Fermi National Accelerator Laboratory

FERMILAB-Pub-89/267-A
December 1989

A NEW PROBE OF THE HOMOGENEITY OF THE UNIVERSE

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Abstract

Long-wavelength density fluctuations along the line of sight to a gravitationally lensed quasar generate a time delay between the images. Based upon the recently measured delay of 415 ± 20 days between the two images in the system 0957+561, we derive a limit to the *rms* mass fluctuation within a sphere of radius r_0, $(\delta M/M)_{r_0} \lesssim (4h^{-1/3}\,\mathrm{Mpc}/r_0)^{3/2}$, assuming a spatially flat Universe. On scales comparable to the present Hubble radius this is competitive with microwave anisotropy limits, and on the scale $8h^{-1}$ Mpc this is in mild conflict with the mass fluctuation *inferred* from the distribution of bright galaxies.

Submitted to the Physical Review Letters.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

One of the underpinnings of the Friedmann-Robertson-Walker (FRW) cosmology is the notion that on sufficiently large scales the Universe is isotropic and homogeneous (see Refs. 1, 2). The evidence for isotropy is quite firm: among other indications, the observed isotropy of the cosmic microwave background radiation (CMBR), of faint radio source counts, of the x-ray background, and of the expansion itself. The evidence for the homogeneity of the Universe is less convincing. The best evidence is the large-angle isotropy[3] of the CMBR, $\delta T/T \lesssim 3 \times 10^{-5}$. At large angles, the CMBR probes the gravitational potential on the last scattering surface on scales comparable to the Hubble radius, $H_0^{-1} \simeq 3000h^{-1}$ Mpc; its isotropy sets a similar limit to the level of mass inhomogeneity.[4] On the scales of clusters and smaller ($\ll 10h^{-1}$ Mpc) the Universe is very inhomogeneous: The overdensities in rich clusters are as large as 10^3 and in bright galaxies as large as 10^5.

Crucial to understanding the evolution of structure in the Universe is the knowledge of the current level of inhomogeneity on intermediate scales, say $10h^{-1}$ to $3000h^{-1}$ Mpc, where we expect that the mass fluctuation $\delta M/M$ is less than unity. Our direct knowledge of $\delta M/M$ on these scales is fragmentary at best. Based upon the distribution of bright galaxies in the CfA survey, Davis and Peebles[5] *infer* that $(\delta M/M)_{r_0} \simeq 1$ for $r_0 = 8h^{-1}$ Mpc. However, since it is by no means established that "light traces mass," determination of the level of inhomogeneity based upon galaxy counts is suspect.[6]

(The quantity $(\delta M/M)$ is the *rms* mass fluctuation within a specified volume and averaged over the Universe. When the volume is defined by a sphere of radius r_0 with a sharp surface, $(\delta M/M)_{r_0} \simeq C(k^{3/2}|\delta_k|/\sqrt{2\pi^2})|_{k=r_0^{-1}}$, where the density contrast $\delta\rho(\vec{x})/\rho = \int d^3k\delta_k \exp(-i\vec{k}\cdot\vec{x})/(2\pi)^3$ and C is a numerical coefficient which depends upon the form of the power spectrum. For $|\delta_k|^2 \propto k^n$, $C = 1.4$ to 4.2 for $n = -2.9$ to 0.9.)

