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Minimal Dynamical Symmetry Breaking of the Electroweak Interactions and m_{top}

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Abstract

We review the recent idea of a mechanism for breaking the electroweak interactions which relies upon the formation of condensates involving the conventional quarks and leptons. In particular, such a scheme would indicate that the top quark is heavy, greater than or of order 200 GeV, and gives further predictions for the Higgs boson mass. It may be imbedded either into a GUT setting using supersymmetry or applied to a fourth generation with new strong TEV scale flavor–interactions.

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1. Theoretical Implications of a Heavy Top Quark

We now know from CDF that $m_{top} > 89$ GeV. From the perspective of the theoretical structure of the Standard Model a large top quark mass, of order the weak scale ~ 175 GeV, implies a strongly coupled theory with a Higgs–Yukawa coupling constant of top, $g_{top} \sim 1$. That is, a heavy top quark is strongly coupled to the *agent or dynamics which breaks the electroweak interactions.* A large m_{top}, moreover, implies difficulties for conventional extended technicolor, and even walking technicolor for very large m_{top} requires fine–tuning (see *e.g.*, [1]). This, in turn, suggests that the top quark might, itself, play a fundamental role in the breaking of electroweak symmetries [2 – 5] by acting as a "techniquark," as a consequence of some new interaction.

Let us first consider the behavior of the parameters of the Standard Model Lagrangian as we evolve upwards in energy scale. Those associated with $d = 4$ terms satisfy the conventional renormalization group equations and evolve logarithmically with scale. For example, the coupling constants g_1, g_2 and g_3 evolve according to the conventional β–functions. g_2 and g_3 are asymptotically free, thus tending toward zero for large energy scales, while g_1 has a Landau singularity at very high energies. On the other hand, the $d = 2$ Higgs boson mass term has an additive contribution and evolves between scale μ and Λ as $m_H^2 \to m_H^2 + c(\Lambda^2 - \mu^2)$ with c determined from vector boson and fermion loops.

In the behavior of Higgs–Yukawa couplings, which are associated with the terms leading to fermion masses, there occurs a special trajectory which we shall refer to as the "Pendleton–Ross trajectory" [6]. This trajectory defines a particular $g_{PR}(\mu)$ such that if $g_{top} < g_{PR}(\mu)$ ($g_{top} > g_{PR}(\mu)$) then g_t is asymptotically free (asymptotically diverges at some scale). This is equivalent to the asymptotic smoothness criteria discussed by Kubo *et al.* [6]. To one loop precision one obtains this trajectory by combining the RG equations for g_3 and for g_{top} and demanding the combined β-function vanishes:

$$16\pi^2 \frac{d}{dt} \ln(g_{top}/g_3) = \frac{9}{2} g_{top}^2 - (8 - b_0) g_3^2 = 0 \tag{1}$$

The physical top quark mass associated with the Pendleton–Ross trajectory is the solution to $m_{PR} = g_{PR}(m_{PR}) \times (175\ GeV)$ and Marciano [5] has recently given a precise estimate of $m_{PR} \approx 98$ GeV for $N_g = 3$ [4]. Remarkably, we are experimentally on the verge of crossing the Pendleton–Ross trajectory and possibly observing a second coupling constant in the Standard Model, g_{top}, that has a Landau singularity!

If $m_{top} > m_{PR}$ and if the Standard Model is a valid effective Lagrangian up to a given large energy scale Λ then the top quark mass is bounded from above by the so-called "triviality bound" [7]. Moreover, this corresponds to an infrared "quasi-fixed point" in the sense that over a large range of initial values of at Λ, g_{top} is swept to a universal physical value at low energies [8]. Thus, on purely probabilistic grounds one might expect $m_{top} \approx 230$ GeV for $\Lambda \approx 10^{15}$ GeV. Similiar results have been obtained for Higgs bosons in the Standard Model and in multi-Higgs boson generalizations [8]. We will see below that *this fixed point actually corresponds to a dynamically broken Standard Model by a top condensate and a composite Higgs boson composed of* $\bar{t}t$.

