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ASPECTS OF DYNAMICAL SYMMETRY BREAKING IN GAUGE FIELD THEORIES

by

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ABSTRACT

The dynamics of chiral and scale symmetry breaking in quenched, planar QED is further examined. Particular attention is focused on the renormalization properties of the model and the relevance of the four-fermion operators. In accord with our previous results, the scale symmetry is explicitly broken and the dilaton is not found at either strong or weak coupling. An effective Lagrangian simultaneously realizing both spontaneously broken scale and chiral symmetries is constructed.

1 Introduction

Nonperturbative phenomena such as dynamical symmetry breaking play a central role in many current areas of physics research. Due to the inherent difficulties in attempting to study such phenomena, advances have been limited to lattice simulations or certain crude analytic approximation schemes. In this paper, we continue our investigations into the dynamical structure of quenched, planar quantum electrodynamics.[1] The original motivation [2] leading to the study of this model stemmed from the observation that gauge theories with slowly running coupling constants possess an approximate scale symmetry. The dynamical breaking of chiral symmetry in such theories should therefore be accompanied by the dynamical breaking of this approximate scale symmetry, resulting in the appearance in the physical spectrum of the dilaton, the pseudo-Goldstone boson of the spontaneously broken scale symmetry. QED in the quenched, planar limit contains the basic dynamical structure of such gauge theories and thus provides a useful laboratory for the study of chiral and scale symmetry breaking in these theories. Recent numerical [3] and analytical [4] studies have confirmed that the quenched, planar approximation is in many ways a reasonable approximation to the full theory.

Unfortunately, the scale symmetry breaking in the quenched, planar limit of QED was such that the dilaton did not emerge in the physical spectrum.[1] Recently, there have been claims concerning the existence of the dilaton in this model.[5] We have thus reanalyzed the model and indeed found a spurious vanishing of the bubble sum scalar denominator. However, this vanishing does not lead to a dilaton pole in the S-matrix elements, consistent with our previous results. This and related topics will be addressed in section 3.

In our earlier study [1] of quenched, planar QED, we found that consistent chiral symmetry breaking solutions existed only if chirally invariant four-fermion interactions were included in the analysis. This resulted in a novel fixed point structure of the theory. The necessity for including the four-fermion operators followed from their large anomalous dimensions at strong coupling so that they become mass dimension four interactions at the fixed point. It was subsequently realized [6] that such a nonperturbtive fixed point with large anomalous dimensions provides a realization of an earlier conjecture [7] to ease flavor changing neutral current problem in extended technicolor theories. This had led in turn to revived interest [8] in technicolor theories. In section 4, we elucidate the role of these operators by analyzing the renormalization flow of the four-fermion coupling. We find that the running of this coupling is essential in order to give quenched, planar QED a sensible interpretation.

Finally, in section 5, we construct an effective lagrangian that simultaneously realizes both scale and chiral symmetry in the Nambu-Goldstone mode, thus explicitly demonstrating that no conflict arises between the low energy theorems of scale and chiral symmetry.[9] This and other results have been reported by one of us (W. A. B.) in a recent workshop.[10]

2 Chiral Symmetry Breaking

We begin by recalling the solutions of the Schwinger-Dyson equation for the fermion self-energy, $\Sigma(p)$, in quenched, planar QED. In Landau gauge, this equation reads

$$\Sigma(p) = m_0 + \frac{3e^2}{(2\pi)^4} \int d^4q \, \frac{\Sigma(q)}{q^2 + \Sigma^2(q)} \frac{1}{(p-q)^2}, \tag{2.1}$$

where m_0 is the bare fermion mass. Solutions to the Schwinger-Dyson equation (2.1) have been analyzed extensively over the years.[11, 12, 13] It was estabished that nontrivial solutions to the homogeneous equation ($m_0 = 0$) exist only in the strong coupling phase $\alpha \geq \alpha_c \equiv \frac{\pi}{3}$.[12]

It is easiest to see this by transforming the integral equation (2.1) into a differential equation with appropriate boundary conditions. Using the scale symmetry, the solution can be written in the form [1, 12]

$$\Sigma(p) = e^t u(t + t_0), \qquad t = \log\ p, \tag{2.2}$$

where t_0 is related to the dynamically generated fermion mass scale by

$$\Sigma(0) \equiv \Sigma_0 = e^{-t_0} \tag{2.3}$$

and the function $u(x)$ satisfies

$$u''(x) + 4u'(x) + 3u(x) + \frac{\alpha}{\alpha_c} \frac{u(x)}{1 + u^2(x)} = 0 \tag{2.4}$$

The infrared boundary condition is such that $e^x u(x) \rightarrow 1\ as\ x \rightarrow -\infty$, while the ultraviolet boundary condition is

$$m_o = \frac{\Lambda}{2}[u'(t_\Lambda + t_0) + 3u(t_\Lambda + t_0)], \qquad t_\Lambda = \log\ \Lambda, \tag{2.5}$$

with Λ being an ultraviolet cutoff. The ultraviolet behavior of $u(x)$ depends on the strength of the fixed coupling constant α. For strong coupling $(\alpha > \alpha_c)$,

$$u(x) \sim \tilde{A}(\alpha) e^{-2x} \sin[\sqrt{\alpha/\alpha_c - 1}(x + \delta(\alpha))]/\sqrt{\alpha/\alpha_c - 1}, \tag{2.6}$$

where $\tilde{A}(\alpha) \approx 1.04$ and $\delta(\alpha) \approx 0.715$ for $\alpha \approx \alpha_c$, while for weak coupling $(\alpha < \alpha_c)$,

$$u(x) \sim \tilde{A}(\alpha) e^{-2x} \sinh[\sqrt{1-\alpha/\alpha_c}(x+\delta(\alpha))]/\sqrt{1-\alpha/\alpha_c}. \tag{2.7}$$

Note that this solution is analytic in α near α_c for fixed x.

The weak coupling solution contains two terms and can be written as

$$\Sigma(p) = p\ u(\log(p/\Sigma_0)) \to m/p^\gamma + <\bar{\psi}\psi>_o /p^{2-\gamma}, \tag{2.8}$$

with

$$\gamma = 1 - \sqrt{1-\alpha/\alpha_c}. \tag{2.9}$$

The first term corresponds to the explicit breaking of chiral (and scale) symmetry, while the second term corresponds to the spontaneous breaking. Both have pure power behavior reflecting the scaling structure [14] of the operator product expansion of the fermion propagator. Here γ is the anomalous dimension of the fermion mass operator, $\bar{\psi}\psi$. The presence of both terms in (2.8) confirms the result that there are no spontaneous chiral symmetry breaking solutions in the weak coupling phase.

For strong coupling, there exists a massive solution in the chiral limit $(m_0 = 0)$. However, the resulting dynamical fermion mass scale is proportional to the cutoff and given by

$$\Sigma_0 = e^{\delta} \Lambda e^{-\theta/\sqrt{\alpha/\alpha_c - 1}}, \qquad 0 < \theta < \pi. \tag{2.10}$$

In order for this to be a dynamical chiral symmetry breaking solution, the mass scale Σ_0 must remain finite in the continuum limit. This can only occur in the Miransky solution [15] which requires the coupling constant to have a specific cutoff dependence given by

$$\alpha = \alpha(\Lambda) \to \alpha_c[1 + \frac{\theta^2}{\log^2(\Lambda/\kappa)}], \ \ \theta \to \ \pi, \ as\ \Lambda \to \ \infty. \tag{2.11}$$

Here κ is an infrared mass scale proportional to Σ_0.

This required tuning of α with the cutoff immediately calls into question the renormalization properties of the theory. This will be discussed in more detail in Sec. 4. An important implication of such considerations is that one must take into account the induced four-fermion interactions in order to study the continuum limit of the theory. To study the dynamical running of the coupling constants, one integrates out the high momentum behavior of the theory and studies the renormalization flow of the coupling constants. Consider, for example, the ladder diagram in Fig. 1. It is evident that integrating out the high momentum part of a given rung will generate a four-fermion interaction. These four-fermion interactions must be included in the study of the renormalization flow of the coupling constants if they are relevant operators. (This is analogous to the induced ϕ^4-interactions in the $\eta\phi^6$ theory in 2 + 1 dimensions [16]). Indeed, in the planar approximation, the four-fermion operators have twice the dimension of the mass operator, $d_{(\bar{\psi}\psi)^2} = 2(3-\gamma) \rightarrow 4$ as $\alpha \rightarrow \alpha_c$, and are relevant (marginal) operators in the continuum limit. We must include them in a chirally invariant fashion in the study of dynamical chiral and scale symmetry breaking in quenched, planar QED.

We are thus led to consider the lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \mu_0)\psi + \frac{G_0}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2] \tag{2.12}$$

which has the form of a gauged Nambu-Jona-Lasinio model (NJL). [17] The bare fermion mass is now denoted by μ_o. This model has been studied in Ref. 1. In the planar approximation, the addition of the four-fermion interactions amounts to a simple modification of the Schwinger-Dyson equation in which the bare mass term for the pure gauge theory receives an extra contribution from the fermion tadpole so that

$$m_0 = \mu_0 - G_0 < \bar{\psi}\psi >_0, \tag{2.13}$$

where

$$< \bar{\psi}\psi >_0 = \frac{\Lambda^3}{2\pi^2}\frac{\alpha_c}{\alpha}[u'(t_\Lambda + t_0) + u(t_\Lambda + t_0)]. \tag{2.14}$$

Determining m_0 consistently with the boundary condition (2.5) yields the gap equation

$$\mu \equiv \mu_0\Lambda = \frac{\Lambda^2}{2}[(1+G)u'(t_\Lambda + t_0) + (3+G)u(t_\Lambda + t_0)], \tag{2.15}$$

where we have introduced the renormalized four-fermion coupling, $G \equiv (G_0\Lambda^2/\pi^2)(\alpha_c/\alpha)$.

