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Chern-Simons and Anyonic Superconductivity*

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ABSTRACT

Anyons are particles with fractional statistics. They can exist as point particles in a 2+1 dimensions, or as quasiparticles in quasiplanar condensed matter systems in the real world. Anyonic particles can be modelled by ordinary bosons or fermions coupled to a "statistical" Chern–Simons abelian gauge field. For certain values of the statistics phase, a plasma of anyons in the Chern–Simons description is a superconductor. Anyonic superconductivity may represent an idealized limit of a new type of superconductor in real systems, perhaps encompassing the recently discovered high T_c copper oxides.

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1. Introduction

"Anyon" is the name coined by Wilczek[1] to denote particles or composite systems which display fractional statistics and angular momentum. The most familiar example of such an object is a point charge bound to an infinite solenoid; interchanging two such composite objects produces in the wavefunction an Aharonov-Bohm phase $exp(i\theta)$ given[2] by the product of the charge and the flux: $\theta = q\Phi$. By tuning the flux θ can take on fractional values. Of course anyonic *particles* cannot exist in 3+1 dimensions due to the nature of the three-dimensional rotation group, which restricts the statistics parameter θ to two values: 0 (bosons) and π (fermions). But in 2+1 dimensions this restriction is absent, and point particle anyons are allowed.[1,3,4] Regarded as elementary particles in a 2+1 dimensional world, anyons lie within the purview of particle physics, and can be studied as we study other exotic (i.e. nonexistent) beasts in the particle physics "zoo".

So far all this has the makings of a cute little topic with rather skimpy motivation. On the contrary, through a felicitous circumstance, the theory of anyons has important applications in condensed matter physics. In fact anyons are now known to exist[5] as quasiparticles in at least one quasiplanar condensed matter system: the semiconductor heterojunctions that exhibit the fractional quantum Hall effect.[6] They may also appear in films of liquid helium–3.[7] Furthermore Laughlin has put forth the provocative hypothesis[8,9,10] that anyons turn up in least one other "hot" area of condensed matter physics: high T_c copper oxide superconductors. In Laughlin's scenario, the Hubbard model describing the antiferromagnetic spin interactions in the copper–oxygen planes has a P and T violating phase with a disordered ground state and anyonic quasiparticles. These anyons have statistics parameter $\theta = \pi/2$, half the value for fermions, and go by the name "semions". They can also acquire unit charge by binding to holes. A bound pair of charged semions is a charge 2 boson (the interchange phase will be 4 times $\pi/2$) analogous to a Cooper pair in BCS theory. High T_c superconductivity, in this picture, is anyon superconductivity coinciding with condensation of these pairs.

There are two broad but quite distinct questions that arise from Laughlin's work. The first is whether, and under what conditions, a plasma of anyons is a superconductor. This question is best attacked by regarding anyons as point particles in 2+1 dimensions, and it can be formulated in the language of particle theory. The second broad question is whether high T_c superconductors are anyonic superconductors. This question[8–18] lies almost entirely within the province of condensed matter theory and is, thus, none of our business. I will, nevertheless, say a few words about the current experimental situation later in this talk.

A variety of different approaches have been used to describe anyons. The direct approach simply treats a gas of point particle anyons with no interactions other than the nonlocal effects of their statistical phases. Computer simulations[19] can keep track of the relative windings of anyon trajectories, though only for rather small ensembles. Indirect theoretical approaches model anyons as conventional bosons or fermions carrying fictitious "statistical" charges and fluxes. The 2+1 dimensional Chern–Simons term[20,21] is the key to such descriptions.[22,23–26] It is not obvious that all these various approaches are equivalent. Anyons are not really free in these models but rather have differing short range quantum dynamics. Nonperturbative effects can also vary from model to model. Furthermore the status of the field–theoretic Chern-Simons description of anyons has recently been attacked by Boyanovsky[27] and by Jackiw[28] on formal grounds. Thus, even the "idealized" description of anyons as fundamental particles is fraught with messy subtleties.

The first dynamical calculations relevant to anyon superconductivity were carried out by Fetter, Hanna, and Laughlin;[29] these were then extended and expanded upon by Chen, Halperin, Wilczek, and Witten.[30] These authors examined a free gas of uncharged point particle anyons with statistics parameter $\theta{=}\pi(1-\frac{1}{N})$, where N is an integer. They showed that, in the random phase approximation, the current–current correlator at zero temperature has a massless pole. This massless collective mode presumably signals superfluidity in the uncharged anyon gas and, by extension, superconductivity for the anyon plasma. This result, though approximate, strongly suggested that some anyon systems, including semions, may exhibit superconductivity.