The most direct probe of the density field of the Universe is the peculiar velocity field: If the matter is not distributed homogeneously, then test particles (galaxies) will be accelerated relative to the Hubble expansion. Roughly, the peculiar velocity on a given scale is related to the density contrast on that scale by $(\delta v/c)_{r_0} \sim \Omega_0^{0.6}(r_0/H_0^{-1})(\delta M/M)_{r_0}$. While data exists on scales up to $60h^{-1}$ Mpc,[7] with measured peculiar velocities ranging from 100 to 800 km sec^{-1}, the interpretation of the observations is still being clarified. Finally, there are a number of more qualitative measures of large-scale structure that suggest that the Universe is inhomogeneous even on relatively large scales (of order 100 h^{-1} Mpc).[8] These observations include the extension of the CfA red shift survey,[9] where galaxies appear to be concentrated on the surfaces of bubbles of radius up to $30h^{-1}$ Mpc, the giant void in Bootes,[10] the structure in the faint galaxy "pencil-beam" surveys,[11] and the clustering of rich clusters and superclusters.[12]

In this *Letter*, we discuss a new probe of the large-scale density field of the Universe that utilizes the double QSO 0957+561. It was the first gravitational lens system found and is the most comprehensively studied.[13] The lensed QSO has a measured red shift of $z_Q = 1.41$, while the lens is a bright galaxy in a cluster with red shift $z_L = 0.36$. There are two images, separated by an angle $\delta\theta = 6.1''$ ($= 3 \times 10^{-5}$ rad). The light from the two images travels a distance $D \sim H_0^{-1}$ on paths that are separated by a distance of order $\delta\theta H_0^{-1} \simeq 0.1h^{-1}$ Mpc. Recently, the time delay between the flux variation of the two images was measured to be 415 ± 20 days.[14] A delay of this magnitude is expected to arise from two effects associated with the lens itself: the geometric difference in length between the two light paths, $\Delta t \sim (\delta\theta)^2 D$, and the gravitational time delay induced as the light paths traverse different parts of the lens potential.[15] In addition, a time delay between the two light paths will arise due to gravitational perturbations associated with density (scalar metric) perturbations along the line of sight. Since the observed time delay is 415 days, and the predicted time delay[16] due to the lens is about 1 ± 1 yr, the extra time delay introduced by any intervening mass inhomogeneity (not associated with the lens) must be less than about a year. We will use this fact to derive an upper limit to the level of mass fluctuations. (Allen[17] has recently considered this effect for tensor metric perturbations to set a limit to the density of long-wavelength gravitational waves.)

The perturbation of a photon's trajectory due to metric perturbations was first discussed by Sachs and Wolfe.[4] We will follow their treatment closely. The background metric is most conveniently given in conformal form: $ds^2 = R(\eta)^2[d\eta^2 - \delta_{ij}dx^i dx^j]$, where i, j run from 1 to 3, and the four vector $dx^\mu = (d\eta, d\vec{x})$. The quantity $R(\eta)$ is the cosmic scale factor; for simplicity we shall assume the spatially flat ($\Omega_0 = 1$) FRW model. The conformal-time variable and cosmic-scale factor are normalized so that $R(\eta) = 2\eta^2 H_0^{-1}$, $t = \frac{2}{3}\eta^3 H_0^{-1}$, and $\eta_0 = 1$, where subscript zero indicates the present epoch. Note that the $x^\mu = (\eta, \vec{x})$ are comoving coordinates and that $R_0 = 2H_0^{-1}$. The perturbed metric is

$$ds^2 = R(\eta)^2[d\eta^2 - (\delta_{ij} - h_{ij})dx^i dx^j], \tag{1}$$

where synchronous gauge has been used ($h_{00} = h_{0j} = 0$). The choice of synchronous gauge means that clock time t and conformal time η are *always* related by $t = 2\eta^3/3H_0$, even in the presence of metric perturbations.

We assume that the deviations from homogeneity are small, and work to linear order. The growing mode density perturbations are characterized by:[1,2,4] $h_{ij}(\eta, \vec{x}) = -\eta^2 \partial^2 B(\vec{x})/\partial x^i \partial x^j$, where $\delta_k(\eta) = \eta^2 k^2 B_k/2$. The scalar field $B(\vec{x})$ describes the metric perturbations today, B_k is its Fourier transform and $k = |\vec{k}|$ is the comoving wavenumber, related to the present physical wavenumber by $k_{\rm phys} = k/R_0 = kH_0/2$.

