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On Thick Domain Walls in General Relativity

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ABSTRACT

Planar scalar field configurations in general relativity differ considerably from those in flat space. We show that static domain walls of finite thickness in curved space-time do not possess a reflection symmetry. At infinity, the space-time tends to the Taub vacuum on one side of the wall and to the Minkowski vacuum (Rindler space-time) on the other. Massive test particles are always accelerated towards the Minkowski side, i.e. domain walls are attractive on the Taub side, but repulsive on the Minkowski side ("Taub-vacuum cleaner"). We also prove that the pressure in all directions is always negative. Finally we briefly comment on the possibility of infinite, i.e. bigger than horizon size, domain walls in our universe. All our results are independent of the form of the potential $V(\Phi) \geq 0$ of the scalar field Φ .

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1. Introduction

Domain walls like other topological defects can be produced in a phase transition in the universe. However, it is well known^{1,2} that stable domain walls in the early universe would soon have dominated the energy density and therefore were discarded. The renewed interest in domain walls is due to a scenario of galaxy formation proposed by Hill, Schramm and Fry³ in which these topological defects form after recombination and provide the seeds for the clustering of baryons without destroying the isotropy of the microwave background radiation. In this model the thickness δ of these walls can be of the order of Mpc's and is related to the neutrino mass m_ν and the GUT-symmetry breaking scale Λ by $\delta \sim \Lambda/m_\nu^2$. It has been speculated⁴ that the "Great Attractor" might be such a domain wall.

Numerical simulations of the dynamics of a network of such walls show^{5,6} that small closed walls decay and only one infinite wall per horizon survives.

In general, a domain wall is a topologically stable configuration where the scalar field attains different vacuum expectation values on different sides of the wall. The gravitational effects of domain walls were studied in the approximation of infinitely thin walls, i.e. where the energy momentum tensor becomes proportional to a δ -function. Under the assumption that the metric functions are reflection ($z \rightarrow -z$) symmetric, no static solutions were found^{7,8,9}. This led to the study of thin^{8,9} and thick^{10,11} (but still reflection symmetric) static domain walls in non-static space-times. However, if the assumption of reflection symmetry is dropped, infinitely thin, static walls exist¹². The remarkable result of these studies was that the gravitational field of these walls is repulsive.

If domain walls exist in the present universe, they could only be detected by their gravitational effects. We therefore investigate thick, static domain walls in the framework of General Relativity. Since the form of the potential $V(\Phi)$ is not known, we analyze the general properties of static planar domain walls for an arbitrary potential. In spite of their complexity the coupled Einstein-scalar field equations allow to extract some remarkable new features. The only assumptions we make are planar symmetry (i.e. with Killing-vectors $\partial_x, \partial_y, x\partial_y - y\partial_x$), a positive scalar field potential ($V(\Phi) > 0$) for finite z and a vanishing energy momentum tensor at infinity $|z| \rightarrow \infty$.

We will show that static domain wall solutions of the Einstein-equations are not reflection symmetric. The analysis of Einstein's equations in a suitable coordinate system reveals that the asymptotic structure of the space-time is in

fact different on the two sides of the wall. Although the coupled Einstein-scalar field equations are symmetric with respect to a $z \rightarrow -z$ transformation, no solutions with this symmetry exist (besides the Minkowski vacuum). We show that the gravitational field of a domain wall always approaches the Minkowski vacuum on one side of the wall and the Taub vacuum on the other side. The Minkowski- and Taub vacua are the only two static planar vacuum solutions of the Einstein equations. Also the equations of motion for test particles are different on the two sides of the wall: All massive particles are accelerated towards the Minkowski side. This is compared with a static wall of perfect fluid which is attractive and admits only bound states for massive test particles. Also, we find that the pressure perpendicular to the wall is always negative and possesses only one minimum. This pressure is entirely due to gravitational effects, since for a wall in Minkowski space-time the pressure is zero everywhere. Finally, we make some remarks on the effects of infinite domain walls in the cosmological context.

