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Dynamical Symmetry Breaking of the Electroweak Interactions and the Renormalization Group[†]

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Abstract

We discuss dynamical symmetry breaking with an emphasis on the renormalization group as the key tool to obtaining reliable predictions. In particular we discuss the mechanism for breaking the electroweak interactions which relies upon the formation of condensates involving the conventional quarks and leptons. Such a scheme indicates that the top quark is heavy, greater than or of order 200 GeV, and gives further predictions for the Higgs boson mass. We also briefly describe recent attempts to incorporate a 4th generation in a more natural scheme.

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1. Role of the Renormalization Group in DSB

Recently there has been considerable interest in dynamical symmetry breaking of the electroweak interactions in which a top quark condensate plays the role of the order parameter [1-4]. The simplest models discussed thus far are generalizations of the Nambu-Jona-Lasinio model, though some effort has commenced to address naturalness issues, and attempts have been made to place the “new dynamics” at the accessible \sim TeV scale, as briefly described here.

We wish, however, to emphasize the application of the renormalization group to this scheme [4, 5]. The renormalization group can be used as a dynamical tool to include *all of the effects of the full theory* and generate reliable, precise predictions of its consequences. This goes beyond the limited approaches of large- N_c bubble sums, or planar QCD calculations. Moreover, the results of these “brute force” analyses can be easily reproduced by including only those terms in the renormalization group equations that correspond to effects included in the “brute force” calculations. The important element which makes the renormalization group applicable is the fact that *the compositeness of certain dynamically generated multiplets, e.g., the Higgs multiplet in the minimal case, implies UV boundary conditions on the renormalization group equations of the effective field theory* [4].

In Section 2 and 3 we will compare three levels of sequentially improved approximation in the context of the renormalization group (RG). These are: (i) the fermion bubble approximation; (ii) ladder QCD (internal gluon lines); (iii) the full RG equation including the effects of the propagating dynamical Higgs boson. The first two cases are presented essentially for illustration of the technique, while the last case represents the correct leading log (one loop RG) results. We will further explore the universality of the results and sensitivity to the boundary conditions in the presence of “irrelevant operators.” The full RG results are found to be robust in the presence of these effects. Moreover the renormalization group predictions are more general than

any particular choice of the high energy Lagrangian, and can represent generally the physics of compositeness in relativistic field theory.

More generally, the renormalization group will be a relevant tool in any dynamical symmetry breaking scheme when (i) there exists a linear realization of the symmetry group in the effective Lagrangian over a large range of intermediate scales $\Lambda \gtrsim \mu \gtrsim (G_F)^{-1/2}$, and (ii) where Λ , the scale of the new physics, is very much larger than the electroweak scale $(G_F)^{-1/2}$. The presence or absence of a linear composite representation of the broken symmetry is a general feature that distinguishes between various schemes of dynamical symmetry breaking. For example, in technicolor theories which attempt to solve all naturalness problems the symmetry is only nonlinearly realized; there is no Higgs boson *per se* (the 0^+ boundstate lies too near to Λ). In such a scheme the low energy physics is less controlled by the renormalization group than by decoupling theorems. Alternatively, in the Nambu-Jona-Lasinio model (by which we generally mean the conventional fermion bubble large- N_c limit analysis of a Lagrangian such as eq.(1) below) a composite Higgs boson does exist. We emphasize that even though Λ may be close to $G_F^{-1/2}$, as in the case of a fourth generation generalization of the $\bar{t}t$ idea, the renormalization group is still a useful tool in analyzing the model.

When $\Lambda \gg (G_F)^{-1/2}$ it is of central importance to understand the structure of physics in the intermediate range of scales, to which we shall refer as a "desert." The effective Lagrangian \mathcal{L} will be, in part, controlled by the compositeness conditions which require that it merge onto the new dynamics at Λ . However, as we scan over the scales in the desert the structure of \mathcal{L} will be determined by the exact field-theoretic renormalization group. Indeed, it must be true that the compositeness conditions are expressible as *boundary conditions* upon \mathcal{L} at the scale Λ . This description is inescapable, and is highly restrictive. The full dynamics of the theory as contained in \mathcal{L} must be operant in any precise loop calculation.