We need to compute the time delay that a photon suffers in traveling from its point of emission to here and now. The solution to the null-geodesic equation can be expanded as $x^\mu(w) = {}_0x^\mu(w) + {}_1x^\mu(w)$, where w is the affine parameter of the photon's path, ${}_0x^\mu$ is the zeroth-order trajectory, and ${}_1x^\mu$ is the first-order (in h_{ij}) correction to the trajectory. Ignoring for the moment the lensing galaxy, the zeroth-order trajectory can be written as: ${}_0\eta = \eta_E + w$, ${}_0x^i = w\, e^i$, where the emission event is $(\eta_E, \vec{0})$, the reception event is $[\eta_R, (\eta_R - \eta_E)\hat{e}]$, and w runs from 0 to $\eta_R - \eta_E$. For simplicity the emission event is taken to be at the origin of the spatial-coordinate system, and the constant unit vector $\hat{e}$ is the zeroth-order-spatial direction of the photon. (Taking $\eta_R = \eta_0 = 1$, $\eta_E = (1+z_Q)^{-1/2}$ and the present physical distance to the QSO is $L_{\rm phys} = R_0 L = 2H_0^{-1}[1-(1+z_Q)^{-1/2}]$.)

The time delay due to density inhomogeneities along the trajectory is contained in ${}_1\eta(w)$: The total (conformal) time delay $\Delta\eta = {}_1\eta(w = \eta_R - \eta_E)$, and the total (clock) time delay $\Delta t = 2H_0^{-1}\Delta\eta$. From the geodesic equation it follows that[4]

$$\frac{d_1\eta}{dw} \left(\equiv \frac{d_1 x^0(w)}{dw} \right) = \frac{1}{2} \int_0^w \left(\frac{\partial h_{ij}}{\partial \eta} e^i e^j \right) dw', \tag{2}$$

where the integrand is to be evaluated along the *unperturbed* trajectory. Using the fact that $dB/dw = (\partial B/\partial x^i)e^i$, it follows $d_1\eta/dw = (-\eta dB/dw + B)|_0^w$. Since we will compare the time delay accumulated along two paths that start and end at the same points, we can drop the path-independent terms. We then find

$$\Delta\eta = 2\int_0^{\eta_R-\eta_E} B dw = \frac{4}{(2\pi)^3}\int \frac{\delta_k}{k^2} d^3k \int_0^{\eta_R-\eta_E} \exp(-i\vec{k}\cdot\vec{x})dw, \tag{3}$$

where $\delta_k \equiv \delta_k(\eta = 1)$. Since $B(\vec{x})$ plays the role of the Newtonian potential,[4] we recognize the first expression as the usual gravitational time delay.

In order to find the observable *difference* in time delays between the two images of the lensed QSO we employ the simplified geometry shown in Fig. 1: The two light paths are taken to be symmetrically perturbed about the straight-line trajectory by an angle $\delta\theta$.[18] The gravitational time delay difference between the two images is

$$\Delta T \equiv 2H_0^{-1}\Delta\eta\Big|_{\rm path\ 2}^{\rm path\ 1} = \frac{8H_0^{-1}}{(2\pi)^3}\int \frac{\delta_k}{k^2}d^3k \int_0^L (\Delta\vec{s}\cdot\vec{\nabla})\exp(-iw\vec{k}\cdot\hat{e})dw, \tag{4}$$

where the unperturbed trajectory is $\vec{x} = \hat{e}w$ ($w = 0$ to $L \equiv 1-\eta_E$) and the separation between the paths of the two images is $\Delta\vec{s} = w\delta\theta\,\hat{\theta}$ for $0 < w < L/2$ and $\Delta\vec{s} = (L-w)\delta\theta\,\hat{\theta}$ for $L/2 < w < L$. Evaluating the integral and generalizing the result by taking the position of the QSO to be $\vec{x}_Q$, the difference in time delay between the two paths is