In the chiral limit, $\mu = 0$, the gap equation for $\alpha > \alpha_c$ reads

$$\tan\theta = (\frac{G+1}{G-1})\sqrt{\alpha/\alpha_c - 1} \tag{2.16}$$

with

$$\theta = \sqrt{\alpha/\alpha_c - 1}\,\log(\frac{\Lambda e^{\delta}}{\Sigma_0}).$$

The vacuum solution requires that $0 < \theta < \pi$. Thus a nontrivial continuum limit, $\Lambda/\Sigma_0 \to \infty$, dictates that $\alpha \to \alpha_c$. On the other hand, for weak coupling, $\alpha < \alpha_c$, the gap equation becomes

$$\tanh\tilde{\theta} = (\frac{G+1}{G-1})\sqrt{1 - \alpha/\alpha_c} \tag{2.17}$$

with

$$\tilde{\theta} = \sqrt{1 - \alpha/\alpha_c}\,\log(\frac{\Lambda e^{\delta}}{\Sigma_0}).$$

This equation exhibits nontrivial solutions provided $G > 1$. Thus, as emphasized by several authors [18], the Schwinger-Dyson equation admits massive solutions even for weak gauge coupling when four-fermion interactions are present. In fact, there is a critical curve along which a nontrivial continuum limit exists. In this limit, $\tilde{\theta} \to \infty$, so that the gap equation relates G to α as

$$G\frac{\alpha}{\alpha_c} = (1 + \sqrt{1 - \alpha/\alpha_c})^2 \tag{2.18}$$

This curve extends from $(\alpha, G) = (\alpha_c, 1)$ to $(\alpha, G) = (0, \infty)$, with the point at $\alpha = 0$ corresponding to the Nambu-Jona-Lasinio model. The physical interpretation is that a weaker gauge interaction can be compensated by a stronger, attractive ($G > 0$) four-fermion interaction in the gauged NJL model. The fact that a finite $\Lambda \to \infty$ limit is found at weak gauge coupling is certainly unexpected since one anticipates the four-fermion interaction to be an irrelevant operator there. The behavior may in fact result from the oversimplification of the quenched, planar approximation, although it may also indicate the existence of an interesting renormalizable phase of the gauged NJL model.[19] We shall elaborate on this possibility a bit more in the next section. It would be worthwhile to investigate this phenomenon in lattice simulations.

A consequence of dynamical symmetry breaking is the appearance of Goldstone bosons in the physical spectrum. To study the symmetry structure of the above solutions we look for the appearance of such Goldstone bosons (pion or dilaton) which would be magnifested as poles in the fermion-antifermion scattering amplitudes. For the gauged NJL model, we anticipate the Goldstone poles to appear in the induced bubble diagrams as zeroes of the renormalized denominator functions. Fortunately these bubble diagrams including the ladder QED radiative corrections can be evaluated at zero momentum transfer.[1]

The renormalized vertex function dressed with the QED ladders for fermion matrix elements containing a pseudoscalar insertion at zero momentum transfer was computed to be

$$\Gamma_p^R(p,p) = e^{(t+t_0)}u(t+t_0) = Z_p\Gamma_p^0(t), \tag{2.19}$$

where $\Gamma_p^0(t)$ is the bare pseudoscalar vertex and

$$Z_p = \frac{1}{2}e^{(t_\Lambda+t_0)}[u'(t_\Lambda+t_0)+3u(t_\Lambda+t_0)]. \tag{2.20}$$

Moreover the bare bubble integral was evaluated as

$$B_p^0(0) = \frac{1}{2\pi^2}\frac{\alpha_c}{\alpha}e^{(3t_\Lambda+t_0)}\frac{1}{Z_p}[u'(t_\Lambda+t_0)+u(t_\Lambda+t_0)]. \tag{2.21}$$

The Goldstone boson of chiral symmetry should then appear as a zero in the renormalized pseudoscalar denominator function, $D_p^R(0)$. Following the computation of Ref. 1, this is given by

$$
\begin{aligned}
D_p^R(0) &= Z_p^2[G_0^{-1} + B_p^0(0)] \\
&= \begin{cases}
(\tilde{A}^2/4\pi^2)\Sigma_0^2(\alpha_c/\alpha)G^{-1}[\sin\theta/\sqrt{\alpha/\alpha_c - 1} + \cos\theta] & \\
\times[(1-G)\sin\theta/\sqrt{\alpha/\alpha_c - 1} + (1+G)\cos\theta], & \alpha > \alpha_c \\
 & \\
(\tilde{A}^2/4\pi^2)\Sigma_0^2(\alpha_c/\alpha)G^{-1}[\sinh\tilde{\theta}/\sqrt{1 - \alpha/\alpha_c} + \cosh\tilde{\theta}] & \\
\times[(1-G)\sinh\tilde{\theta}/\sqrt{1 - \alpha/\alpha_c} + (1+G)\cosh\tilde{\theta}], & \alpha < \alpha_c.
\end{cases}
\end{aligned}
\tag{2.22}
$$

Upon imposing the gap equations (2.16) and (2.17), we see that $D_p^R(0)$ does indeed vanish in the chiral limit for all α. We thus conclude that the gauged NJL model exhibits dynamical chiral symmetry breaking for both strong and weak gauge coupling.

3 Scale Symmetry Breaking

Since its perturbative beta functions vanishes, quenched planar QED provides an attractive arena to study the relationship between the dynamics of scale symmetry breaking and chiral symmetry breaking. We have already established that the gauged NJL model exhibits a phase in which chiral symmetry is spontaneously broken and a fermion mass scale, Σ_0, is dynamically generated. If the scale symmetry is not explicitly broken, it will also be spontaneously broken by the dynamical generation of Σ_0. In this case, the dilaton pole will appear as a zero in the renormalized scalar denominator function. Although the dynamical breaking of chiral symmetry can occur for all values of α with $G = G(\alpha, \Lambda/\Sigma_0)$, we expect the scale invariance to be preserved only for specific values of the induced four-fermion coupling, as is the case for the induced couplings in the scale invariant $\eta\phi^6$ theory.[16] We have analyzed the case for strong gauge coupling and did not find such a scale invariant fixed point [1]. It seems that the apparent scale symmetry of quenched, planar QED is expicitly broken even when the induced four-fermion interactions are taken into account.

Recent claims have suggested the possible existence of a scale invariant fixed point along the critical line for weak gauge coupling.[5] We have thus reanalyzed the model for both strong and weak gauge coupling and confirmed our previous result that the dilaton pole does not exist. There is a spurious vanishing of the scalar denominator function arising from the bubble sum, but this is cancelled by a related pole in the ladder diagrams yielding no overall dilaton pole in the fermion-antifermion scattering amplitudes.

By computations similar to that for the pseudoscalar case, the QED ladder dressed renormalized vertex function for the fermion matrix elements containing a scalar $\bar{\psi}\psi$ insertion was obtained at zero momentum transfer as [1]

$$\Gamma_s^R(p,p) \;=\; -e^{(t+t_0)}u'(t+t_0) \;=\; Z_s\Gamma_s^0(t), \tag{3.1}$$

where Γ_s^0 is the bare vertex and

$$Z_s = -\frac{1}{2}e^{(t_\Lambda+t_0)}[u''(t_\Lambda+t_0)+3u'(t_\Lambda+t_0)]. \tag{3.2}$$

The zero momentum scalar bare bubble function was also evaluated as

$$B_s^0(0) = -\frac{1}{2\pi^2}\frac{\alpha_c}{\alpha}e^{(3t_\Lambda+t_0)}\frac{1}{Z_s}[u''(t_\Lambda+t_0)+u'(t_\Lambda+t_0)]. \tag{3.3}$$

With these ingredients, the renormalized scalar denominator function can then be obtained as

$$\begin{aligned} D_s^R(0) &= Z_s^2[G_0^{-1}+B_s^0(0)] \\ &= \begin{cases} (\tilde{A}^2/4\pi^2)\Sigma_0^2\frac{\alpha_c}{\alpha}G^{-1}[(1+\frac{\alpha}{\alpha_c})\sin\theta/\sqrt{\alpha/\alpha_c-1}+\cos\theta] \\ \times\{[(2-2G)+(1+G)(\frac{\alpha}{\alpha_c}-1)]\sin\theta/\sqrt{\alpha/\alpha_c-1} \\ +(1+3G)\cos\theta\}, \qquad \alpha>\alpha_c \\ (\tilde{A}^2/4\pi^2)\Sigma_0^2\frac{\alpha_c}{\alpha}G^{-1}[(1+\frac{\alpha}{\alpha_c})\sinh\tilde{\theta}/\sqrt{1-\alpha/\alpha_c}+\cosh\tilde{\theta}] \\ \times\{[(2-2G)-(1+G)(1-\frac{\alpha}{\alpha_c})]\sinh\tilde{\theta}/\sqrt{1-\alpha/\alpha_c} \\ +(1+3G)\cosh\tilde{\theta}\}, \qquad \alpha<\alpha_c. \end{cases} \end{aligned} \tag{3.4}$$