2. Dynamical Symmetry Breaking

We [4] have straightforwardly implemented a BCS or Nambu–Jona-Lasinio mechanism in which a new fundamental interaction associated with a high energy scale, Λ, is used to trigger the formation of a low energy condensate, $\langle \bar{t}t \rangle$. The bootstrapping of the symmetry breaking mechanism to the top quark introduces no *fundamental* Higgs scalar bosons (though a dynamical 0^+ boundstate emerges) and, by virtue of its economy, leads to new predictions which are in principle testable, or which constrain or rule out the mechanism altogether. In particular, we are able to derive predictions for m_{top} and m_{Higgs} in this scheme. The usual Cabibbo–Kobayashi–Maskawa structure and fermion mass spectrum is readily accomodated, but *bona fide predictions* of mixing angles and light quark masses are not derivable until one specifies the dynamics at the scale Λ more precisely. The usual single–Higgs–doublet Standard Model emerges as the low energy effective Lagrangian, but with new constraints that lead to nontrivial predictions.

If we consider, for discussion, the approximation in which all quarks and leptons other than the top quark are massless we may then define the theory at the scale Λ to be:

$$L = L_{kinetic} + G(\bar{\Psi}_L^{ia} t_{Ra})(\bar{t}_R^b \Psi_{Lib}) \tag{2}$$

Here $\Psi_L = (t,b)_L$ and i runs over $SU(2)_L$ indices, (a,b) run over color indices. $L_{kinetic}$ contains the usual gauge invariant fermion and gauge boson kinetic terms.

We have first considered a solution based upon the effects of the fermionic determinant alone, *i.e.*, a fermion bubble approximation. This is equivalent to a large–N_{color} expansion in the limit in which the QCD coupling constant is set to zero, and it captures nonperturbative features of the theory from the point of view of a small–coupling constant expansion.

We demand a solution to the gap equation for the induced top quark mass:

$$m_t = -\frac{1}{2} G \langle \bar{t}t \rangle = 2GN_c m_t \frac{i}{(2\pi)^4} \int d^4l \, (l^2 - m_t^2)^{-1} \tag{3}$$

or:

$$1 = \frac{GN_c}{8\pi^2} \left(\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right). \tag{4}$$

which has solutions for sufficiently strong coupling, $G \geq G_c = 8\pi^2/N_c\Lambda^2$ where G_c is the "critical" coupling constant. We regard G and Λ as fundamental parameters of the theory and we solve for m_t. Normally, for very large Λ, perhaps of order the GUT scale 10^{15} GeV, we would expect the solution of this equation to produce a large mass, $m_t \sim \Lambda$ in the broken symmetry phase. We see that a solution for $m_t \sim M_W$ for such large Λ constitutes a fine–tuning problem in that $G^{-1} - G_c^{-1}$ must then be very small. This is, indeed, the usual fine–tuning or gauge hierarchy problem of the Standard Model. The gap equation contains a quadratic divergence, corresponding to the usual Higgs mass quadratic divergence in the Standard Model. *However, the fine–tuning problem will be isolated in the gap equation, i.e., once we tune G to admit the desirable solution we need cancel no other quadratic divergences in other amplitudes.*

If we now consider the sum of leading large-N_c scalar channel fermion bubbles generated by the interaction eq.(2) we find:

$$\Gamma_s(p^2) = \frac{1}{2N_c}\left[(p^2 - 4m_t^2)(4\pi)^{-2}\int_0^1 dx \log\left\{\Lambda^2/(m_t^2 - x(1-x)p^2)\right\}\right]^{-1} \tag{5}$$

Γ_s is the propagator for a dynamically generated 0^+ boundstate, a scalar composite particle composed of $\bar{t}t$. In particular, owing to the pole at $p^2 = 4m_t^2$, we see that the theory predicts the boundstate mass of $2m_t$. This is a standard result for the Nambu–Jona-Lasinio model. We emphasize that this boundstate is the physical, observable, low energy Higgs boson. The prediction holds here only to leading order in $1/N_c$ in the absence of gauge boson corrections. We can also infer from eq.(5) that this particle is described by a field with a wave–function renormalization constant, Z_H, given by:

$$Z_H = \frac{N_c}{8\pi^2}\int_0^1 dx \log\left\{\Lambda^2/(m_t^2 - x(1-x)p^2)\right\} \tag{6}$$

This is *a relativistic boundstate*, and normal intuition from nonrelativistic potential models does not apply. In fact, the compositeness of this state is reflected by the behavior of Z_H:

$$Z_H \to 0 \qquad \text{as} \qquad -p^2 = \mu^2 \to \Lambda^2. \tag{7}$$

In fact, the essential point is embodied in eq.(7) and this allows us to give a more precise determination of the top mass upon considering the full renormalization group behavior of the complete theory.