The existence of the desert corresponds to our insistence of lying on (or very near to) the critical line of the theory. This is equivalent to the fine-tuning of quadratic divergences in the gap equation leading to $\Lambda \gg G_F^{-1/2}$. It is, therefore, not surprising that infra-red renormalization group quasi-fixed points correspond to the low energy solution of the theory. By “quasi-fixed points” we mean that, if it were not for some explicit breaking of scale invariance in the far infra-red, these would be exact RG fixed points. Indeed, the prediction for m_{top} is an exact fixed point in the limit in which the QCD coupling constant g_3 does not run (i.e., setting the QCD- $\beta(g)$ function to zero). The precise prediction of m_{top} is sensitive, however, to any new physics which acts over the desert through the RG equations. Thus, if we ultimately find a value of $m_{top} < 200$ GeV, we should still look for theories that give new interactions in the RG equations capable of predicting this value. It is not obvious that such models exist (in a recent paper T.K. Kuo, U. Mahanta, and G. T. Park [6] considered an imbedding of $SU(2) \times U(1)$ into $SP(6)_L \times U(1)$ and find the minimal prediction, $m_t \sim 230$ GeV, to be fairly resilient). The issue of “naturalness” will be addressed in section (4).

2. Conventional Analysis of NJL from the Perspective of the Renormalization Group

2.1 Fermion Bubble Approximation

If we consider, for discussion, the approximation in which all quarks and leptons other than the top quark are massless we may then define a theory at the scale Λ to be:

$$L = L_{kinetic} + G(\bar{\Psi}_L^{ia} t_{Ra})(t_R^b \Psi_{Lib}) \quad (1)$$

Here $\Psi_L = (t, b)_L$ and i runs over $SU(2)_L$ indices, (a, b) run over color indices. $L_{kinetic}$ contains the usual gauge invariant fermion and gauge boson kinetic terms.

Alternatively, we may introduce a non-dynamical auxiliary field to rewrite eq.(1)

equivalently as:

$$L = L_{kinetic} + \left(\bar{\Psi}_L^{ia} t_{Ra} H_i + h.c. \right) - M_0^2 H^\dagger H \quad (2)$$

where we identify

$$G = 1/M_0^2 \quad (3)$$

Note that eq.(2) must be viewed as an effective Lagrangian at the scale Λ . By “effective Lagrangian at a scale μ ” we mean that all the dynamics above the scale μ has been integrated out, but all dynamics below μ must be computed.

The structure of eq.(2) will change significantly, due to radiative corrections, when we consider the effective Lagrangian at any other scale, $\mu < \Lambda$. The technique for descending from Λ to μ is known as the “block-spin renormalization group,” [8] and consists in the present case of integrating out all loops with internal momenta $\Lambda \geq l \geq \mu$. We will see that eq.(2) defines the renormalization group boundary conditions for the full solution to the theory of eq.(1). The auxiliary field introduced at the scale Λ will become the propagating physical Higgs field at low energies $\mu \ll \Lambda$.

Let us summarize the results of a full block spin renormalization group transformation performed on eq.(2) to generate the effective Lagrangian at a scale μ [4]. We must include the induced gauge invariant kinetic terms of the Higgs doublet, with a wave-function normalization constant, Z_H , and the induced quartic interaction coming from top quark loops. We obtain:

$$\begin{aligned} L = & L_{kinetic} + (\bar{\Psi}_L t_{RH} + h.c.) \\ & + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2 + \dots \end{aligned} \quad (4)$$

where the result of the calculation of the parameters is [4]:

$$Z_H = \frac{N_c}{(4\pi)^2} \ln (\Lambda^2/\mu^2) \quad (5)$$

$$\lambda_0 = \frac{2N_c}{(4\pi)^2} \ln(\Lambda^2/\mu^2) \quad (6)$$

We note that the mass M_μ^2 has a quadratic divergence leading to running as in:

$$M_\mu^2 = M_0^2 - \frac{N_c}{8\pi^2} (\Lambda^2 - \mu^2) + O(m_t^2/\mu^2) \quad (7)$$

The evolution of the mass term is ultimately dictated by our choice of a symmetry breaking phase at very low energies, which is equivalent to the fine-tuning of the solution of the gap equation. This fine tuning is equivalent to demanding a cancellation between the large terms, M_0^2 and $N_c\Lambda^2/8\pi^2$ in eq.(7). However, in what follows we will simply assume that the theory has a broken phase at low energies; the logarithmic RG evolution will suffice to obtain the predictions.