In the chiral limit, $\mu=0$, this takes the form

$$D_s^R(0) = (\frac{\tilde{A}^2}{4\pi^2})\Sigma_0^2(\frac{2\alpha_c}{\alpha}+1+G^{-1}) \quad for\ all\ \alpha. \tag{3.5}$$

For attractive four-fermion interactions $(G>0), D_s^R(0)$ does not vanish at either strong or weak gauge coupling. On the other hand, $D_s^R(0)$ does have a zero for repulsive four-fermion couplings, $G<0$. This zero in $D_s^R(0)$ is due to the vanishing of the scalar vertex renormalization factor,

$$Z_s = \begin{cases} \frac{\tilde{A}}{2}\frac{\Sigma_0}{\Lambda}[(1+\frac{\alpha}{\alpha_c})\sin\theta/\sqrt{\alpha/\alpha_c-1}+\cos\theta], & \alpha>\alpha_c \\ \frac{\tilde{A}}{2}\frac{\Sigma_0}{\Lambda}[(1+\frac{\alpha}{\alpha_c})\sinh\tilde{\theta}/\sqrt{1-\alpha/\alpha_c}+\cosh\tilde{\theta}], & \alpha<\alpha_c \end{cases} \tag{3.6}$$

which in the chiral limit is given by

$$Z_s = \frac{\tilde{A}\,\Sigma_0}{2\,\Lambda}\left(\frac{G}{G-1}\right)\frac{\alpha}{\alpha_c}\left(\frac{2\alpha_c}{\alpha}+1+\frac{1}{G}\right)\begin{cases}\cos\theta, & \alpha > \alpha_c \\ \cosh\tilde{\theta}, & \alpha < \alpha_c\end{cases}. \tag{3.7}$$

The vanishing of Z_s when $G = -1/(1+2\alpha_c/\alpha)$ indicates the presence of a pole in the bare scalar vertex function, $\Gamma_s^o(p,p) = \Gamma_s^R(p,p)/Z_s$, or equivalently that the renormalized scalar vertex function satisfies a homogeneous Bethe-Salpeter equation. But this vertex function is obtained from a sum of ladder graphs only. Thus if a bound state pole exists, it must also appear in the ladder diagrams for the fermion-antifermion scattering amplitude. In the vicinity of the bound state, the ladder sum will be dominated by this pole term and can be accurately approximated by the diagram of Fig. 2. That is,

$$\begin{aligned} S_{ff}(ladder) &\rightarrow \Gamma_s^0(p,p)\frac{1}{B_s^0(0)}\Gamma_s^0(p',p') \\ &= \Gamma_s^R(p,p)\frac{1}{Z_s^2\;B_s^0(0)}\Gamma_s^R(p',p'). \end{aligned} \tag{3.8}$$

When this pole contribution of the ladder diagrams is combined with the scalar denominator of the bubble sum, the scalar channel of the full fermion-antifermion scattering amplitude is detemined as

$$\begin{aligned} S_{ff}(full) &= \Gamma_s^R(p,p)[\frac{1}{Z_s^2\;B_s^0(0)} - \frac{1}{D_s^R(0)}]\Gamma_s^R(p',p') \\ &= \Gamma_s^R(p,p)T_s(0)_{ff}\Gamma_s^R(p',p'), \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} T_s(0)_{ff} &= [G_0 B_s^0(0) D_s^R(0)]^{-1} \\ &= [(\tilde{A}/4\pi^2)\Sigma_0^2(G+1-6\frac{\alpha_c}{\alpha})]^{-1}, \end{aligned} \tag{3.10}$$

and the last equality holds in the chiral limit. We see that the spurious zero in the bubble denominator has disappeared from the inverse S-matrix element. Hence it does not correspond to a true dilaton pole.

It is interesting to note that the scalar denominator remains finite in the continuum limit even in the case of weak coupling, $\alpha < \alpha_c$. For this coupling regime, using dimensional analysis and taking into account the anomalous dimensions calculated in the pure gauge model (cf. Eq. 2.9), the four-fermion interactions should be irrelevant for weak coupling. One anticipates that the scalar denominator should diverge with the cutoff as

$$D_s^R(0) \to \Lambda^{2\sqrt{1-\alpha/\alpha_c}} \to \infty, \qquad \alpha \ < \ \alpha_c. \tag{3.11}$$

In fact one finds that the leading Λ dependence in Z_s which is of order $\Lambda^{\sqrt{1-\alpha/\alpha_c}-1}$ actually cancels against the same order term in $G_0 Z_s B_s^0(0)$. Since $D_s^R(0) = (Z_s/G_0)(Z_s + G_0 \ Z_s B_s^0(0))$, it was this cancellation that originally led the authors of ref. [5] to erroneously conclude that $D_s^R(0)$ vanished along the critical line [18]

$$G \ \frac{\alpha}{\alpha_c} = (1 + \sqrt{1 - \alpha/\alpha_c})^2. \tag{3.12}$$

This was in turn interpreted as the appearance of the dilaton. However, when the subdominant $\Lambda^{-\sqrt{1-\alpha/\alpha_c}-1}$ terms in $G_o Z_s B_s^o$ and Z_s are retained, they cancel the $\Lambda^{\sqrt{1-\alpha/\alpha_c}+1}$ dependence in Z_s/G_o yielding a finite $D_s^R(0)$.

This finiteness of $D_s^R(0)$ for weak coupling may result simply as an artifact of the factorization properties of the quenched planar approximation. Alternatively, it may imply an interesting weak coupling, renormalizable phase of the full theory. In fact, for the gauged NJL model at weak gauge coupling and with $G = (\alpha_c/\alpha)(1 + \sqrt{1 - \alpha/\alpha_c})^2$ so that the chiral symmetry is spontaneously broken, there has recently appeared in the literature the claim [19] that the anomalous dimension of the $\bar{\psi}\psi$ operator is given by

$$\tilde{\gamma} = 1 + \sqrt{1 - \alpha/\alpha_c} \ . \tag{3.13}$$

If this is true, then the dimension of the fermion mass operator is

$$d_{\bar{\psi}\psi} = 2 - \sqrt{1 - \alpha/\alpha_c} \tag{3.14}$$

and consequently the chirally invariant four-fermion operator would be a relevant operator for $\alpha < \alpha_c$ in this phase of the theory.

Let us scrutinize this claim a bit further. We thus compute the full vertex function for a fermion matrix element containing the scalar $\bar{\psi}\psi$ insertion including both the QED ladders and the effects of the four-fermion interactions. The net effect of the extra graphs is to rescale the $\Gamma^o_s(p,p)$ vertex so that at zero momentum transfer, the full bare scalar vertex, $\tilde{\Gamma}^0_s(p,p)$, is given by

$$
\begin{aligned}
\tilde{\Gamma}^0_s(p,p) &= \frac{1}{1+G_0 B^0_s(0)}\Gamma^0_s(p,p) \\
&= \frac{1}{1+G_0 B^0_s(0)}\frac{1}{Z_s}\Gamma^R_s(p,p) \qquad (3.15)\\
&= \frac{1}{\tilde{Z}_s}\Gamma^R_s(p,p).
\end{aligned}
$$

Using Eqs. (3.2 - 3.3) in conjunction with the gap equation (2.17) gives

$$
\begin{aligned}
\tilde{Z}_s &= Z_s + G_0 Z_s B^0_s(0) \\
&= \frac{\tilde{A}}{2}\frac{\Sigma_0}{\Lambda}\sqrt{(G-1)^2-(G+1)^2(1-\alpha/\alpha_c)}.
\end{aligned} \qquad (3.16)
$$

It follows that in the spontaneous chiral symmetry breaking phase at fixed weak gauge coupling we secure the result

$$
-\Lambda\frac{d}{d\Lambda}\log\tilde{Z}_s|_{G\frac{\alpha}{\alpha_c}=(1+\sqrt{1-\frac{\alpha}{\alpha_c}})^2} = 1+\sqrt{1-\alpha/\alpha_c}. \qquad (3.17)
$$

This is the quantity that the authors of reference [19] refer to as the anomalous dimension of the $\bar{\psi}\psi$ operator. We have already established that in the unbroken chirally symmetric phase, the anomalous dimension of the mass operator is $1-\sqrt{1-\alpha/\alpha_c}$. If, in the chirally broken phase, this anomalous dimension is $1+\sqrt{1-\alpha/\alpha_c}$, then the function has a jump discontinuity across the critical line $G(\alpha/\alpha_c) = (1+\sqrt{1-\alpha/\alpha_c})^2$. Such a result is quite puzzling since the anomalous dimension should depend on the operator structure but not on the particular vacuum configuration. Thus the operator product expansion should be valid in both phases. From the explicit weak coupling solution to the Schwinger-Dyson equation, we constructed the operator product expansion for $\Sigma(p)$ in Eq. (2.8). The two terms appearing in the expression were identified with an explicit mass term and a fermion vacuum condensate. The anomalous dimension, γ, of the $\bar{\psi}\psi$ operator was then extracted. This identification was consistent with the perturbative result. If, on the other hand, $\tilde{\gamma}$ is identified with the anomalous dimension in the chirally broken phase, then since $\tilde{\gamma} = 2-\gamma$, the form of the operator product expansion still holds except that now the roles of the bare mass term and fermion condensate have to be interchanged. We find this new interpretation to be rather unconventional.