Since this mechanism is indeed a dynamical breaking of the continuous $SU(2) \times U(1)$ symmetry, it implies the existence of Goldstone modes. Moreover, the symmetry breaking transforms as $I = \frac{1}{2}$ and will produce the same spectrum of Goldstone bosons as in the Standard Model Higgs–sector. Of course, we have a dynamical Higgs–mechanism and the gauge bosons acquire masses by "eating" the dynamically generated Goldstone poles. We obtain a second prediction of the theory in the form of a relation between the W boson mass and the top quark mass as follows.

Consider now the inverse propagator of the gauge bosons. We rescale fields to bring the gauge coupling constants into the gauge boson kinetic terms, *i.e.*, we write the kinetic terms in the form $(-1/4g^2)(F_{\mu\nu})^2$. It is useful to write the induced inverse W boson propagator in the form:

$$\frac{1}{g_2^2} D^W_{\mu\nu}(p)^{-1} = (p_\mu p_\nu / p^2 - g_{\mu\nu}) \left[\frac{1}{\bar{g}_2^2(p^2)} p^2 - \bar{f}^2(p^2) \right]. \tag{8}$$

The W boson mass is the solution to the the mass–shell condition:

$$M_W^2 = p^2 = \bar{g}_2^2(p^2) \bar{f}^2(p^2) \tag{9}$$

while the Fermi constant is the zero–momentum expression:

$$\frac{G_F}{\sqrt{2}} = \frac{1}{8\bar{f}^2(0)} \tag{10}$$

In the bubble approximation we find:

$$\begin{aligned}\frac{1}{\bar{g}_2^2(p^2)} &= \frac{1}{g_2^2} + N_c(4\pi)^{-2}\int_0^1 dx\ 2x(1-x) \\ &\qquad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\} \qquad (11)\end{aligned}$$

and:

$$\begin{aligned}\bar{f}^2(p^2) &= N_c(4\pi)^{-2}\int_0^1 dx\ (xm_b^2 + (1-x)m_t^2) \\ &\qquad \times \log\left\{\Lambda^2/(xm_b^2 + (1-x)m_t^2 - x(1-x)p^2)\right\} \qquad (12)\end{aligned}$$

A quantitative prediction for m_t in terms of G_F results when eq.(10) is combined with eq.(12):

$$\begin{aligned}\bar{f}^2(0) = \frac{1}{4\sqrt{2}G_F} &\approx N_c(4\pi)^{-2}\int_0^1 (1-x)m_t^2 \log\left\{\Lambda^2/((1-x)m_t^2)\right\} \\ &\approx \frac{1}{2}N_c(4\pi)^{-2}m_t^2\log\{\Lambda^2/m_t^2\} \qquad (13)\end{aligned}$$

For example, with $\Lambda = 10^{15}$ GeV one finds $m_t \approx 165$ GeV.

To what extent is this an accurate prediction for m_t? For one, it is valid only in leading order of $1/N_c$ with $g_3 = 0$. This result, moreover, neglects the full dynamical effects of gauge bosons and the composite Higgs boson, which should be included in the renormalization group running below the scale Λ. We note that this result is substantially less than the full RG–improved Standard Model result as described below.

Analogous results are obtained for the neutral gauge boson masses, but they contain no additional information beyond that described here, a consequence of the conventional $I = \frac{1}{2}$ breaking mode. Moreover, the usual ρ parameter relationship for m_t emerges.

3. Effective Lagrangian

The dynamically generated scalar boundstates are described by the following effective Lagrangian:

$$\begin{aligned} L &= L_{kinetic} + (\bar{\Psi}_L t_R H + h.c.) \\ &\quad + Z_H |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 \end{aligned} \tag{14}$$

We include here the gauge invariant kinetic terms of the Higgs doublet and its induced quartic interaction coming from top quark loops, as well as the wave-function normalization constant, Z_H.