2. Einstein equations

The coupled Einstein- scalar field equations are obtained from the action

$$S = \int d^4x \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi) - \frac{1}{2} R \right], \quad g \equiv |\det(g_{\mu\nu})| \quad (2.1)$$

by variation with respect to the metric $g_{\mu\nu}$ and the scalar field Φ . (We use the units $8\pi G = 1$.) This gives the Einstein-equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu} \quad (2.2)$$

with the Ricci-tensor $R_{\mu\nu}$ and the energy momentum tensor

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu} \left[\frac{1}{2} g^{\rho\sigma} \partial_\rho \Phi \partial_\sigma \Phi - V(\Phi) \right] . \quad (2.3)$$

and the Euler-Lagrange equation for the scalar field

$$g^{-1/2} \partial_\mu \left[\sqrt{g} g^{\mu\nu} \partial_\nu \Phi \right] + \frac{dV(\Phi)}{d\Phi} = 0 . \quad (2.4)$$

This last equation can also be viewed as a consequence of the Bianchi-identity $T_\nu{}^\mu{}_{;\mu} = 0$. Since we are interested in static and plane-symmetric configurations

we can take the metric of the form

$$ds^2 = e^{2A(z)} dt^2 - e^{2B(z)} dz^2 - e^{2C(z)} (dx^2 + dy^2) . \quad (2.5)$$

We use the freedom to choose the coordinate system by imposing the condition $2C = B - A$. The energy-momentum tensor then reads:

$$\begin{aligned} T_t^t = T_x^x = T_y^y &= + \frac{1}{2} e^{-2B} \Phi'^2 + V(\Phi) \equiv \rho \\ T_z^z &= - \frac{1}{2} e^{-2B} \Phi'^2 + V(\Phi) \equiv -p \end{aligned} \quad (2.6)$$

and the Einstein-equations become:

$$\begin{aligned} G_t^t &= - e^{-2B} \left[4B'' - B'^2 - 2A'B' - 4A'' + 3A'^2 \right] / 4 = \rho \\ G_z^z &= - e^{-2B} \left[B'^2 + 2A'B' - 3A'^2 \right] / 4 = -p \\ G_x^x = G_y^y &= - e^{-2B} \left[2B'' - B'^2 - 2A'B' + 2A'' + 3A'^2 \right] / 4 = \rho , \end{aligned} \quad (2.7)$$

where the prime denotes the derivative $\partial/\partial z$. The scalar field equation (2.4) simplifies to:

$$e^{-2B} \Phi'' - \frac{dV(\Phi)}{d\Phi} = 0 . \quad (2.8)$$

From (2.7) one immediately finds that

$$A'' = -e^{2B} V(\Phi) \quad (2.9)$$

$$A'' = B''/3 . \quad (2.10)$$

Eqs. (2.8) - (2.10) are equivalent to the Einstein-equations (2.7) and are sufficient to determine the functions A , B and Φ for a given $V(\Phi)$.

3. Properties of planar scalar field configurations

The simple form of eqs. (2.8) -(2.10) allows us to draw some interesting conclusions about static planar scalar field configurations for an arbitrary potential $V(\Phi)$. The properties of thick walls are radically different from those of infinitely thin walls discussed so far. Our assumptions, besides planar symmetry, are only that (i) pressure and density vanish for $|z| \rightarrow \infty$ and (ii) $V(\Phi) > 0$ for finite z . The assumption (ii) requires the Φ field to take a ground state value at $|z| \rightarrow \infty$.