2. Chern–Simons theory

One way of describing an anyon gas employs a finite–density 2+1 dimensional field theory of fermions minimally coupled to an abelian Chern–Simons (CS) gauge field. A suitable action is:

$$S = \int d^3x \left[i\bar{\Psi}(\not{D} - m)\Psi + \tfrac{1}{2}\mu\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \right]$$

Here a_μ denotes the fictitious "statistics" gauge field, and μ is the Chern–Simons coupling. In the absence of a Maxwell term the statistics gauge field is not dynamical. The second class contraint

$$e\rho = \mu\mathcal{B}$$

where ρ is the fermion density and $\mathcal{B}=\epsilon^{ij}\partial_i a_j$, implies that the Chern–Simons term attaches a statistical flux e/μ to each fermion, which also carries statistical charge e. This can be seen by spatially integrating both sides of the constraint equation:

$$e\int d^2x\rho = \mu \oint \vec{a} \cdot d\vec{\ell} = \mu\Phi$$

The statistics of the fermions acquires an anyonic contribution given by

$$\theta = \frac{e^2}{2\mu}$$

It is important to note that the CS coupling appearing in this equation, which determines the anyon statistics, is the *renormalized vacuum value*. For either vacuum or finite density Chern–Simons field theory, μ is only finitely renormalized.

For example, using a cutoff regularization in the vacuum theory, one obtains[20,31]

$$\mu_{ren} = \mu_{bare} + \frac{m}{|m|}\frac{e^2}{4\pi}$$

The "special" anyons of refs [29,30] correspond to the CS coupling taking values

$$\mu = \frac{Ne^2}{2\pi}$$

The case $N=2$ corresponds to semions. If our two-dimensional spatial manifold is taken to be compact, then the CS coupling may already be restricted to quantized values, but we do not consider this in our discussion.

Although we excluded a Maxwell term for the statistics gauge field, one-loop radiative corrections will generate it. Thus in the full quantum CS theory the statistics gauge field *is* dynamical. These dynamics are strictly short range, since the CS term gives the statistics photon a gauge invariant (P and T violating) mass.

3. Criterion for anyonic superconductivity

We can give a simple criterion for anyonic superconductivity in the language of Chern–Simons field theory.[32] Let's give our fermions a real charge e in addition to their statistical charge e. They are then minimally coupled to the sum, $a_\mu + A_\mu$, of the statistics gauge field and the real gauge field. Integrating out the fermions yields a low energy effective action of the form

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{1}{4}(1+\Pi_e)F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}\Pi_o\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \\ &+ \mu_R\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda - \frac{1}{4}\Pi_e f_{\mu\nu}f^{\mu\nu} \\ &- \tfrac{1}{2}\Pi_e F_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}\Pi_o\epsilon_{\mu\nu\lambda}(A_\mu\partial_\nu a_\lambda + a_\mu\partial_\nu A_\lambda)\end{aligned}$$

Here $\Pi_e = \Pi_e(0)$, $\Pi_o = \Pi_o(0)$ come from the parity even and odd parts of the vacuum polarization:

$$\Pi_{\mu\nu}(k^2) = \Pi_e(k^2)(k_\mu k_\nu - g_{\mu\nu}k^2) - \Pi_o(k^2)\epsilon_{\mu\nu\lambda}k^\lambda$$

and to simplify the discussion we have assumed 2+1 dimensional Lorentz invariance.

The parameter μ_R is the renormalized value of the *finite density* CS coupling. We can now state our criterion for anyonic superconductivity:

$$\mu_R = 0$$

One can easily demonstrate that this is at least a *sufficient* condition for anyonic superconductivity. One considers the path integral for the effective theory, changing variables from a_μ to the dual field f^μ, $f^\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda}f_{\mu\nu}$. This is accompanied by imposing the Bianchi identity on f^μ via a Lagrange multiplier scalar field ϕ. If and only if $\mu_R=0$, one can then perform the Gaussian path integral over f^μ and obtain an equivalent local effective action:

$$\begin{aligned}\mathcal{L}_{eff}(A_\mu, \phi) =& \tfrac{1}{2}(\partial_\mu\phi + CA_\mu)^2 + a(\partial_\mu\phi + CA_\mu)F^\mu \\ & \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}\Pi_o\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda\end{aligned}$$

where I have rescaled ϕ and introduced new parameters a and C.

The first term on the right hand side of the above expression is the conventional Higgs mechanism; it gives the Meissner effect and the London supercurrent. Of course, just as in refs [29,30], a massless collective mode is the signal of superconductivity. In the present description this mode is just the statistics photon, which has become dynamical via an induced Maxwell term and has become massless due to our dynamical assumption $\mu_R=0$.