Finally, let us note that there is also the somewhat remote possibility that the observed behavior of the scalar denominator at zero momentum represents a decoupled dilaton, much like the pion in the ordinary NJL model [17]. In this case we could have $F_D^2 \rightarrow \infty$ and $m_D^2 \rightarrow 0$ as $\Lambda \rightarrow \infty$. Here F_D is the dilaton decay constant and m_D is its mass. To check this possibility, we must compute the momentum derivative of the scalar denominator function, $\partial_{p^2} D_s^R(p^2)|_{p^2 = 0} \approx F_D^2$, to see whether it is finite or divergent. It is not possible to obtain an exact analytic expression for the result, but a rough estimate of the diagrams indicates that F_D^2 remains finite and the decoupled dilaton scenario is not viable.

To summarize, the scale symmetry in the gauged NJL model is explicitly broken, as in the pure gauge theory. No dilaton is observed. It is of course possible that a scale invariant theory may exist beyond the quenched, planar limit when additional relevant interactions are present.

4 Renormalization

As mentioned before, the Miransky solution [15] for the pure gauge theory requires the gauge coupling constant to have the specific cutoff dependence given by Eq. (2.11). The origin of such a cutoff dependence is, however, somewhat mysterious as the diagrammatic structure of quenched, planar QED does not seem to allow for a dynamical running of the gauge coupling constant. To clarify the situation we now study the renormalization properties of the theory. In the chirally broken phase of the strong coupling theory, the four-fermion interactions are relevant operators in the continuum limit. Moreover, the four-fermion coupling is renormalized even in the quenched, planar approximation. It will be shown that it is this dynamical running of the four-fermion coupling constant that leads to an infrared sensible theory when $G \to 1$ and $\alpha \to \alpha_c$ in the continuum limit. In this sense, the four-fermion interactions play a similar role as the induced ϕ^4 interactions in the scale invariant $\eta\phi^6$ theory [16], although the ensuing ultraviolet fixed point, $G \to 1$ as $\alpha \to \alpha_c$, is not a scale invariant one.

The study of the renormalization properties of the theory is restricted by our inability to compute explicitly the momentum dependence of specific diagrams. Fortunately, since the theory is defined with an ultraviolet momentum cutoff, Λ, we can effectively study the renormalization flow of the coupling constants by varying the cutoff while keeping the infrared behavior unchanged. The dynamically generated fermion mass scale, Σ_0, and the renormalized fermion condensate are examples of such infrared stable quantities. By holding low energy physics fixed, we must require Σ_0 and $<\bar\psi\psi>_R$ to be invariant as Λ is varied. Moreover, for moderate momentum $\Sigma_0 < p << \Lambda$, and for gauge couplings near the critical value, the fermion self energy has the nearly universal behavior (cf. Eqs. 2.6 - 2.7)

$$\Sigma(p) \to \tilde{A}(\alpha)\frac{\Sigma_0^2}{p}[\log\frac{p}{\Sigma_0} + \delta(\alpha)]. \tag{4.1}$$

Thus it also can be treated as a low energy quantity..

Consider the pure gauge theory. The renormalized fermion condensate is given for all α as

$$<\bar\psi\psi>_R = Z_s <\bar\psi\psi>_0 = -(\tilde{A}^2/2\pi^2)(\alpha_c/\alpha)\Sigma_0^3\,, \tag{4.2}$$

where we have used (2.14), (3.2), and the gap equations (2.16) and (2.17) with G = 0. An identical result is obtained in the presence of non-vanishing four fermion coupling. In that case, the fermion condensate is renormalized using $\tilde{Z}_s = Z_s + G_o Z_s B_s^o$ (cf. Eq. 3.16) so that for all α we again find

$$< \bar{\psi}\psi >_R = \tilde{Z}_s < \bar{\psi}\psi >_0 = -(\tilde{A}^2/2\pi^2)(\alpha_c/\alpha)\Sigma_0^3. \tag{4.3}$$

The right hand side of Eq. (4.3) has the explicit α dependence $\tilde{A}^2(\alpha)(\alpha_c/\alpha)$. Fitting the coefficient $\tilde{A}(\alpha)$ to a numerical integration of the Schwinger-Dyson equation for $\Sigma(p)$ yields, in the vicinity of α_c, the form $\tilde{A}^2(\alpha)(\alpha_c/\alpha) \simeq 1.086 - 1.083(\alpha/\alpha_c - 1) + 1.050(\alpha/\alpha_c - 1)^2$. Hence $\tilde{A}^2(\alpha)(\alpha_c/\alpha)$ does indeed vary with α and since Σ_0 and $< \bar{\psi}\psi >_R$ can both be chosen as fixed low energy quantities, it follows that the bare α is in fact physical and is not renormalized in ladder approximation of the gauged NJL model. This is in accord with the diagram structure of this model. The non-running of α also follows if one calculates an observable which probes only high partial waves. Such a quantity is insensitive to the contact four-fermion interaction but does depend on the gauge coupling. For it to correspond to fixed low energy physics, α must remain constant.

As a final comment with regard to the possible running of the gauge coupling in quenched, planar approximation, we note that since, in this approximation, the wave function renormalization of the photon is trivial, $Z_3 = 1$, then the gauge coupling gets renormalized only if the Ward identity relating the fermion propagator to the QED vertex function is violated. Indeed there is a weak violation of the gauge Ward identity in the chirally broken phase. However, we do not believe this is responsible for the running of α required for the continuum limit in the strong coupling phase. There is no physical mechanism which has been proposed to achieve this. The collapse phenomenon introduced by Miransky et al. [15] occurs at asymptotic momentum scales and thus would not account for any running of the coupling at intermediate momentum scales.

On the other hand, the four-fermion coupling is renormalized and a physical running ensues. The renormalization flow of the four-fermion coupling can be computed from the gap equations. For $\alpha \approx \alpha_c$ and fixed cutoff

Λ, $\tan\theta \simeq \theta$ (or $\tanh\tilde{\theta} \simeq \tilde{\theta}$) and the gap equations (2.16) and (2.17) give

$$\begin{aligned} \Sigma_0 &= e^{\delta}\Lambda\, e^{-\theta/\sqrt{\alpha/\alpha_c - 1}} \\ &\to e^{\delta}\Lambda\, e^{-(G+1)/(G-1)}, \quad \tan\theta \simeq \theta \end{aligned} \tag{4.4}$$

or

$$\begin{aligned} G &= (\tan\theta/\sqrt{\alpha/\alpha_c - 1} + 1)/(\tan\theta/\sqrt{\alpha/\alpha_c - 1} - 1) \\ &\to 1 + \frac{2}{\log(\Lambda/\Sigma_0) + \delta - 1}, \quad \tan\theta \simeq \theta. \end{aligned} \tag{4.5}$$

From this, we can define a β function describing the running of the four-fermion coupling as

$$\beta_G(G) = \Lambda\frac{dG}{d\Lambda} = -\frac{1}{2}(G-1)^2. \tag{4.6}$$

This renormalization flow of the four-fermion coupling is shown in Fig. 3 where $\tilde{G} = G \cdot (\alpha/\alpha_c)$ is plotted as a function of the inverse log of the cutoff. The flow is given for various values of the gauge coupling α which, as stated above, does not run with the cutoff. It indicates the existence of an apparent ultraviolet fixed point, $G \to 1$ as $\alpha \to \alpha_c$, which is also reflected in the beta function of Eq. (4.6).

We see that a physical running of the four-fermion coupling is required in order to maintain fixed low energy physics and that when $G \to 1$ for $\alpha = \alpha_c$, this is sufficient to obtain a smooth continuum limit. On the other hand, there does not appear to be a smooth continuum limit for $0 < G < 1$ and $\alpha = \alpha_c$. For $\alpha > \alpha_c$, the curves terminate at $G = -1$ where θ obtains its maximum value of $\theta = \pi$. As seen in Fig. 3a, this occurs for a finite value for Λ. Thus the renormalization flow cannot be maintained to the continuum. This is due to short distance effects which destabilize the vacuum.

It is interesting to compare Eq. (4.4) with Eq. (2.10) in the Miransky limit where $\theta = \pi$. In the pure gauge case, an infrared sensible theory requires Σ_0 to remain finite in the continuum limit, which in turn requires the running of the gauge coupling constant, Eq. (2.11). When the induced four-fermion interactions are incorporated, Eq. (4.4) indicates that only the running of the four-fermion coupling constant is required for a sensible, finite low energy behavior. The strong gauge coupling continuum limit corresponds to the fixed point, $G = 1$ and $\alpha = \alpha_c$. The Miransky solution [15] which requires the tuning of α to α_c as $\Lambda \to \infty$ can thus be viewed as an attempt to mock up the effect of the physics arising from the four-fermion interaction. When this interaction is properly accounted for, the gauge coupling need no longer be required to vary.

In addition to the strong coupling fixed point, the quenched, planar model also appears to exhibit a weak gauge coupling continuum limit along the critical curve [18] $G(\alpha/\alpha_c) = (1 + \sqrt{1 - \alpha/\alpha_c})^2$. This is indicated by the flow of $\tilde{G}$ in Fig. 3b. However, further study of the renormalization properties of this solution is required to establish whether it is a new renormalizable phase of the theory or merely an artifact of the quenched planar approximation.

5 Effective Lagrangian for Spontaneously Broken Chiral and Scale Symmetries

As mentioned in the introduction, our original motivation leading to the current investigation stemmed from the possibility that in certain gauge models the spontaneous breaking of chiral symmetry may trigger the spontaneous breakdown of a scale symmetry. Although this does not turn out to be the case in quenched, planar QED, it remains a viable possibility for other models. For such a gauge model which does realize the simultaneous spontaneous breakdown of both chiral and scale symmetry, it should be possible to construct an effective Lagrangian solely in terms of the Nambu-Goldstone bosons of these spontaneously broken symmetries which satisfies the associated (spontaneously broken) Ward identities.