Eq.(4) completely summarizes the set of relevant operators that are present on all scales $\mu \ll \Lambda$. We thus see from eq.(5) and (6) the following compositeness conditions hold as $\mu \rightarrow \Lambda$:

Compositeness Conditions:

$$Z_H \rightarrow 0|_{\mu \rightarrow \Lambda}. \quad (8)$$

$$\lambda_0 \rightarrow 0|_{\mu \rightarrow \Lambda} \quad (9)$$

Notice also that $M_\mu^2 \rightarrow M_0^2|_{\mu \rightarrow \Lambda}$, and in the NJL model we see that $\lambda_0/Z_H \rightarrow 2$. These conditions reflect the consistency between the effective Lagrangian and eq.(2) at the scale Λ . Note that all of the results of eq.(5-7) can be inferred from the "brute force" bubble summation [4]. The only subtlety here is that the Lagrangian of eq.(4) is not written in the conventional normalization form. Let us now reexpress things in a more conventional formalism.

Conventionally one normalizes the kinetic terms of a field theory at any scale, μ , with a condition that they have free-field theory normalization. Indeed, this is an intermediate step in the block-spin RG transformation as described by Kogut and Wilson [8]. In the previous discussion we chose not to insist upon this because of the singular behavior of Z_H as in eq.(8). However, we can transfer this singularity to a condition on coupling constants in the conventional normalization. That is, we may exercise our freedom of rescaling the various fields, H , Ψ_L , t_R , *etc.*, to define the coefficient of $|D_\mu H|^2$ to be unity. In the present case $H \rightarrow H/\sqrt{Z_H}$. The conventionally normalized Lagrangian becomes:

$$L = L_{kinetic} + (g_t \bar{\Psi}_L t_R H + h.c.) + |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 + \dots \quad (10)$$

where the physical coupling constants, m_H^2 , g_t , and λ , are given by:

$$m_H^2 = M_H^2/Z_H; \quad g_t = \frac{1}{\sqrt{Z_H}}; \quad \lambda = \frac{1}{Z_H^2} \lambda_0 \quad (11)$$

It is clear from eqs.(11) that as $\mu \rightarrow \Lambda$ then g_t and λ diverge, while $g_t^2/\lambda \rightarrow constant$. Hence, in conventional normalization we have:

Compositeness Conditions (conventional normalization):

$$g_t(\mu) \rightarrow \infty|_{\mu \rightarrow \Lambda} \quad (12)$$

$$\lambda \rightarrow \infty|_{\mu \rightarrow \Lambda} \quad (13)$$

We now will show that all of the usual large- N_c results are easily recovered directly from the conventional, differential renormalization group equations, supplemented with the compositeness conditions of eqs.(12, 13). To obtain the renormalization

group description of the NJL model we may utilize the partial β -functions which reflect only the presence of fermion loops:

$$16\pi^2 \frac{dg_t}{dt} = N_c g_t^3 \quad (14)$$

$$16\pi^2 \frac{d\lambda}{dt} = (-4N_c g_t^4 + 4N_c g_t^2 \lambda) \quad (15)$$

Solving eq.(14) gives:

$$\frac{1}{g_t^2(\mu)} = \frac{N_c}{16\pi^2} \ln(\Lambda^2/\mu^2) \quad (16)$$

where we use the boundary condition, $1/g_t^2(\Lambda) = 0$. Eq.(15) may then be solved by hypothesizing an ansatz of the form $\lambda = c g_t^2$. Substituting into eq.(15) one finds:

$$16\pi^2 \frac{dg_t}{dt} = \frac{1}{2c}(4c - 4)N_c g_t^3 \quad (17)$$

and demand that this must be consistent with eq.(14). Thus one finds: $c = 2$ and:

$$\frac{1}{\lambda(\mu)} = \frac{N_c}{32\pi^2} \ln(\Lambda^2/\mu^2) \quad (18)$$

Note that the solutions eqs.(16,18) are equivalent to those of eq.(5,6) with the identifications of eq.(11).

Now, to obtain the usual phenomenological results of the NJL model we examine the low energy Higgs potential:

$$V(H) = -m_H^2 H^\dagger H + \frac{\lambda}{2} (H^\dagger H)^2 \quad (19)$$

as contained in eq.(10). Let $H^0 = v + \phi/\sqrt{2}$. Here the value of v can be derived by tracking the evolution of M_μ^2 . But we can obtain the usual results directly from the RG equation solutions, simply assuming that v has been fine-tuned to a physical value of $v^2 = 1/2\sqrt{2}G_F = (175 \text{ GeV})^2$.