3.1 PLANE SYMMETRIC VACUUM SOLUTIONS

Before we analyze the properties of scalar field configurations we shall have a look at the asymptotic vacuum states that are possible far away from the wall, where the density and pressure vanish. From (2.7) we find for $\rho, p \rightarrow 0$

$$A'' = 0, \quad B'' = 0, \quad B'^2 + 2A'B' - 3A'^2 = 0, \quad (3.1)$$

with the two solutions

$$\begin{aligned} (a): \quad & \alpha \equiv A' = B' = \text{const} \\ (b): \quad & \alpha \equiv A' = -B'/3 = \text{const} \end{aligned} \quad (3.2)$$

In case (a) the metric (2.5) becomes

$$ds^2 = e^{2\alpha z} (dt^2 - dz^2) - (dx^2 + dy^2), \quad (3.3)$$

where we absorbed the integration constants by a rescaling of the coordinates. Of course, for $\alpha = 0$ this gives the Minkowski vacuum. For $\alpha \neq 0$ (3.3) is the metric of a Rindler space¹³, that is Minkowski space in an accelerated coordinate system. This can be seen by applying the transformation

$$\hat{t} = e^{\alpha z} \sinh(\alpha t) / \alpha, \quad \hat{z} = e^{\alpha z} \cosh(\alpha t) / \alpha, \quad (3.4)$$

so that eq. (3.3) becomes

$$ds^2 = d\hat{t}^2 - d\hat{z}^2 - dx^2 - dy^2. \quad (3.5)$$

An observer at rest at $z = \text{const}$ receives an acceleration along the hyperbola $\hat{z}^2 - \hat{t}^2 = e^{2\alpha z} \alpha^2$ in Minkowski-space.

In case (b) the metric (2.5) becomes

$$ds^2 = e^{2\alpha z} dt^2 - e^{-6\alpha z} dz^2 - e^{-4\alpha z} (dx^2 + dy^2) \quad (3.6)$$

where we again absorbed the integration constants by a rescaling of the coordinates. Again, for $\alpha = 0$ the metric reduces to the Minkowski vacuum. For $\alpha \neq 0$ after the coordinate transformation $\hat{z} = e^{-4\alpha z}/(4|\alpha|)$ and a further rescaling $(\hat{t}, \hat{x}, \hat{y}) = 2\sqrt{|\alpha|} (t, x, y)$ one finds

$$ds^2 = \hat{z}^{-1/2} (d\hat{t}^2 - d\hat{z}^2) - \hat{z} (d\hat{x}^2 + d\hat{y}^2), \quad (3.7)$$

that is the Taub-vacuum¹⁴.

Now, knowing the possible asymptotic vacuum space-times far away from the wall, we show that the quite general assumptions stated above, i.e. planar symmetry, $V(\Phi) > 0$ at finite z and $p, \rho \rightarrow 0$ for $|z| \rightarrow \infty$, are indeed sufficient to determine the actual vacuum states on both sides of the wall.

3.2 FORM OF THE METRIC COEFFICIENTS

Here we examine the general form of the metric functions A and B . The relative sign of their derivatives, A' and B' , determines by eq. (3.2) the nature of the vacua at $z \rightarrow \pm\infty$. The assumption $V(\Phi) > 0$ for finite z yields (see eq. (2.9))

$$A'' < 0, \quad B'' < 0 \quad \text{for } z \text{ finite.} \quad (3.8)$$

Thus, A' and B' can change their signs at most once. Since we demand $p \rightarrow 0$ as $|z| \rightarrow \infty$ and $p \neq 0$ at some points ($p \equiv 0$ would be equivalent to Minkowski vacuum everywhere by eqs. (2.7)), p must have an extremum. This implies that

$$p' = -B'\Phi'^2 e^{-2B} \quad (3.9)$$

has to change its sign at least once and, because $\Phi'^2 \exp(-2B) \geq 0$, also B' changes its sign at *least* once. On the other hand, since by $B'' < 0$, B' can change sign at *most* once. So there exists a single point z_0 where $B'(z_0) = 0$. $B(z)$ has a single maximum and must therefore tend to $-\infty$ for $|z| \rightarrow \infty$. The general shape of $B(z)$ is illustrated in Fig.1. It is also obvious that the sign of B' must be different for $z \rightarrow +\infty$ and for $z \rightarrow -\infty$:

$$\begin{aligned} \text{for } z \rightarrow +\infty: \quad B' < 0 &\Leftrightarrow p' \geq 0 \\ \text{for } z \rightarrow -\infty: \quad B' > 0 &\Leftrightarrow p' \leq 0 \end{aligned} \quad (3.10)$$

Since the space-time tends to a vacuum as $|z| \rightarrow \infty$, B' becomes constant (see (3.2) and Fig.2).