In particular, we envision an underlying fermionic gauge theory in which the only explicit scale and chiral symmetry breaking arises from a soft fermion mass term. In that case, the scale and SU(N) axial current softly broken conservation laws take the term

$$\partial_\mu D^\mu = (1+\gamma)\bar{\psi} m \psi \tag{5.1}$$

$$\partial_\mu A^{a\mu} = \bar{\psi}\{\frac{\lambda^a}{2}, m\} i\gamma_5 \psi, \tag{5.2}$$

where m is the fermion mass matrix and γ is the anomalous dimension of the fermion mass operator. The model exhibits the flavor symmetry chiral $SU(N)_L \times SU(N)_R$ as well as scale invariance, which is assumed to be dynamically broken by fermion condensate formation to a vectorial $SU(N)_V$. The $\lambda^a/2, a = 1, 2, ..., N^2-1$ appearing in Eq.(5.2) are the SU(N) generators in the representation carried by the fermions.

The Nambu-Goldstone bosons resulting from the spontaneously broken scale and chiral symmetries, the dilaton ($D(x)$) and the pions ($\pi^a(x)$), will carry scale dimension and chirality. Following standard techniques, we introduce the combinations

$$S(D) = e^{D(x)/F_D} \tag{5.3}$$

$$U(\pi) = e^{i\lambda^a \pi^a(x)/F_\pi}, \tag{5.4}$$

so that $U(\pi)$ is a dimensionless matrix with $SU(N)_L \times SU(N)_R$ flavor symmetry, while the chirally invariant S(D) carries scale dimension one. If the Nambu-Goldstone bosons are to saturate the low energy theorems, then the fermion bilinear operator will have a Nambu-Goldstone realization given by

$$\bar{\psi}_{R_j}\psi_{L_i} = -r_0 F_\pi^2 [S(D)]^{3-\gamma}[U(\pi)]_{ij}. \tag{5.5}$$

Here the $S^{3-\gamma}$ factor generates the correct scale dimension and $U(\pi)$ gives the correct chirality, while the coefficient r_o is the order parameter for dynamical symmetry breaking and is given by $r_0 = - < \bar{\psi}\psi >_0 /(2F_\pi^2)$.

The effective Lagrangian which satisfies the softly broken scale and chiral Ward identities takes the form

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}F_D^2 \partial_\mu S \partial^\mu S + \frac{1}{4}F_\pi^2 S^2 tr(\partial_\mu U^+ \partial^\mu U) \\ & -\frac{1}{2} r_0 F_\pi^2 (3-\gamma) S^4 tr(m) + r_0 F_\pi^2 S^{3-\gamma} tr(U^+ m + mU). \end{aligned} \tag{5.6}$$

The soft symmetry breaking arises only from the term $r_0 F_\pi^2 S^{3-\gamma} tr(U^+ m + mU)$ which corresponds to a fermion mass term $\bar{\psi} m \psi$ (cf. Eq.(5.5)). On the other hand, the scale and chiral invariant term proportional to S^4 with its specified coefficient is needed to insure that the classical vacuum corresponds to the Nambu-Goldstone realization, namely, $< D >_0 = < \pi^a >_0 = 0$ or equivalently $< S_0 > = 1, < U_{ij} > = \delta_{ij}$. Building on this vacuum, we see that the pseudo-Goldstone boson masses are given by

$$m_\pi^2 = 2r_0(m_i + m_j) = -\frac{1}{F_\pi^2}(m_i + m_j) < \bar{\psi}\psi >_0 \tag{5.7}$$

$$m_D^2 = 2r_0(\frac{F_\pi}{F_D})^2(3-\gamma)(1+\gamma)tr(m) = -\frac{1}{F_D^2}(3-\gamma)(1+\gamma)tr(m) < \bar{\psi}\psi >_0, \tag{5.8}$$

where m_i, m_j are eigenvalues of the fermion mass matrix m.

The fact that the coefficient of the scale and chirally invariant S^4 term in the Lagrangian (5.6) depends on the explicit breaking mass parameter, m, may appear somewhat puzzling and thus warrants some further elaboration. If we had taken an arbitrary coefficient λ so this term appears as

λS^4 then the resulting vacuum configuration is given by $< U_{ij} >_0 = \delta_{ij}$ and $< S >_0^{1+\gamma} = (3-\gamma) r_0 f_\pi^2 tr(m)/(2\lambda)$. However, a rescaling of the field S to give $< S >_0 = 1$ accompanied by a rescaling of the dimensionful parameters in $\mathcal{L}$ fixes the S^4 coupling constant and reproduces the Lagrangian of Eq.(5.6). The necessity for this value of the S^4 coupling can also be established by expanding $\mathcal{L}$ in powers of the dilaton field D. The elimination of the term linear in D is then accomplished by fixing $\lambda = r_0 f_\pi^2 (3-\lambda) tr(m)/2$ as in Eq.(5.6). As we have noted, the S^4 coefficient is proportional to the scale and chiral symmetry breaking parameter m even though S^4 is a scale and chirally invariant operator. This dependence is dictated by requiring a Nambu-Goldstone realization of the symmetry. The vanishing of the S^4 coupling in the chiral limit, $m \to 0$, is required since a potential of the form λS^4 with λ nonvanishing gives a classical vacuum corresponding to $< S >_0 = 0$ which drives $< D >_0 \to -\infty$. This instability signals that the corresponding effective Lagrangian realizes the symmetry a la Wigner-Weyl. Consequently, a Nambu-Goldstone realization of the symmetry requires that the coefficient of the S^4 term vanishes in the chiral limit as is the case in Eq.(5.6). We cannot simply ignore the S^4 term entirely since in its absence, the dilaton becomes tachyonic for $m \neq 0$. Such a model was in fact considered by Miransky et al. [9] who then concluded that spontaneous scale symmetry breaking was inconsistent with PCAC dynamics. We now see that the inclusion of the S^4 term as in Eq.(5.6) alleviates this difficulty and provides a consistent effective Lagrangian.

To study the properties of the effective theory given by the Lagrangian of Eq.(5.6), we compute the symmetric energy-momentum tensor as

$$\begin{aligned} \theta_{\mu\nu} = & F_D^2(\partial_\mu S \partial_\nu S - \frac{1}{2} g_{\mu\nu} \partial_\lambda S \partial^\lambda S) \\ & + \frac{1}{4} F_\pi^2 S^2 tr(\partial_\mu U^+ \partial_\nu U + \partial_\nu U^+ \partial_\mu U - g_{\mu\nu} \partial_\lambda U^+ \partial^\lambda U) \\ & - g_{\mu\nu} r_0 F_\pi^2 S^{3-\gamma} tr(U^+ m + mU) \\ & + \frac{1}{2} g_{\mu\nu} r_0 F_\pi^2 (3-\gamma) S^4 tr(m) \\ & - \frac{1}{6} F_D^2 (\partial_\mu \partial_\nu - g_{\mu\nu} \partial^2) S^2, \end{aligned} \tag{5.9}$$

where the last term is a necessary improvement factor.

Defining the dilatation current as $D^\mu = x_\nu \theta^{\mu\nu}$ and using the classical field equations derived from the Lagrangian (5.6), we secure the trace of the energy-momentum tensor as

$$\begin{aligned}\partial_\mu D^\mu &= \theta^\mu_\mu = -(1+\gamma) r_0 F_\pi^2 S^{3-\gamma} tr(U^+ m + mU) \\ &= (1+\gamma)\bar{\psi} m \psi\end{aligned} \tag{5.10}$$

in agreement with Eq.(5.1). It can also be shown that using the classical field equations also produces the divergence of the axial current as given by Eq.(5.2).

We are now ready to verify the low energy theorems for both the scale and chiral symmetries. Using the axial current divergence to interpolate for the pseudoscalar Goldstone bosons, we can evaluate matrix elements of both the energy-momentum tensor trace of Eq.(5.9) and the divergence of the scale current as given by Eq.(5.10). Defining the pseudoscalar field

$$\phi^a = \frac{1}{F_\pi m_\pi^2}\bar{\psi}\{\frac{\lambda^a}{2}, m\}\gamma_5\psi \to \pi^a[1 + \frac{1}{F_D}(3-\gamma)D + ...] \tag{5.11}$$

we compute the insertion of θ^μ_μ in the ϕ^a two-point function from the diagrams of Fig. 4 to be

$$\begin{aligned}\Gamma &= < \phi(p)\phi(p')\theta^\mu_\mu(p'-p) >_0 \\ &= (p'^2 - m_\pi^2)^{-1}(p^2 - m_\pi^2)^{-1}\{(4m_\pi^2 - 2p\cdot p') \\ &\quad -q^2(q^2 - m_D^2)^{-1}[(3-\gamma)m_\pi^2 - 2p\cdot p']\} \\ &\quad -(3-\gamma)(p'^2 - m_\pi^2)^{-1}q^2(q^2 - m_D^2)^{-1} \\ &\quad -(3-\gamma)(p^2 - m_\pi^2)^{-1}q^2(q^2 - m_D^2)^{-1},\end{aligned} \tag{5.12}$$

where $q^\mu = (p'-p)^\mu$ and m_D, m_π are given in Eq.(5.7-5.8). Note that it is essential to keep the contributions of the dilaton poles.