Therefore we use the implied results for the top mass from eq.(10):

$$m_t = g_t v; \quad (20)$$

and the ϕ mass implied from eq.(19):

$$m_\phi^2 = 2v^2 \lambda \quad (21)$$

and so:

$$m_\phi^2/m_t^2 = 2\lambda/g_t^2 = 4 \quad (22)$$

where in eq.(22) we use the explicit solutions eq.(16) and eq.(18). This is the familiar NJL result, $m_\phi = 2m_t$. Moreover, we have:

$$v^2 = m_t^2/g_t^2 = m_t^2 \frac{N_c}{16\pi^2} \ln(\Lambda^2/m_t^2) = \frac{1}{2\sqrt{2}G_F} \quad (23)$$

which is equivalent to the prediction obtained from a direct fermion bubble approximation computation of the decay constant [4]. For example, with $\Lambda = 10^{15}$ GeV one finds $m_t \approx 165$ GeV. The results of fermion bubble approximation are given in Table I. We have seen that the RG directly and simply reproduces the result of a “brute force” summation of fermion bubbles.

2.3 Ladder QCD

We can now take a step closer to the full theory by including the effects of gluons in the RG equations. Indeed, King and Mannan, and independently Mahanta and Barrios [9] consider the “brute force” solutions to the Schwinger–Dyson equations with the four–fermion interaction in ladder approximation to QCD. Mahanta and Barrios also show that the results are equivalent to those obtained by using the renormalization group with the inclusion of terms representing the gluon effects. This analysis illustrates again the power of the renormalization group in reproducing the

results of this approximation. However, we do not accept the statements of these authors that this represents an improvement over the *full* RG solution obtained in [4,5] and in Section (3), since obviously the ladder QCD calculations omit the propagating Higgs boson, as well as electroweak effects which are included in the full RG equations below.

We now have, including only the effects of fermion loops and gluons in loops:

$$16\pi^2 \frac{dg_t}{dt} = N_c g_t^3 - (N_c^2 - 1)g_3^2 g_t \quad (24)$$

$$16\pi^2 \frac{dg_3}{dt} = -(11 - 2n_f/3)g_3^3; \quad (25)$$

$$16\pi^2 \frac{d\lambda}{dt} = (-4N_c g_t^4 + 4N_c g_t^2 \lambda) \quad (26)$$

Notice the additional QCD term in eq.(24) in comparison to eq.(14). The equation for Λ is unchanged at this one-loop (leading log) order. Again, the UV boundary conditions on the theory are as in eq.(12,13). It is most convenient to obtain these results numerically, and they are indicated in Table I for various values of Λ . We first see the appearance of the nontrivial RG infrared fixed point for g_t at this stage [5].

3. Fully Improved Renormalization Group Solution

To obtain the full renormalization group improvement over the Nambu-Jona-Lasinio model we may utilize the boundary conditions on g_t and λ and the full β -functions (neglecting light quark masses and mixings) of the Standard Model. To one-loop order we have:

$$16\pi^2 \frac{dg_t}{dt} = \left((N_c + \frac{3}{2})g_t^2 - (N_c^2 - 1)g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \right) g_t \quad (27)$$

and, for the gauge couplings:

$$16\pi^2 \frac{dg_i}{dt} = -c_i g_i^3 \quad (28)$$

with

$$c_1 = -\frac{1}{6} - \frac{20}{9}N_g; \quad c_2 = \frac{43}{6} - \frac{4}{3}N_g; \quad c_3 = 11 - \frac{4}{3}N_g \quad (29)$$

where N_g is the number of generations and $t = \ln \mu$. The principle difference in eq.(27) relative to eq.(24) is (i) inclusion of the propagating dynamically generated Higgs boson in loops (the additional 3/2 in the coefficient of g_t^3) and (ii) the inclusion of electroweak effects.

The precise value of the top quark mass is determined by running $g_t(\mu)$ from very high values at a given compositeness scale Λ down to the mass-shell condition $g_t(m_t)v = m_t$. We will not discuss possible low energy corrections associated with the extrapolation of the symmetric three-point function to a zero-momentum Higgs line. The nonlinearity of eq.(27) focuses a wide range of initial values into a small range of final low energy results [5]. The solution for $m_{quark} = g_t(\mu)v$ is shown in Fig.(1) for $\Lambda = 10^{15}$ GeV case (A) and $\Lambda = 10^{19}$ GeV (case B) respectively.

The "quasi" or moving fixed point would be an exact fixed point if g_3 were constant. Thus, this is a reflection of approximate scale invariance of the theory as we tune the gap equation to produce $m_t \ll \Lambda$. The scale invariance is explicitly broken by Λ_{QCD} . The uncertainties of higher orders can be viewed as an uncertainty in the precise position of the UV cut-off, Λ , and the fixed point behavior implies that m_t is determined up to $O(\ln \ln \Lambda/m_t)$ sensitivity to Λ . In Table I we give the resulting physical m_{top} obtained by a numerical solution of the renormalization group equations as a function of Λ . Note the sensitivity to Λ is reduced when the nontrivial IR fixed point is present.