Next, we want to determine the sign of A' . Since $\rho(z)$ and $V(\Phi)$ are supposed to vanish for $|z| \rightarrow \infty$ and $B(z) \rightarrow -\infty$ the gradient $\Phi'(z)$ must also go to zero for $|z| \rightarrow \infty$. From (2.10) we have

$$B' = 3A' + q, \quad q = \text{const.} \quad (3.11)$$

and (2.7) yields then:

$$\Phi'^2 = 6A'^2 + 4qA' + \frac{1}{2}q^2 + 2Ve^{2B}. \quad (3.12)$$

The information about the sign of A' , contained in the term $4qA'$ of eq. (3.12), allows one to draw conclusions about the reflection symmetry of solutions and the asymptotic vacuum spacetimes on the different sides of the wall. This will be discussed in the following two sections for the case $q = 0$ and $q \neq 0$, respectively.

3.3 NO REFLECTION SYMMETRIC SOLUTIONS

We show here that in the case $q = 0$ of eq. (3.11) there are no solutions (besides the Minkowski vacuum, $A' = B' = 0$). Only for $q = 0$ both metric coefficients, A and B , could be symmetric functions of z . The choice of the integration constant $q = 0$ implies that for $|z| \rightarrow \infty$ $A' \rightarrow 0$, since we have assumed $\Phi' \rightarrow 0$ and $V \rightarrow 0$. Then, on account of (3.11), also B' has to vanish at infinity which contradicts the result about the shape of $B(z)$ derived at the beginning of this section. Since the case $q = 0$ is not compatible with the imposed boundary conditions and for $q \neq 0$ eq. (3.11) is not invariant under $z \rightarrow -z$, i.e. at least one metric function cannot be invariant under reflections, we conclude that *reflection symmetric static walls are not admitted by the coupled Einstein scalar field equations*. This is also corroborated by the fact, which we are now going to prove, that the two vacua at $z \rightarrow \pm\infty$ are different.

3.4 DIFFERENT VACUA OUTSIDE THE WALL

For the remaining case $q \neq 0$ we now analyze the shape of the metric function A to determine the asymptotic vacuum states on the different sides of the wall. With $V \rightarrow 0$ and $\Phi' \rightarrow 0$ at $|z| \rightarrow \infty$ eq. (3.12) becomes

$$6A'^2 + 4qA' + \frac{1}{2}q^2 \rightarrow 0 \quad \text{for } |z| \rightarrow \infty. \quad (3.13)$$

However, this is only possible if

$$qA' < 0 \quad \text{at } z \rightarrow +\infty \text{ and } z \rightarrow -\infty, \quad (3.14)$$

since $6A'^2 + \frac{1}{2}q^2$ is positive. The important point is that qA' , and therefore also

A' , have the same sign at $z \rightarrow +\infty$ and $z \rightarrow -\infty$. Taking into account $A'' < 0$ (eq. (3.8)), A' cannot change its sign at all and therefore $A(z)$ must have a shape as depicted in Fig.1 .

We have seen in (3.1) , (3.2) that the two possible vacuum solutions are given by $B' = A' = \text{const}$ (Minkowski, i.e. Rindler) and $B' = -3A' = \text{const}$ (Taub), i.e. the two vacua can be distinguished by the relative sign of the asymptotic values of A' and B' . Since we arrived at the conclusion that B' must have different signs at $z \rightarrow +\infty$ and $z \rightarrow -\infty$ and A' cannot change sign at all, *the vacuum spacetimes at $z \rightarrow +\infty$ and at $z \rightarrow -\infty$ must be different*. If $q < 0$ then, on account of (3.14) , $A' > 0$ everywhere and since $B' < 0$ at $z \rightarrow +\infty$ the vacuum at $z \rightarrow +\infty$ must be the Taub vacuum and since $B' > 0$ for $z \rightarrow -\infty$ the space-time tends to the Minkowski vacuum at $z \rightarrow -\infty$. For $q > 0$ the Taub space is at $z \rightarrow -\infty$ and Minkowski space at $z \rightarrow +\infty$.