The matrix elements for the divergence of the scale current are also obtained using Fig. 4 to be

$$\begin{aligned}\Gamma_m &= < \phi(p)\phi(p')(1+\gamma)\bar{\psi} m \psi(p'-p) >_0 \\ &= (p'^2 - m_\pi^2)^{-1}(p^2 - m_\pi^2)^{-1}\{(1+\gamma)m_\pi^2 \\ &\quad -m_D^2(q^2 - m_D^2)^{-1}[(3-\gamma)m_\pi^2 - 2p\cdot p')]\} \\ &\quad -(3-\gamma)(p'^2 - m_\pi^2)^{-1}m_D^2(q^2 - m_D^2)^{-1} \\ &\quad -(3-\gamma)(p^2 - m_\pi^2)^{-1}m_D^2(q^2 - m_D^2)^{-1}\end{aligned} \tag{5.13}$$

Using Eqs.(5.12) and (5.13), the scale identity is determined as

$$\Gamma = \Gamma_m - \frac{(3-\gamma)}{p'^2 - m_\pi^2} - \frac{(3-\gamma)}{p^2 - m_\pi^2} . \tag{5.14}$$

Since $(3-\gamma)$ is the scale dimension of the pseudoscalar field ϕ, the identity (5.14) takes precisely its expected form.

We can use these results to evaluate the on-shell meson matrix elements yielding

$$\begin{aligned} < \pi(p')|\theta^\mu_\mu|\pi(p) > &= \lim_{p^2, p'^2 \to m_\pi^2} (p'^2 - m_\pi^2)(p^2 - m_\pi^2)\Gamma \\ &= (q^2 + 2m_\pi^2) - q^2(q^2 - m_D^2)^{-1}[q^2 + (1-\gamma)m_\pi^2] \\ &= < \pi(p')|(1+\gamma)\bar{\psi}m\psi|\pi(p) > . \end{aligned} \tag{5.15}$$

Clearly, the inclusion of the dilaton pole is essential to the consistent evaluation of the low energy theorems for the meson matrix elements.

We have constructed an effective lagrangian for which the low energy theorems for both scale and chiral symmetries are satisfied. Furthermore, there appears to be no constraint on the value of the anomalous dimension of the fermion mass operator.[20] Hence, there is no inconsistency for the simultaneous Nambu-Goldstone realization of the scale and chiral symmetries. The results of the previous sections have shown that the spontaneously broken chiral symmetry phase of quenched, planar QED is associated with a hard, explicit breaking of the scale symmetry. Thus the scale current algebra cannot be applied to this system.

Acknowledgements

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Figure Captions

Fig. 1. Diagrammatic origin of four fermion coupling.

Fig. 2. Ladder diagram contribution to fermion-antifermion scattering amplitude near bound state.

Fig. 3a. Renormalization flow of $\tilde{G} = G(\alpha/\alpha_c), \quad \alpha > \alpha_c$.

Fig. 3b. Renormalization flow of $\tilde{G} = G(\alpha/\alpha_c), \quad \alpha < \alpha_c$.

Fig. 4. Feynman graphs contributing to matrix elements containing the trace of the energy-momentum tensor.

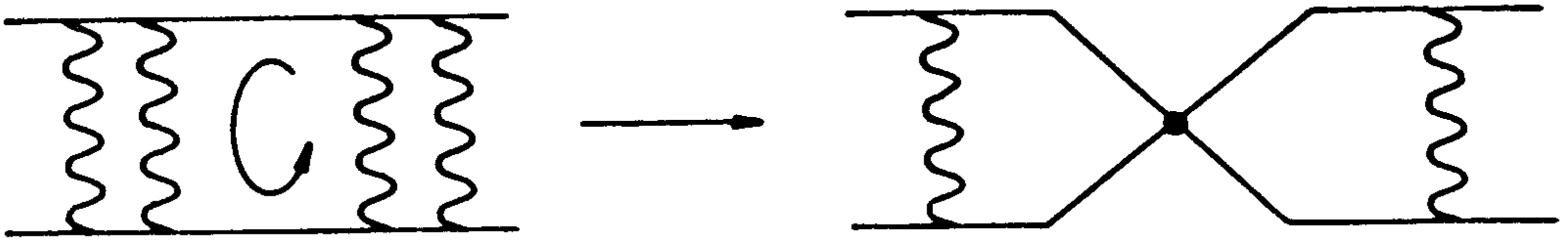


Fig. 1

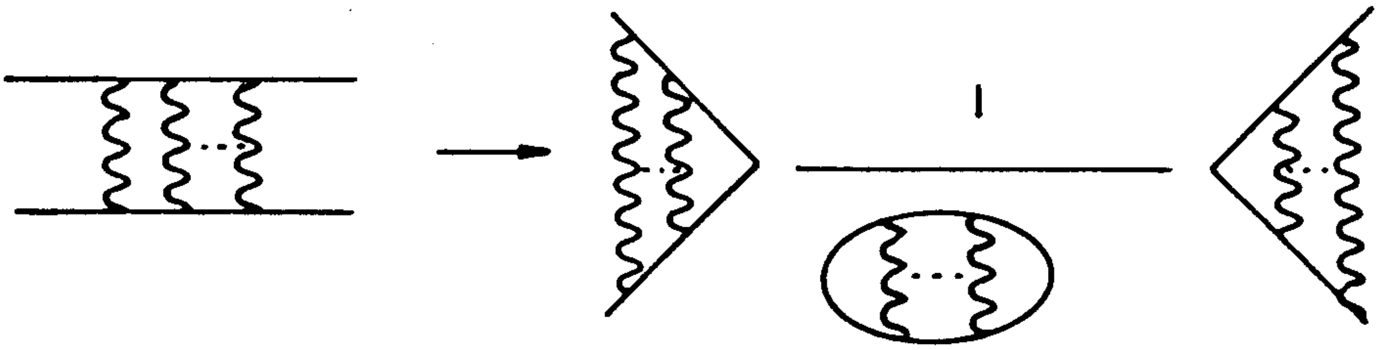
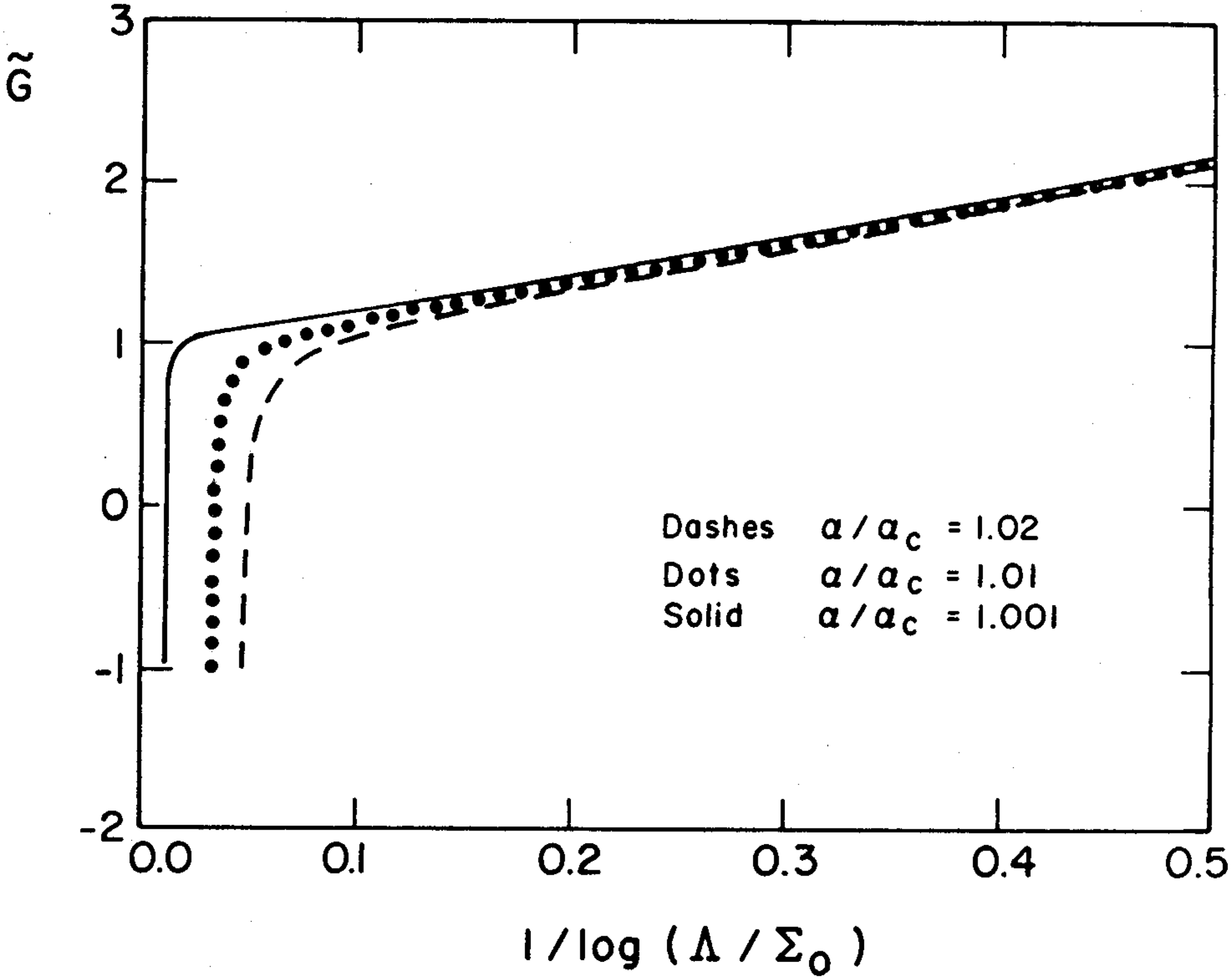