The Higgs boson mass will likewise be determined by the evolution of λ now given

by:

$$16\pi^2 \frac{d\lambda}{dt} = 12(\lambda^2 + (g_t^2 - A)\lambda + B - g_t^4) \quad (30)$$

where:

$$A = \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2; \quad B = \frac{1}{16}g_1^4 + \frac{1}{8}g_1^2g_2^2 + \frac{3}{16}g_2^4 \quad (31)$$

There are now significant modifications in eq.(30) relative to eq.(26) due to the inclusion of Higgs propagation (the first term on the *rhs*) and electroweak interactions. The joint evolution of g_t and λ to the RG fixed point is shown in Fig.(2), and m_H is given in Table I including the full RG effects.

3.2 Sensitivity to Irrelevant Operators

The action of the effective fixed-point appears to make the top quark mass prediction insensitive to the precise initial high values of the coupling constant close to Λ due to the aforementioned focusing [5]. Indeed, there will be potential real physical effects here which modify the boundary conditions somewhat, and may be viewed as due to irrelevant or higher dimension operators. How sensitive are we to such model-dependent effects?

In an interesting analysis Suzuki has included the effects of some operators which involve higher derivatives. We will show that these ‘‘Suzuki effects’’ are in fact rather small for a reasonable range of the coefficients of these new operators. The present analysis is due to W. Bardeen.

We take our starting point Lagrangian, eq.(1), to be modified as

$$L = L_{kinetic} + G \left(\bar{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2} (D_\mu \bar{\Psi}_L^{ia})(D^\mu t_{Ra}) \right) \left(\bar{t}_R^b \Psi_{Lib} + \frac{\chi}{\Lambda^2} (D_\mu \bar{t}_R^b)(D^\mu \Psi_{ibL}) \right) \quad (32)$$

hence eq.(2) is similarly modified:

$$L = L_{kinetic} + \left((\bar{\Psi}_L^{ia} t_{Ra} + \frac{\chi}{\Lambda^2} (D_\mu \bar{\Psi}_L^{ia})(D^\mu t_{Ra})) H_i + h.c. \right) - M_0^2 H^\dagger H \quad (33)$$

Now, we perform the block-spin RG transformation as in section 2.1. we obtain the low energy effective Lagrangian in analogy with eq.(4):

$$L = L_{kinetic} + \left((\bar{\Psi}_L^{i\alpha} t_{R\alpha} H_i + h.c.) \right. \\ \left. + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2 + O(1/\Lambda^2) \dots \right) \quad (34)$$

where now the parameters transform as:

$$Z_H = \frac{N_c}{8\pi^2} \left(\ln(\Lambda/\mu) - \chi + \chi^2/8 \right) \quad (35)$$

$$\lambda_0 = \frac{N_c}{4\pi^2} \left(\ln(\Lambda/\mu) - 2\chi + \frac{3}{2}\chi^2 - \frac{2}{3}\chi^3 + \frac{1}{8}\chi^4 \right) \quad (36)$$

and M_μ^2 has additive terms which we will fine-tune as above.

To obtain the low energy predictions we see that for large χ we cannot use the usual renormalization group equations up to scale $\mu \sim \Lambda$. Thus, in this regime we are forced to adopt the exact results in large-N. However, once the logarithm becomes large the usual RG equations become exact.

The following procedure has been adopted to explore the sensitivity to χ : (i) from $\mu = \Lambda$ to $\mu = \mu^* = \Lambda/5$ we use eq.(35) and eq.(36) directly to evolve Z_H and λ_0 ; (ii) from $\mu = \mu^*$ to $\mu = m_t$ we use the RG equations. The sensitivity of the low energy predictions is shown in Figs.(3,4) for the three cases: (1) fermion bubble approximation; (2) ladder QCD; and (3) full Standard Model. The most sensitive case is that of fermion loop approximation since we see that there is no real nontrivial fixed point to the RG equations of eq.(14). For a wide range of χ the planar QCD and full Standard Model predictions are very insensitive owing to the nontrivial fixed point for large g_t which is rapidly approached.