$$\Delta T = \frac{-4i\delta\theta H_0^{-1}}{(2\pi)^3}\int \delta_k \exp\left(-i\vec{k}\cdot\hat{e}\frac{L}{2}\right)\frac{H_0^3}{k_{\rm phys}^3} g(\vec{k})\exp(-i\vec{k}\cdot\vec{x}_Q)d^3k, \tag{5}$$

where $g(\vec{k}) \equiv (\hat{k}\cdot\hat{\theta})\sin^2(\hat{k}\cdot\hat{e}\frac{kL}{4})/(\hat{k}\cdot\hat{e})^2$ is the "response function" to the mode k and $\hat{k} = \vec{k}/|\vec{k}|$. From this expression by using Parseval's theorem we obtain the expectation for $(\Delta T)^2$ averaged over all possible QSO positions $\vec{x}_Q$:

$$\langle(\Delta T)^2\rangle = (4\delta\theta H_0^{-1})^2\int_0^\infty |g(\vec{k})|^2 (H_0^3/k_{\rm phys}^3)^2 (k^2|\delta_k|^2/2\pi^2)dk. \tag{6}$$

The response function $g(\vec{k})$ depends upon both k and the direction of $\vec{k}$ in relation to the lens system. Averaging $g(\vec{k})$ over all directions we find that $\bar{g} \equiv [\int |g(\vec{k})|^2 d\Omega/4\pi]^{1/2} \rightarrow$

$3.6 \times 10^{-2}(kL)^2$ for $kL \ll 1$ and $\bar{g} \to 0.1(kL)^{3/2}$ for $kL \gg 1$. The quantity kL can be written as $kL = 2[1-(1+z_Q)^{-1/2}](k_{\rm phys}/H_0)$.

If we assume that QSO 0957+561 occupies a "typical" position in the Universe, we can infer that $\langle(\Delta T)^2\rangle^{1/2} \lesssim 1$ yr. This leads to the following bound: $9h^2 \times 10^{-13} \gtrsim \int_0^\infty |g(\vec{k})|^2 (H_0^3/k_{\rm phys}^3)^2 (k^3|\delta_k|^2/2\pi^2) d\ln k$. Assuming no logarithmic decade contributes more than $9h^2 \times 10^{-13}$ to the integral (and setting $C = 3$), we obtain the bounds

$$(\delta M/M)_{r_0} \lesssim (4h^{-1/3}\,{\rm Mpc}/r_0)^{3/2} \qquad (r_0 \ll 3000h^{-1}{\rm Mpc}), \tag{7a}$$

$$(\delta M/M)_{r_0} \lesssim 1.4 \times 10^{-4}(3000\,{\rm Mpc}/r_0) \qquad (r_0 \gg 3000h^{-1}{\rm Mpc}). \tag{7b}$$

How does our result compare with other observations? The large-angle isotropy of the CMBR sets a limit[8] $(\delta M/M)_{r_0} \lesssim 3 \times 10^{-2}(100h^{-1}{\rm Mpc}/r_0)^2$, so our bound is more sensitive than the microwave anisotropy limit on scales $r_0 \lesssim 2000h^{-3}$ Mpc. On small scales our bound is in mild conflict with the inference that $(\delta M/M)_{8h^{-1}\,{\rm Mpc}} \simeq 1$. This could indicate that bright galaxies are more clustered than the underlying mass density (as predicted in the biased scenarios[6]). To compare with a specific model, consider the Harrison-Zeldovich spectrum with cold dark matter; the power spectrum is given by $|\delta_k|^2 = Ak/(1+\beta k+\omega k^{3/2}+\gamma k^2)^2$ where $\beta = 1.7h^{-2}$ Mpc, $\omega = 9.0h^{-3}$ Mpc$^{3/2}$, $\gamma = 1.0h^{-4}$ Mpc^2, and the overall normalization $A = 4.4 \times 10^6$ Mpc^4 ($h = 0.5$), 7.5×10^4 Mpc^4 ($h = 1$) based upon $(\delta M/M)_{8h^{-1}\,{\rm Mpc}} = 1$. Using this expression for $|\delta_k|^2$ we obtain $\langle(\Delta T)^2\rangle^{1/2} = 28$ yr ($h = 0.5$), 10 yr ($h = 1$), in conflict with the observed delay.