Fig. 2



$\tilde{G}$
3
2
1
0
-1
-2
0.0
0.1
0.2
0.3
0.4
0.5
$1/\log(\Lambda/\Sigma_0)$
Dashes α/α_c = 1.02
Dots α/α_c = 1.01
Solid α/α_c = 1.001

Fig. 3a

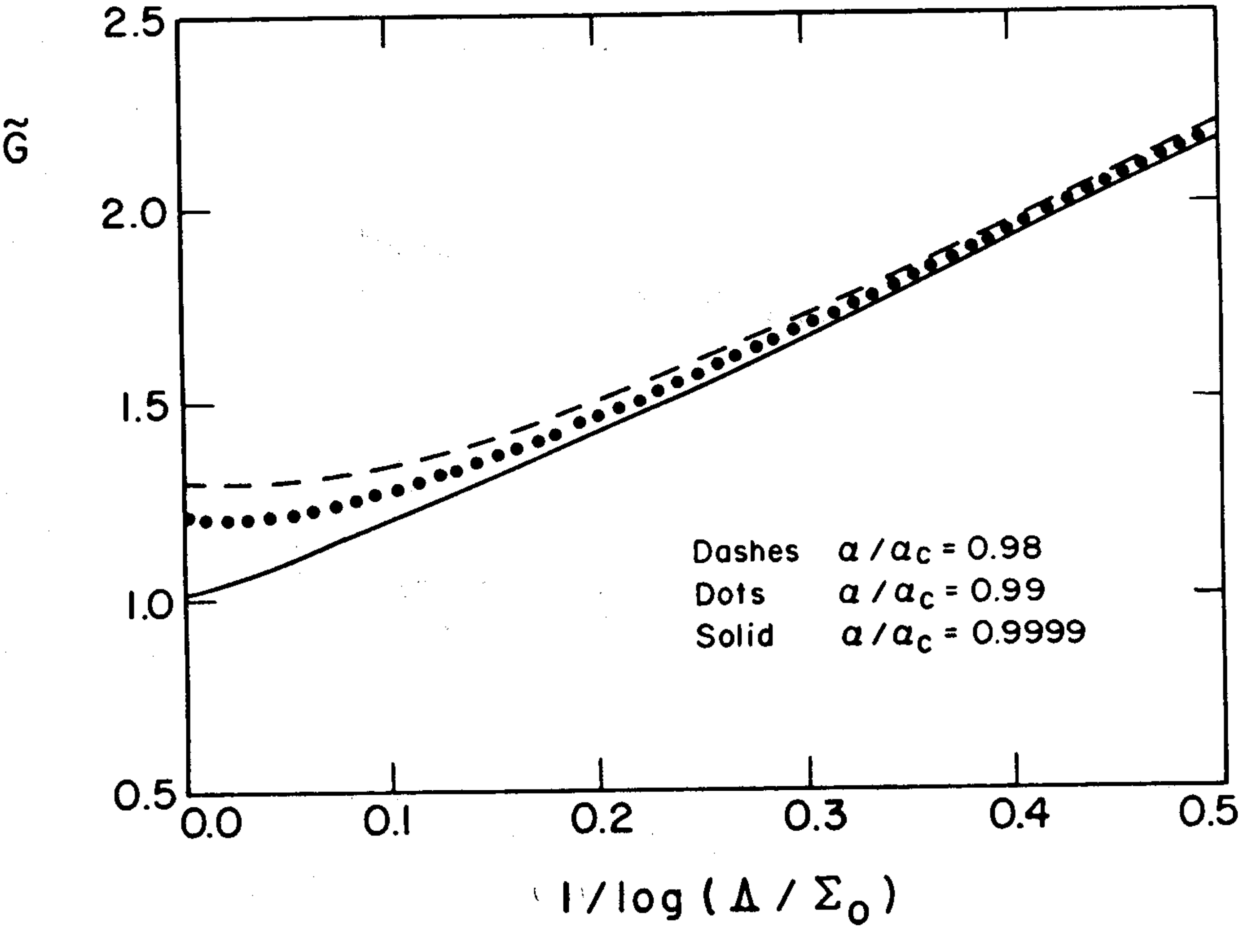

2.5
2.0
1.5
1.0
0.5
$\tilde{G}$
0.0
0.1
0.2
0.3
0.4
0.5
$1/\log(\Lambda/\Sigma_0)$
Dashes $\alpha/\alpha_c = 0.98$
Dots $\alpha/\alpha_c = 0.99$
Solid $\alpha/\alpha_c = 0.9999$

Fig. 3b

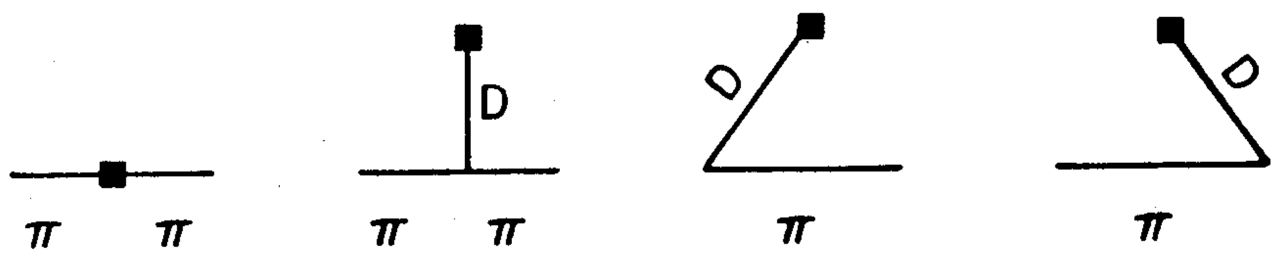


Fig. 4

field equations derived from the Lagrangian (5.6), we secure the trace of the energy-momentum tensor as

$$\begin{aligned}\partial_\mu D^\mu &= \theta^\mu_\mu = -(1+\gamma)r_0 F_\pi^2 S^{3-\gamma} tr(U^+ m + mU) \\ &= (1+\gamma)\bar{\psi} m \psi\end{aligned} \tag{5.10}$$

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Figure Captions

Fig. 1. Diagrammatic origin of four fermion coupling.

Fig. 2. Ladder diagram contribution to fermion-antifermion scattering amplitude near bound state.

Fig. 3a. Renormalization flow of $\tilde{G} = G(\alpha/\alpha_c)$, $\alpha > \alpha_c$.

Fig. 3b. Renormalization flow of $\tilde{G} = G(\alpha/\alpha_c)$, $\alpha < \alpha_c$.

Fig. 4. Feynman graphs contributing to matrix elements containing the trace of the energy-momentum tensor.