We note that taking χ larger than unity is unphysical, since then the higher dimension operators dominate the lower dimension ones at scales $\mu < \Lambda$. This implies

generally a unitarity breakdown, and a full unitarization of the theory would modify the assumed Lagrangian at Λ from that displayed in eq.(2).

4. Naturalness

One might object to this scheme on the basis of naturalness and the fine-tuning that is implicit in demanding a solution in the limit $m_t \ll \Lambda$. Of course, all known physical quantum field theories have a naturalness problem in association with the cosmological constant, and whatever mechanism controls this problem commutes with many successful predictions. In fact, in a specific proposal to remedy the cosmological constant, *i.e.*, Coleman's "wormhole calculus," scalars become light and the RG fixed points for fermion masses are favored [10]! However, we should investigate whether there exist natural generalizations of the above mechanism and what kinds of natural theories might exist.

(i) Supersymmetric Extension

A supersymmetric extension of the model described above has been studied by Clark, Love and Bardeen [11]. One imagines an effective supersymmetric four-fermion interaction to exist on scales $\mu \ll \Lambda$ and supersymmetry is broken softly on a scale Δ . Here the quadratic divergence of the gap equation is essentially replaced by the SUSY soft-breaking scale Δ . Thus, if $\Delta \sim m_t$ and $G \sim 1/\Delta^2$ there is no large hierarchy. One generates a low energy effective Lagrangian which now contains the two Higgs bosons as demanded by supersymmetry and chirality. One of these (the one associated with top) is now composite with analogous compositeness conditions as above. The renormalization group improvement is thus similar to the preceding case the net result for $\Delta \sim 100$ GeV, $\Lambda \sim 10^{19}$ GeV is $m_t \approx 200$ GeV.

There is, however, a potential problem with schemes like this. In particular, solutions to the gap equation require $G \sim 1/\Delta^2$ while the four-fermion effective

Lagrangian is viewed as valid up to scales $\mu \sim \Lambda$. This implies that G is extremely large on scales $\mu \gg \Delta$ and thus there may be unitarity violations on scales large compared to Δ but small compared to Λ . While the fermion bubble sum implies that a partial unitarization has been performed in some channels, there could presumably be large violations in more complicated processes.

(ii) A Fourth Generation Scenario

Perhaps the simplest solution to the naturalness problem is to consider theories in which $\Lambda \sim 1$ to 100 TeV. Then the top can probably no longer be upheld as the condensate since we see that m_t becomes ~ 500 GeV and unacceptably large. However, a fourth generation is then workable. We emphasize that such has not been ruled out by neutrino counting at LEP and in fact, it is very reasonable in such a scheme to consider the see-saw mechanism to be operant at the electroweak scale. In this case a remarkable thing happens: light neutrinos go down to their experimental limits while heavy neutrinos go up to the electroweak scale [12]! Thus, we will consider a dynamical generation of the neutrino Majorana mass scale in the following. In fact, this is just a pure, ungauged BCS theory.

Consider a Lagrangian for right-handed neutrinos in isolation:

$$L = \bar{\nu}_R i \not{\partial} \nu_R + G(\bar{\nu}_{iR}^c \nu_{iR})(\bar{\nu}_{jR} \nu_{jR}^c) \quad (37)$$

where c refers to charge conjugation, (i, j) are summed from 1 to N . In analogy to eq.(2) we introduce an auxiliary field so that:

$$L = \bar{\nu}_R i \not{\partial} \nu_R - M_0^2 \Phi^\dagger \Phi + (\bar{\nu}_{iR}^c \nu_{iR} \Phi + h.c.) \quad (38)$$

becomes the equivalent effective Lagrangian at the scale Λ . Note that this possesses an $SO(N) \times U(1)$ symmetry. As we descend to the scale μ by block-spin RG the

effective Lagrangian in conventional notation takes the form:

$$L = \bar{\nu}_R i \not{\partial} \nu_R + |\partial\Phi|^2 - M^2 \Phi^\dagger \Phi - \frac{\Lambda}{2} (\Phi^\dagger \Phi)^2 + (\kappa \bar{\nu}_{iR}^c \nu_{iR} \Phi + h.c.) \quad (39)$$

Now the RG equations are found to be [12]:

$$16\pi^2 \frac{d}{dt} \kappa = 2N\kappa^3 + 4\kappa^3 \quad (40)$$

$$16\pi^2 \frac{d}{dt} \lambda = 8N\kappa^2 \lambda - 32N\kappa^4 + 8\lambda^2 \quad (41)$$