Our limit is based upon the assumption that QSO 0957+561 is at a "typical" location in the Universe; if the perturbations arise from a gaussian-random process (e.g., as in most inflationary models), then the deviations of ΔT from the mean are gaussian distributed and not likely to differ from the above estimate by more than a factor of a few. On the other hand, if the perturbations are non-gaussian, much larger excursions from the mean *could* occur. In addition, we have assumed that our ability to *observe* a lensed QSO is not strongly biased toward extremely smooth lines of sight. Since surveys have turned up about as many lenses as expected (albeit only a handful), this effect cannot be large.

The formalism developed above is only strictly valid for linear perturbations. On small scales, $r_0 \lesssim 8h^{-1}$ Mpc, we expect structure to be nonlinear. In this regime we can instead consider the time delay due to individual, bound stationary clumps along the line of sight; clearly, we cannot tolerate more than one or two gravitational potential wells comparable to those of the brightest cluster galaxies very near the line of sight. This is consistent with the observed absence of bright field galaxies near the QSO images, and it also implies that there are not many dark potential wells along the line of sight. This has implications for models of biased galaxy formation:[6,19] In these scenarios, one expects up to 4 or 5 times as many "dark" galaxies as normal galaxies. Even if they have surface densities too small to produce multiple image splittings themselves, such dark galaxies should reveal their presence by their contribution to lens time delays.

Finally, we emphasize that our results were obtained for a flat Universe; the extension to open models ($\Omega_0 < 1$) will be discussed elsewhere.[19] Here we note that, if Ω_0 is close to 1, the time delay due to inhomogeneity is modified, $\Delta T \to \Delta T[1 - 13(1-\Omega_0)/7]$, and that ΔT vanishes as $\Omega_0 \to 0$, $\Delta T \to \Delta T[-6\Omega_0\{\ln(2\Omega_0^{-1/2}) - 2\}]$. Thus, the apparent conflict with non-linear structure on $8h^{-1}$ Mpc could be resolved if Ω_0 is relatively small.

To summarize, the "double" quasar 0957+561 provides a unique probe of the gravitational potential (and underlying mass density) of the Universe on scales of $10h^{-1}$ Mpc to $3000h^{-1}$ Mpc, and provides convincing evidence that the Universe is smooth on these scales. A number of lensed QSO's have now been found, and they should provide additional probes when their time delays are measured. For 0957+561 the observed time delay is about that expected from the lens alone. Since this "intrinsic" lens delay is proportional to $\delta\theta^2$ while the delay due to inhomogeneities scales as $\delta\theta$, the sensitivity of our probe increases as $(\delta\theta)^{-1}$. It is of interest that several lens candidates with $\delta\theta \simeq 2''$ have been proposed; for these systems, the time delay due to inhomogeneities may dominate the intrinsic delay. In this regard, the preliminary evidence for a time delay of order a month for the triple lens PG 1115+08 ($\delta\theta \simeq 2''$) is suprising.[20]

We gratefully acknowledge helpful conservations with P.J.E. Peebles, D.N. Schramm, and C. Alcock. This work was supported in part by the NASA (grant NGW-1340) at Fermilab and by the DoE at Fermilab and Chicago.

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Figure Caption

FIG. 1. The simplified geometry assumed for system 0957+561. The straight line represents the unperturbed path; the straight lines labeled 1 and 2 represent the unperturbed paths when lensing is taken into account.