To summarize the last sections, we have shown that every planar, static solution of the coupled Einstein-scalar field equations with a positive potential and an asymptotically vanishing energy momentum tensor cannot be symmetric with respect to reflections $z \rightarrow -z$ and tends to different vacuum space-times at $z \rightarrow +\infty$ and $z \rightarrow -\infty$.

3.5 NEGATIVE PRESSURE

In the appendix we give a proof that the pressure $p(z)$ perpendicular to the wall must be negative and has a single minimum. This is markedly different from domain walls in flat space-time where this pressure is always zero. The scalar field equation in flat space is

$$\Phi'' = \frac{dV}{d\Phi} \quad (\text{in flat space}) \quad (3.15)$$

which has the first integral

$$p = \frac{1}{2}\Phi'^2 - V = \text{const} = 0 \quad (\text{in flat space}) \quad (3.16)$$

By imposing the same boundary conditions for p as above the constant must be zero. Thus, the pressure perpendicular to a wall in Minkowski space is zero. This shows that $p(z)$ originates entirely from gravitational effects. Since in curved space a force is necessary to counterbalance gravity in order to make a wall static, the pressure in a self gravitating wall cannot vanish. In fact, if p were zero, (2.7) shows that also ρ vanishes, i.e. there is no static wall solution with $p = 0$ in curved space.

3.6 GEODESIC EQUATIONS

The absence of a reflection symmetry also gives rise to different geodesic motions of test particles on the two sides of the wall. All previous treatments of the gravitational effects of domain walls in static and non-static space-times^{7,8,9,10,11} were based on the assumption of reflection symmetry about the center of the wall. So repulsive gravitational effects were assumed to occur on both sides. However, non-reflection symmetric walls will have different gravitational effects on the two sides. We find that a wall is attractive on the Taub side and repulsive on the Minkowski side.

The results from the last sections allow us to determine the general features of the motion of a test particle in the gravitational field of the wall. The geodesic equations for a test particle moving perpendicular to the wall with position vector $x^\mu = (t(\tau), z(\tau), 0, 0)$ (τ is an affine parameter along the geodesic) have the following first integrals (a dot denote differentiation with respect to τ) :

$$\dot{t} = \tilde{E}e^{-2A} \quad (3.17)$$

$$\dot{z}^2 = e^{-2B} \left[\tilde{E}^2 e^{-2A} - \mu^2 \right] \quad (3.18)$$

where \tilde{E} is the energy constant associated with the Killing vector ∂_t and $\mu^2 = 1, 0$ for massive and massless particles, respectively. The local energy \tilde{E}_{loc} a freely falling observer measures is (in units of the mass for massive particles)

$$\tilde{E}_{loc} = \tilde{E}e^{-A} \quad (3.19)$$

The acceleration of the particle measured by an observer that remains at a constant distance from the wall is given by

$$\ddot{z} = -e^{-2B} \left[\tilde{E}^2 e^{-2A} (A' + B') - \mu^2 B' \right] \quad (3.20)$$

In order to determine whether a particle is repelled by the wall one has to know the sign of the acceleration on both sides of the wall. On the side where the space-time asymptotically tends to the Minkowski vacuum we have $A' = B' = -q/2$. We choose $q > 0$, so that the Minkowski vacuum is at $z \rightarrow +\infty$. Then the acceleration for $z \rightarrow +\infty$ is

$$\ddot{z} = -2B'e^{-2B} \left[\tilde{E}^2 e^{-2A} - \frac{1}{2}\mu^2 \right] \quad (3.21)$$