Note that, upon using the low energy effective Lagrangian and demanding that Φ develop a VEV v so that $\Phi = (v + \phi/\sqrt{2}) \exp(i\chi/\sqrt{2}v)$, we see that χ is a massless Nambu-Goldstone mode and the residual Higgs-Majoron, ϕ , will have a mass $m_\phi^2 = 2v^2\lambda$. The neutrinos will have Majorana masses of $m_M = 2\kappa v$

Consider the solution to eqs.(40,41) in the large- N limit. We find:

$$\frac{1}{\kappa^2(\mu)} = \frac{2N}{(4\pi)^2} \log(\Lambda^2/\mu^2); \quad \frac{1}{\lambda(\mu)} = \frac{N}{4(4\pi)^2} \log(\Lambda^2/\mu^2); \quad (42)$$

where we use the compositeness conditions. Hence, again we obtain $m_\phi = 2m_M$, so the usual Nambu-Jona-Lasinio result holds in the Majorana or BCS case as well.

Incorporating this into a realistic theory involves more analysis. In general we will have additional quartic couplings of the dynamically generated Higgs boson and a term of the form $|H^2||\Phi|^2$. The full RG equations are now nonlinear and fully coupled and one must treat them numerically. This analysis is in progress by Hill, Paschos and Luty [12].

Such a theory is a novelty in terms of its dynamics, being a "Strong Broken Horizontal Gauge Symmetry." We have experience with the weak broken symmetries of the standard model and the strong confining gauge force of QCD, but it is unusual (albeit perfectly reasonable) to ponder a force that is, itself, broken yet sufficiently

strong to drive the formation of chiral condensates. In fact, this work [12] suggests that there may be some dynamical possibilities for engineering a natural family hierarchy by “tumbling.” Thus a fourth generation with $\Lambda \sim TeV$ is an intriguing possibility and we expect $m_{quarks} \sim 500$ GeV. The details are under investigation [12]. We further note that nonminimal schemes can lead to multiple Higgs doublets in low energy effective theory, as analyzed by M. Suzuki and M. Luty [13].

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Λ (GeV)	10^{19}	10^{15}	10^{11}	10^7	10^5
m_t (GeV); Fermion Bubble ^a	144	165	200	277	380
m_t (GeV); Planar QCD ^a	245	262	288	349	432
m_t (GeV); Full RG ^b	218	229	248	293	360
m_H (GeV); Full RG ^b	239	256	285	354	455

Table I. Predicted m_{top} in three levels of increasingly better approximation as described in the text. “Fermion Bubble” refers only to the inclusion of fermion loops, equivalent to the conventional Nambu–Jona-Lasinio analysis, in which case $m_H = 2m_t$. “Planar QCD” includes additional effects of internal gluon lines. All effects, including internal Higgs lines and electroweak corrections, are incorporated in the “Full RG” lines, and we include the m_H results. *Notice that the full renormalization effects cause $m_H \neq 2m_t$.* Results (^a) are from Mahanta and Barrios, ref.[9] and (^b) are from ref.[4].

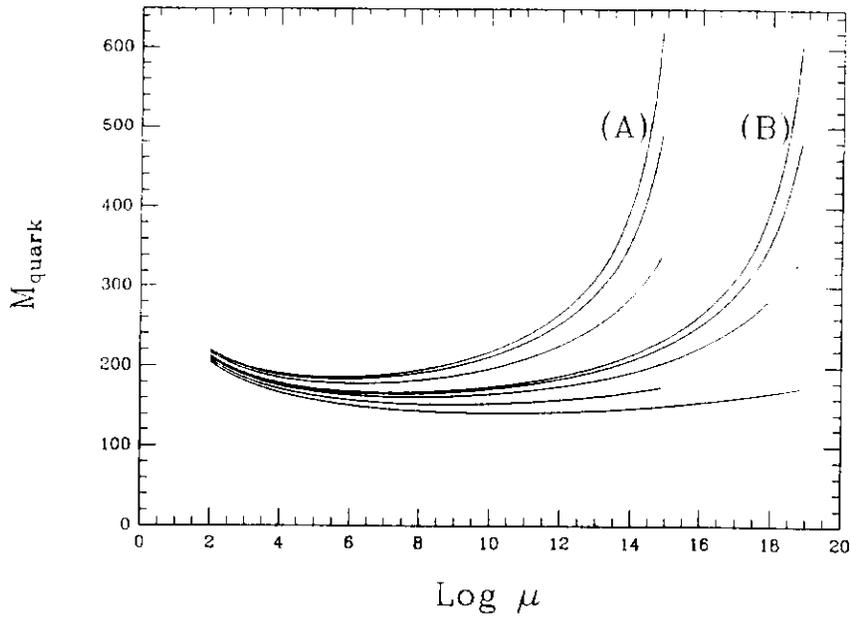


Fig.(1) Full RG trajectories as a function of scale μ . (A) $\Lambda = 10^{15}$ GeV; (B) $\Lambda = 10^{19}$ GeV. The composite trajectories diverge at the corresponding value of Λ . The predicted m_{quark} is controlled by the quasi-infrared fixed point and is very insensitive to Λ .

