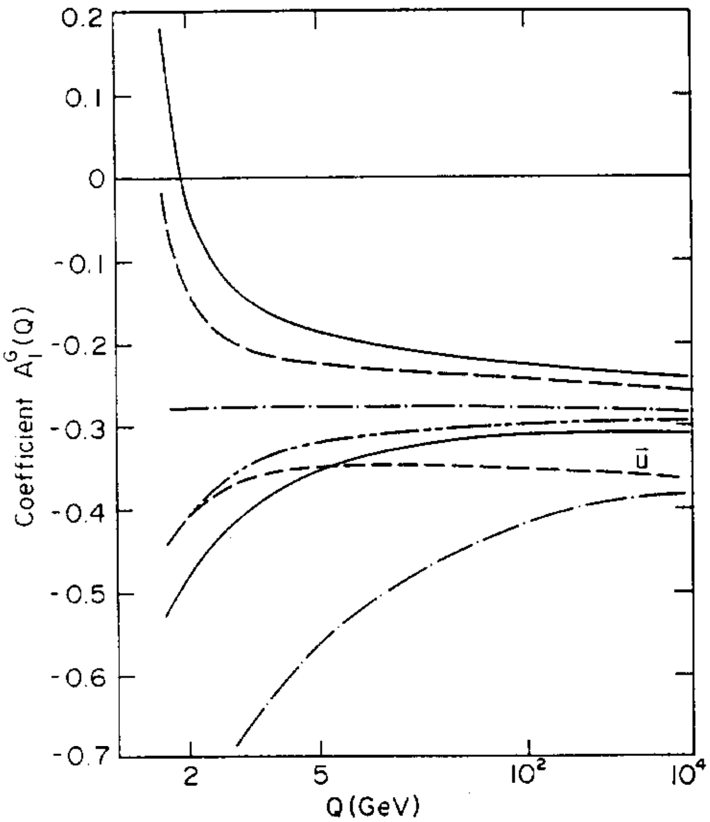

Figure 5: The parameter A_1, Eq. (3.1), for the gluon from the various parton distribution sets plotted against Q

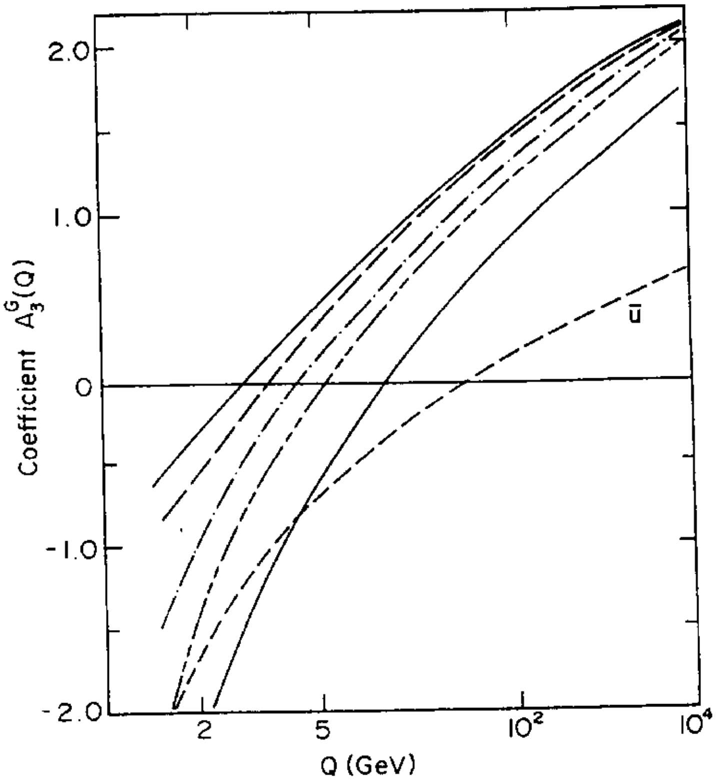

Figure 6: The parameter A_3, Eq. (3.1), for the gluon from the various parton distribution sets plotted against Q

behavior of the parton distributions which is steeper than that implied by resummation of $(\log 1/x)^n$ terms, suggests that the 2-loop evolved distributions could in fact be numerically reliable, as the resummed function may not have a chance to catch up in the physically interesting region. (Cf. [2] and [3])

We also see that the effective B_1 parameter for the sea quark $\overline{u}$, starting from the same value as that of the gluon by assumption, (shown only for one distribution) does not decrease as fast as the gluon. Consequently, the sea quark distributions are less singular then the gluon distribution at high energies.

5.2. Evolution of the Power- and the Logarithmic-law Parameters

We now return to the combined power- and logarithm-law functional form, Eq. (3.1). First, we find that this form can represent the QCD-evolved distributions with good accuracy for all Q values and parton flavors. This represents a substantial improvement over the "conventional" form Eq. (5.1). The resulting effective parameters $A_1(Q)$ and $A_3(Q)$ from the same series of fits presented in Fig. 4 before are shown in Fig. 5 and Fig. 6 respectively. We notice the interesting feature that the parameter $A_1(Q)$ converges to a *constant asymptotic value* around -0.27 *cf.* Fig. 5, even if it starts from a very wide range of values initially. Thus the increasing softening of the parton distributions at small-x is carried almost exclusively by the logarithm factor $log^{A_3}(1/x)$, as is seen in Fig. 6 where the increasing $A_3(Q)$ for all distribution sets represents the growing singular behavior of the functions. We have no theoretical explanation of this rather striking outcome. We should realize, of course, that this phenomenological observation depends somewhat on the range of x used to obtain these effective parameters. Nonetheless, the result is perhaps indicative of something significant since the range of x used is that of practical physical interest. Beyond this range, saturation and other considerations can very well render the parton picture invalid in any case.

One possible way to understand our phenomenological observation is to make use of the known analytic solution to the QCD-evolution equation at the extreme small-x region.[20] With this in mind, it appears very attractive to try the Ansatz:

$$f(x,Q) = f_r(x,Q)\exp\sqrt{C\ln\frac{1}{x}\ln\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}} \quad (5.2)$$

where $C = \frac{144}{11-2n_f}$, the exponential factor is the appropriate asymptotic function form for small x and large Q, and the "regular" part of the function $f_r(x,Q)$ can be parametrized in the form Eq. (3.1) or Eq. (5.1). We tried this approach. Surprisingly, within the same range of x and Q described before, it turned out to be rather difficult to fit numerically this functional form (with either parametrization of the regular part) to the 2-loop QCD evolved distribution functions in a consistent and reliable way, as can be done with the plain function Eq. (3.1). We obtained reasonable fits sometimes, but unacceptable ones other times. This finding needs to be studied further.

6. Summary

We have described results on a phenomenological study of the small x behavior of parton distributions based on global fits to current data in the next-to-leading order perturbative QCD framework. The range of values for these distributions as well as for physically measurable quantities in the small x region accessible to the next generation of experiments are delineated. Finally, we studied the Q-evolution of effective power-law and logarithmic-law parameters for the distributions of a whole series of parton distributions which are consistent with known constraints. We found the functional form Eq. (3.1) to be particularly suitable to parametrize the distribution functions over the entire kinematic region ($10^{-5} < x < 1$ and $10 < Q^2 < 10^8 GeV^2$) which is relevant for physical applications. Interesting features on the Q-dependence of the exponents in this parametrization are presented.

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