In the present case, however, the Higgs field is dynamical with a vanishing wave-function renormalization constant at the scale $\mu \sim \Lambda$. That is, we have the following conditions at Λ (in terms of the unconventional normalization):

$$Z_H \propto N_c \ln(\Lambda/\mu) \rightarrow 0|_{\mu\rightarrow\Lambda}. \tag{15}$$

$$\lambda_0 \propto N_c \ln(\Lambda/\mu) \rightarrow 0|_{\mu\rightarrow\Lambda} \tag{16}$$

Conventionally one normalizes the kinetic terms of a field theory at any scale, μ, with a condition that the kinetic terms have free-field theory normalization. That is, we may exercise our freedom of rescaling the various fields, H, Ψ_L, t_R, *etc.*, to define the coefficient of $|D_\mu H|^2$ to be unity. In the present case $H \rightarrow H/\sqrt{Z_H}$. The physical coupling constants, such as top quark Higgs-Yukawa coupling, $\bar{g}_t$, and the quartic Higgs coupling, $\bar{\lambda}$, are then:

$$\bar{g}_t = \frac{1}{\sqrt{Z_H}}; \qquad \bar{\lambda} = \frac{1}{Z_H^2}\lambda_0 \tag{17}$$

It is clear from eqs.(17) that as $\mu \to \Lambda$ then $\bar{g}_t$ and $\bar{\lambda}$ diverge, while $\bar{g}_t^2/\bar{\lambda} \to$ *constant*.

To obtain a renormalization group improvement over the large-N_c Nambu–Jona-Lasinio model we may utilize these boundary conditions on $\bar{g}_t$ and $\bar{\lambda}$ and the full β-functions (neglecting light quark masses and mixings) of the Standard Model. To one–loop order we have:

$$16\pi^2 \frac{d\bar{g}_t}{dt} = (\frac{9}{2}\bar{g}_t{}^2 - 8\bar{g}_3{}^2 - \frac{9}{4}\bar{g}_2{}^2 - \frac{17}{12}\bar{g}_1{}^2)\bar{g}_t \tag{18}$$

and, for the gauge couplings:

$$16\pi^2 \frac{d\bar{g}_i}{dt} = -c_i\ \bar{g}_i{}^3 \tag{19}$$

with

$$c_1 = -\frac{1}{6} - \frac{20}{9}N_g; \quad c_2 = \frac{43}{6} - \frac{4}{3}N_g; \quad c_3 = 11 - \frac{4}{3}N_g \tag{20}$$

where N_g is the number of generations and $t = \ln \mu$.

The precise value of the top quark mass is determined by running $\bar{g}_t(\mu^2)$ from very high values at a given compositeness scale Λ down to the mass–shell condition $\bar{g}_t(m_t^2)v/\sqrt{2} = m_t$. The nonlinearity of eq.(18) focuses a wide range of initial values into a small range of final low energy results. For an estimate one can assume that the gauge couplings are constant, which indicates why the solutions are attracted toward the effective low energy fixed–point [8]:

$$\bar{g}_t{}^2(\mu^2) \approx \frac{16}{9}\ \bar{g}_3{}^2(\mu^2) \tag{21}$$

The action of the effective fixed–point makes the top quark mass prediction very insensitive to the initial high values of the coupling constant close to Λ. The uncertainties of higher orders can be viewed as an uncertainty in the precise position of Λ, and the fixed point behavior implies that m_t is determined up to $O(\ln\ln\Lambda/m_t)$ sensitivity to Λ. In Table I we give the resulting physical m_{top} obtained by a numerical solution of the renormalization group equations as a function of Λ.

The Higgs boson mass will likewise be determined by the evolution of $\bar{\lambda}$ given by:

$$16\pi^2\frac{d\bar{\lambda}}{dt} = 12(\bar{\lambda}^2 + (\bar{g}_t{}^2 - A)\bar{\lambda} + B - \bar{g}_t{}^4) \tag{22}$$

where:

$$A = \frac{1}{4}\bar{g}_1{}^2 + \frac{3}{4}\bar{g}_2{}^2; \quad B = \frac{1}{16}\bar{g}_1{}^4 + \frac{1}{8}\bar{g}_1{}^2\bar{g}_2{}^2 + \frac{3}{16}\bar{g}_2{}^4 \tag{23}$$

The resulting prediction of the full Standard Model analysis is a top quark mass that might be considered large in comparison to certain published upper limits. Indeed, it has been claimed that the ρ parameter limit implies $m_t \lesssim 180$ to 200 GeV [9], and this is the most stringent quoted limit. However, without doing sufficient justice to any one of them, a number of recent experimental results have central values that are tantalizingly suggestive of a very heavy top quark (*e.g.*, the E-731 measurement of ϵ'/ϵ; recent UA-2 W-mass determination; the LEP combined results on $\Gamma_{Z\to hadrons}$).