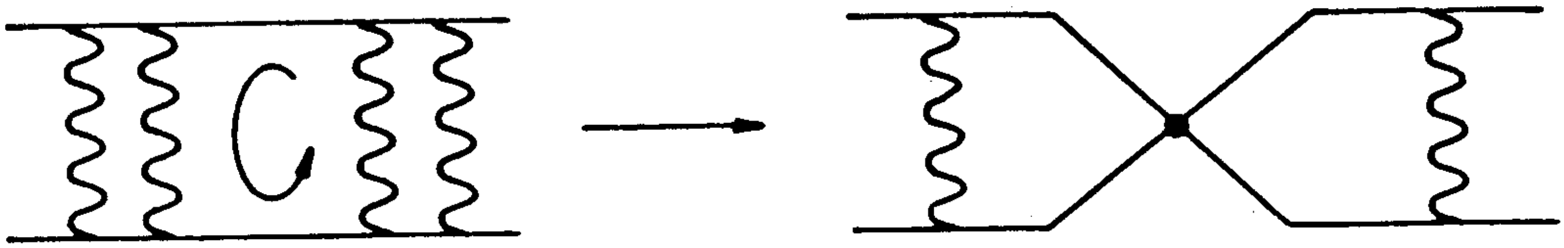


Fig. 1

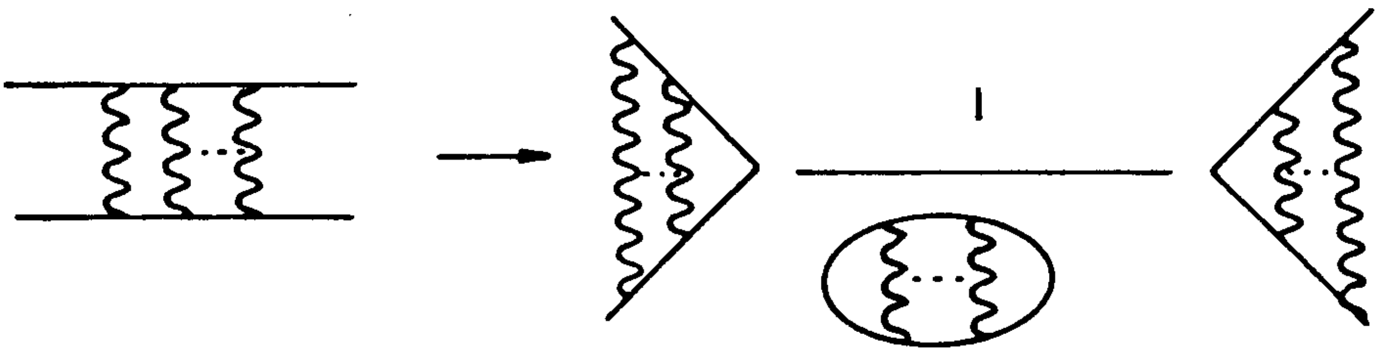
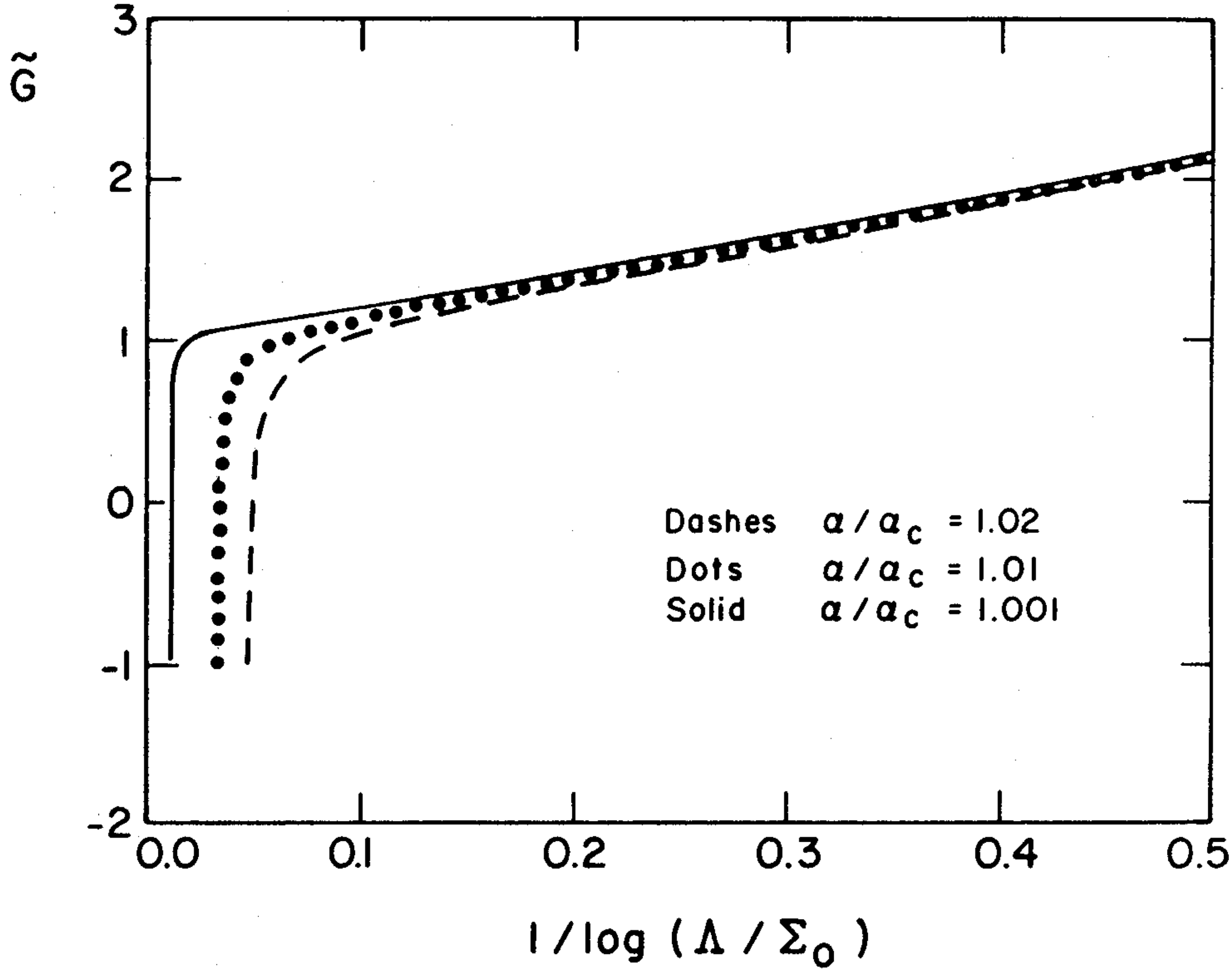


Fig. 2



3
$\tilde{G}$
2
1
0
-1
-2
0.0
0.1
0.2
0.3
0.4
0.5
$1/\log(\Lambda/\Sigma_0)$
Dashes α/α_c = 1.02
Dots α/α_c = 1.01
Solid α/α_c = 1.001

Fig. 3a

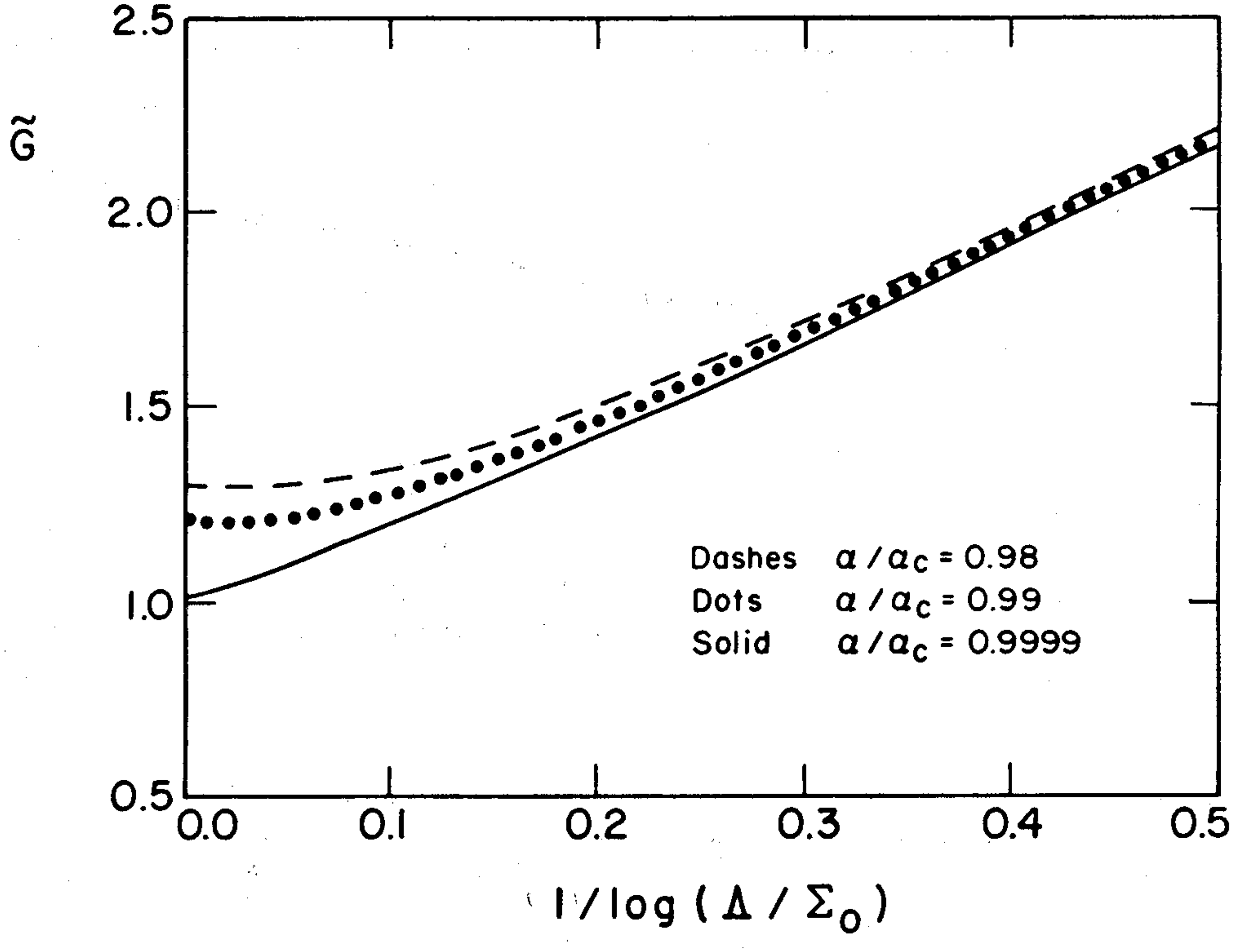

$\tilde{G}$
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.2
0.3
0.4
0.5
Dashes $\alpha / \alpha_c = 0.98$
Dots $\alpha / \alpha_c = 0.99$
Solid $\alpha / \alpha_c = 0.9999$
$1 / \log(\Lambda / \Sigma_0)$

Fig. 3b

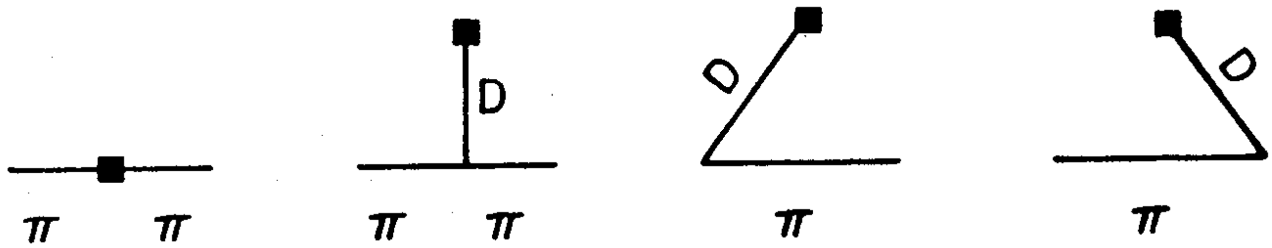


Fig. 4