Since $\dot{z}^2 \geq 0$ implies $\tilde{E}^2 e^{-2A} - \frac{1}{2}\mu^2 \geq 0$ and $B' = -q/2 < 0$ we conclude that $\ddot{z} > 0$, i.e. the wall is repulsive on the Minkowski side at $z \rightarrow +\infty$. For a particle

moving in the part of space which tends asymptotically to the Taub vacuum it is not possible to derive the sign of \ddot{z} in a similar way. However, $A(z)$ is a monotonic function and $\dot{z}^2 \geq 0$ implies by eq. (3.18) that massive particles ($\mu^2 = 1$) can only move in the region $z \geq z_T$ (for $q > 0$), where z_T is the single turning point

$$\tilde{E}^2 - e^{2A(z_T)} = 0 \quad \text{i.e.} \quad \tilde{E}_{loc} = 1, \quad (3.22)$$

see Fig.3 . Thus, any massive particle coming from the Minkowski vacuum and moving towards the wall bounces at $z = z_T$ and is repelled back into the Minkowski vacuum. This means that any test particle is accelerated towards the Minkowski side. Massive particles on the Taub side are attracted by the wall. In this sense the wall may be viewed as a giant ‘‘Taub-vacuum cleaner’’. For photons the possible trajectories are quite different: from (3.21) and (3.10) it follows that massless particles ($\mu^2 = 0$) moving perpendicular to the wall feel a repulsive force on both sides of the wall. However, they can penetrate the wall freely without any turning point. On the other hand, it can be shown that massless particles moving parallel to the wall are always driven towards the Minkowski side.

In the remainder of this section we briefly discuss the essential difference of the gravitational field of these scalar field walls and the gravitational field of a planar static perfect fluid configuration. Taking the metric (2.5) with $2C = (B - A)$ the Einstein equations with an energy momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$ ($u_\mu u^\mu = 1$) are

$$(\rho + 3p) = 2A''e^{-2B} \quad (3.23)$$

$$(\rho - p) = e^{-2B}(A'' - B'') \quad (3.24)$$

and the Bianchi identity is

$$p' + (\rho + p)A' = 0 \quad (3.25)$$

If the fluid satisfies $\rho + 3p > 0$ then $A'' > 0$ and if $p \rightarrow \infty$ as $|z| \rightarrow \infty$ and $\rho + p > 0$ (3.25) implies that A' has to change sign at least once. The equations of motion for a test particle moving along the z -axis are the same as (3.17), (3.18) and (3.20). Since $A(z)$ increases monotonically as $|z| \rightarrow \infty$ and has a single minimum, the condition $\dot{z}^2 \geq 0$ implies that every massive particle can only move within a finite range $z_{T1} \leq z \leq z_{T2}$ where z_{T1}, z_{T2} are the two solutions of $\tilde{E}^2 - e^{2A(z)} = 0$. Thus, every massive test particle is trapped by a perfect fluid wall in a bound state and therefore this wall must be attractive. Only photons can escape the gravitational field since no points with $\dot{z}^2 = 0$ exist for $\mu^2 = 0$.

4. Infinite walls in the cosmological context

A phase transition in the early universe that allows a scalar field to settle down in different vacuum expectation values in different regions of the universe would produce domain walls between these regions. The network of domain walls will initially have a coherence length of the order of the inverse scalar mass, which is also the thickness of the walls. Because of their surface tension, the smaller closed domain walls will gradually shrink and finally decay into scalar bosons; bigger walls will straighten out. Also, walls can collide, annihilate or merge together. A numerical simulation of a network of domain walls in an expanding universe^{5,6}, which, however, did not take into account the gravitational effects of the walls, suggests that only one infinite wall per horizon volume will eventually remain.

An infinite wall in the cosmological context means a wall of a size larger than the horizon. The horizon volume is then cut into two halves by the wall. Even a closed wall, with a curvature radius larger than the horizon, can be treated as an infinite wall, since the information of the topology of the wall can spread only on horizon scales.