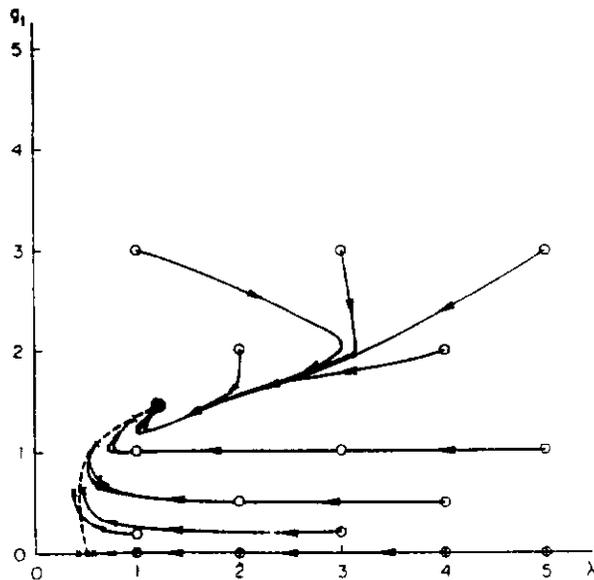


Fig.(2) Full RG trajectories showing joint evolution of g_t and λ for various initial values. Compositeness corresponds to large initial g_t and λ , and these are attracted toward the nontrivial IR fixed point (solid circle).

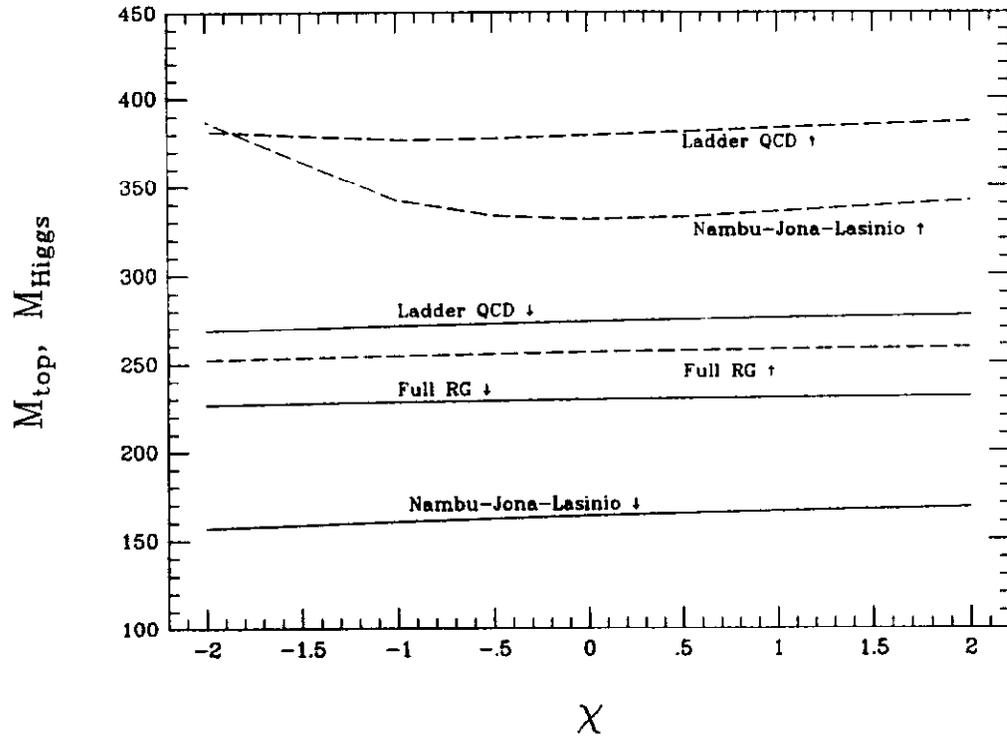


Fig.(3) Sensitivity of predicted m_{top} (solid lines) and m_{Higgs} (dashed lines) to $d = 6$ operator coefficient χ .