We feel that it it is premature to reject theoretical predictions of a very heavy top quark, up to at least ~ 250 GeV, based upon the present status of the precision measurements to date. Note that, by incorporating the data with our prediction, we favor $\Lambda \gtrsim 10^{11}$ GeV. The phenomenological predictions for $\sin^2\theta_W$ (Marciano–Sirlin definition) and M_W are summarized by the following equations:

$$\sin^2\theta_W = 0.215 \pm 0.002 + 0.00017(230 - m_t), \tag{24}$$

$$M_W = 80.73 \pm 0.15 + 0.009(m_t - 230). \tag{25}$$

If, ultimately, the theoretical top quark mass prediction proves to be too high then it is still possible, albeit possibly less compelling, to maintain this mechanism by assuming that the gap equation is saturated by a fourth generation. The top quark then plays no important role itself in the symmetry breaking of the Standard Model and should have a mass between current lower bounds, but presumably much less than the predictions for the masses of the fourth generation. In Table II we include the corresponding predictions for the masses of a degenerate fourth doublet. The resulting modified predictions for the Higgs mass as well as the corresponding errors are also shown.

We note that the two–Higgs boson generalization of this scheme has recently been analyzed in detail by Suzuki and Luty [10]

4. Naturalness and Other Issues

One might object to this scheme on the basis of naturalness and the fine–tuning that is implicit in demanding a solution to eq.(4) in the limit $m_t << \Lambda$. Of course, all known physical quantum field theories have a naturalness problem in association with the cosmological constant, and whatever mechanism controls this problem commutes with many successful predictions. Perhaps such a mechanism is operant for the quadratic divergences that plague theories with scalars. However, we should investigate whether there exist natural generalizations of the above mechanism and what kinds of natural theories might exist.

(i) Supersymmetric extensions of the model described above exist and the most straightforward has been studied by Bardeen, Love and Clark [11]. One imagines an effective supersymmetric four–fermion containg interaction to exist on scale $\mu < \Lambda$ and supersymmetry is broken softly on a scale Δ. Here the quadratic divergence of the gap equation is essentially replaced by the SUSY soft–breaking scale Δ. Thus,

if $\Delta \sim m_t$ and $G \sim 1/\Delta^2$ there is no large hierarchy. One generates a low energy effective Lagrangian which now contains the two Higgs bosons as demanded by supersymmetry and chirality. One of these (the one associated with top) is now composite with analogous compositeness conditions as in eq.(7). The renormalization group improvement is thus similar to the preceding case the net result for $\Delta \sim 100$ GeV, $\Lambda \sim 10^{19}$ GeV is $m_t \approx 200$ GeV.

There is, however, a problem with schemes like this. In particular, solutions to the gap equation require $G \sim 1/\Delta^2$ while the four-fermion effective Lagrangian is viewed as valid up to scales $\mu \sim \Lambda$. This implies that G is unacceptably large on scales $\mu >> \Delta$ and thus there would be unitarity violations on scales large compared to Δ but small compared to Λ. While the fermion bubble sum implies that a partial unitarization has been performed in some channels, there would presumably be large violations in more complicated processes. This defect is also problematic for any "walking" version of this scenario in which a new force is invoked to provide a large anomalous dimension to the four-fermion operator thus reducing the degree of (quadratic) divergence of the gap equation loop integral. We thus feel the following line is more promising.

(ii) Perhaps the simplest solution to the naturalness problem is to consider theories in which $\Lambda \sim 1$ TeV. Then the top can probably no longer be upheld as the condensate since we see that m_t becomes ~ 500 GeV and unacceptably large. However, a fourth generation may be workable. We emphasize that such has not been ruled out by neutrino counting at LEP and in fact, it is very reasonable in such a scheme to consider the see-saw mechanism to be operant at the electroweak scale. In this case a remarkable thing happens: light neutrinos go down to their experimental limits a heavy neutrino goes up to the electroweak scale [12]!