Note that in anyonic superconductivity the real photon acquires mass in two ways: from the Higgs mechanism and from its induced CS term. Indeed if we truely lived in a 2+1 dimensional world all we would need to obtain a Meissner effect is a CS term for the real photon, which would generally be induced by any P and T violation that happens to be hanging around. Such "Chern-Simons superconductivity" probably has no relation to real systems, but it is an interesting topic nonetheless.[33,34]

4. Integer quantum Hall picture

In refs. [29,30] a massless collective mode was exhibited in a mean field approximation to the anyon gas, where the statistics flux is smeared out to a constant mean value. In the language of CS perturbation theory, this is equivalent to summing the finite–density tadpole corrections to the fermion propagator.[35,36] One can define a new "tadpole–improved" perturbation expansion using this tadpole–corrected propagator, which is equivalent to fermion propagation in a background mean field, $\mathcal{B}$, which is fixed by the mean fermion density:

$$\mathcal{B} = \frac{e}{\mu}\rho$$

The mean fermion density can be determined in this same approximation as a function of the chemical potential, μ_c. Solving the relevant Schwinger–Dyson equation gives[35,36]

$$\rho = \frac{e\mathcal{B}}{2\pi}\left[Int\left(\frac{\mu_c^2 - m^2}{2e\mathcal{B}}\right) + \tfrac{1}{2}\right]$$

where I have taken $\mu_c > m > 0$.

If we forget for the moment that $\mathcal{B}$ is not an independent variable, this expression simply exhibits the ρ vs. μ_c behavior of fermions in Landau levels. Each Landau level has degeneracy $e\mathcal{B}/2\pi$ per unit area. The levels are evenly spaced; the spacing is the Landau gap $\Delta = e\mathcal{B}/m$. If we define

$$N = Int\left(\frac{\mu_c^2 - m^2}{2e\mathcal{B}}\right)$$

then the condition to have N *exactly filled* Landau levels is that $(\mu_c^2 - m^2/2e\mathcal{B})$

be equal to N plus a *nonzero* remainder.

Our criterion for superconductivity, $\mu_{ren}=0$, can be rewritten as

$$\Pi_o(k=0)=\mu$$

We can easily compute $\Pi_o(0)$ in the mean field approximation, since cutting tadpoles that appear in diagrams for ρ gives diagrams for Π_o. Thus[35,36]

$$\Pi_o(0)=\frac{\delta e\rho}{\delta B}|_{\mu_c}$$

Using our expression for ρ and the relation between ρ and B, one sees immediately that the criterion for anyonic superconductivity is precisely satisfied in the case where we have N exactly filled Landau levels. Furthermore this requires that the vacuum renormalized CS coupling taking values $Ne^2/2\pi$.[35,36,37,38] Thus the "special values" of the anyon statistics parameter correspond to exactly filled Landau levels.

Now exactly filled Landau levels for planar fermions in a constant perpendicular background magnetic field sounds like the integer quantum Hall (IQH) system.[6] Of course in our case the magnetic field is a fictitious statistics field. Nevertheless this similarity is surprising, since IQH systems are certainly *not* superfluids! In particular, they don't exhibit the linear dispersion at low momenta signalling the presence of a massless collective mode. The crucial difference between the mean field anyon gas and the IQH system is that B is fixed in relation to ρ. A local density perturbation in the IQH system requires exciting fermions across the Landau gap to an unoccupied level. In our system, however, a local increase in density brings a local increase in B, resulting in increased level degeneracy that is just enough to allow a density wave without exciting fermions across the gap.[37] *This is the origin of the massless collective mode in the mean field anyon gas.*

5. Nonrenormalization theorem

Above I have sketched a rather simple and physically intuitive picture of anyon superconductivity in a mean field approximation. However, except for N very large, mean field would not appear to be a good description of the actual anyon gas. In CS language, the above analysis completely neglects all 2–loop and higher radiative corrections which are not pure tadpole.

Never fear, Chern–Simons theory comes to our rescue with a powerful nonrenormalization theorem.[38,39] This theorem states that all radiative corrections to ρ and $\Pi_o(0)$, higher than 1–loop, *vanish* order by order in *tadpole–improved* perturbation theory. Thus *the CS mean field picture discussed above is exact.* We can therefore say with confidence that anyonic superconductivity does exist as a property of finite–density (zero temperature) Chern–Simons theory.

It is amusing to note that the physics of the nonrenormalization theorem is equivalent to the well–known insensitivity of the IQH quantization to impurities.[37,6] Thus condensed matter experiment "confirms" the formal properties of a 2+1 dimensional gauge theory!

6. Conclusion

As we have seen, anyonic superconductivity differs from ordinary (London) superconductivity by the addition of P and T violating effects. This suggests a number of possible experimental signatures for anyonic superconductivity.[12,13,40] These include "orbital ferromagnetism" and optical rotation effects. The experimental evidence for orbital ferromagnetism is distinctly negative,[41,42] while the situation for optical rotation is more promising. In any case the experimental status of P and T violation in high T_c materials should be clarified fairly soon. I must point out that the theoretical predictions themselves have several murky points; these need to be clarified before a serious confrontation with experiment is attempted.

Since anyonic superconductivity exists as a theoretical possibility, I would be surprised if it is not realized in *some* condensed matter system. Thus, even if it turns out that the copper–oxide superconductors are not anyonic, there may be other superconducting anyon systems yet to be discovered.

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