Such a theory is a novelty in terms of its dynamics, being a "Strong Broken Horizontal Gauge Symmetry." We have experience with the weak broken symmetries of the standard model and the strong confining gauge force of QCD, but it is unusual (albeit perfectly reasonable) to ponder a force that is, itself, broken yet sufficiently strong to drive the formation of chiral condensates. In fact, some preliminary work [13] suggests that there may be some dynamical possibilities for engineering a natural hierarchy, $m_t/\Lambda << 1$, though $\sim 10^{-13}$ would seem unlikely. Thus, a fourth generation with $\Lambda \sim TeV$ seems an intriguing possibility and from Table II we expect $m_{quarks} \sim 500$ GeV. The details of the lepton sector are under investigation [12,13].

5. Conclusions and Discussion

Our principal conclusions are as follows:

(1) The gauged Nambu–Jona-Lasinio mechanism within the framework of the Standard Model, dynamically broken by a strongly coupled top quark which forms a condensate $\langle \bar{t}t \rangle$, may be implemented in the fermion loop approximation (or large-N_c with vanishing g_3). It yields primitive relationships between M_W and m_t and the cut-off Λ, and the Higgs mass is determined as $m_H = 2m_t$. The latter relationship has been emphasized by Nambu [2].

(2) We infer that $Z_H \rightarrow 0$ as $\mu \rightarrow \Lambda$ is the general compositeness condition for the Higgs boson of the full theory. The conventional normalization, $Z_H = 1$, implies, equivalently, that g_t and λ diverge as $\mu \rightarrow \Lambda$. This constraint, in turn, implies that the low energy values of these coupling constants are controlled by the renormalization group infra-red fixed-points. Consequently, the low energy results are insensitive to the detailed behavior of g_t and λ as $\mu \rightarrow \Lambda$.

(3) We give our results for the full Standard Model as functions of the scale of new physics (or Higgs boson compositeness scale), Λ. Our favored results for m_t are the

lower values, as constrained by phenomenological considerations, hence $m_{top} \approx 230$ GeV and $m_{Higgs} \approx 260$ GeV with $\Lambda \approx 10^{15}$ GeV. The mechanism may be adapted to a fourth generation, or a multiple dynamical Higgs scheme, though our primary impetus is in the connection with the top quark since the lower mass limits on the top quark are suggestive of a strongly coupled system.

As our discussion has indicated, the compositeness of the auxiliary Higgs field leads to predictions for the top quark and Higgs masses which are equivalent to effective fixed-point arguments. We have in this mechanism a *raison d'etre* for the single-Higgs doublet Standard Model with a heavy top quark.

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$\Lambda[GeV]$	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}
$m_t^{phys}[GeV]$	218	223	229	237	248	255	264	277	293	318	360	455
pert.	± 2	± 3	± 3	± 3	± 5	± 6	± 7	± 9	± 12	± 16	± 25	± 45
$m_H^{phys}[GeV]$	239	246	256	268	285	296	310	329	354	391	455	605
pert.	± 3	± 3	± 4	± 5	± 8	± 9	± 11	± 15	± 21	± 32	± 56	± 142

Table I: The predictions for the physical top–quark and Higgs boson mass for different scales Λ. One loop β–functions are used with $g_1^2(M_Z) = 0.127 \pm 0.009$, $g_2^2(M_Z) = 0.446 \pm 0.020$, $\alpha_S(M_Z) = 0.115 \pm 0.015$ and $M_Z = 91.17$ GeV as input. The numbers are obtained for the central value of these input data and requiring the on-shell condition $\bar{m}(m) = m$. Variation of the gauge couplings within their errors results to a very good approximation in a change of ± 6 GeV for the top mass and ± 4 GeV for the Higgs mass. The rows labeled *'pert.'* show the change in the result if we change the couplings at the cutoff to unity instead of infinity, as a measure of the errors induced by using perturbation theory.

$\Lambda[GeV]$	10^{19}	10^{17}	10^{15}	10^{13}	10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}
$m_4^{phys}[GeV]$	199	202	206	212	220	226	233	243	257	277	312	388
pert.	± 1	± 2	± 2	± 2	± 3	± 4	± 5	± 7	± 10	± 14	± 22	± 39
$m_H^{phys}[GeV]$	235	241	248	258	272	282	294	310	333	365	423	553
pert	± 1	± 2	± 2	± 3	± 4	± 6	± 7	± 10	± 15	± 22	± 39	± 99

Table II: Predictions for a degenerate fourth–generation quark doublet with the same input data as in Table I. The top–quark and the fourth–generation leptons are assumed to be much lighter than this quark doublet. The variation of the gauge couplings results in a change of ± 7 GeV for the quark masses and ± 5 GeV for the Higgs mass.