The assumption of a vanishing energy-momentum tensor at large distance from the wall is obviously not satisfied in the universe. But even the assumption of an isotropic perfect fluid far from the wall would, by eqs. (2.7), lead to $A' = \eta B' + \text{const}$ ($\eta = \text{const}$), so that the metric would still have no reflection symmetry and the space-times on the two sides of the wall would be different! In any case, one can expect that the above results are an approximation for almost planar walls and regions where the energy density of the universe is much smaller than that of the wall.

If there is such an infinite wall between us and the last scattering surface of the microwave background, it could destroy the isotropy of this background by deflecting the photons at the wall. On the other hand, the wall would also influence our motion with respect to the microwave background. From the requirement that the combination of both effects is in accord with the measured isotropy of the microwave background one could derive constraints or indications on the possible existence of these domain walls in the universe. But also the wall-induced perturbations in the density and velocity distributions of ordinary matter could lead to similar effects.

5. Conclusions

We have shown that static domain walls in General Relativity possess no reflection symmetry. This implies some interesting phenomena. The asymptotic vacua are Minkowski (i.e. Rindler) space-time and Taub space-time on the different sides of the wall. Massive test particles are always accelerated towards the Minkowski side; that is, particles coming from the Minkowski side experience a reflection. Massless particles moving perpendicular to the wall are always accelerated away from the wall (and never get reflected), but those moving parallel to the wall are driven towards the Minkowski side. Indications or constraints for the existence of infinite walls in the universe could be obtained by comparing their gravitational effect on our local motion and on the microwave background with the observed isotropy of this background.

ADDED NOTE:

After completing this paper we found an exact solution of eqs. (2.8)-(2.10) for the potential $V(\Phi) = V_0 \cos^{2(1-n)}(\Phi/f(n))$ ($0 < n < 1$) which confirms and illustrates the general results obtained in this paper.

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APPENDIX

Here we show that p is negative and has only a single minimum. By eq. (3.9) p' vanishes only, if either $\Phi' = 0$ or $B' = 0$. The second derivative of $p(z)$ at the point z_0 , defined by $B'(z_0) = 0$, is

$$p''(z_0) = -B''\Phi'^2 e^{-2B} \geq 0. \quad (\text{A1})$$

In order to prove that p is negative and has a single minimum which coincides with z_0 we have to discriminate three possible cases for p' to vanish:

$$B'(z_0) = 0, \Phi'(z_0) \neq 0 :$$

Eqs. (3.9) and (A1) show that z_0 is a minimum of $p(z)$. At all points $z \neq z_0$ where $\Phi' = 0$ (and $B' \neq 0$), p' does not change its sign since we know that B' changes sign only once. Therefore, these points can only be turning points of $p(z)$. It follows that $p(z)$ can have only one minimum and since $p \rightarrow 0$ for $|z| \rightarrow \infty$ eq. (3.10) implies $p(z) < 0$ for finite z .

$$B' \neq 0, \Phi' = 0 \text{ for } z \neq z_0 :$$

Since the sign of p' cannot change at these points, p can have only a turning point and no extremum.

$$B'(z_0) = 0, \Phi'(z_0) = 0 :$$

In this case $p''(z_0) = p'''(z_0) = 0$ and

$$p^{(4)}(z_0) = -6B''\Phi''^2 e^{-2B} \geq 0 \quad (\text{A2})$$

If $\Phi''(z_0) \neq 0$, z_0 is a minimum of $p(z)$. Again it is the single minimum of p , because at all other points where $\Phi' = 0$, B' and p' do not change sign. The boundary condition $p \rightarrow 0$ as $|z| \rightarrow \infty$ then implies $p(z) < 0$ for finite z . If $\Phi''(z_0) = 0$, then, on account of (2.8), $dV/d\Phi = 0$ at z_0 . Upon differentiating (2.8) one can show that if $\Phi'(z_0) = B'(z_0) = \Phi''(z_0) = 0$ then all higher derivatives of $\Phi(z)$ must vanish. However, a function whose derivatives are all zero at some point must be constant. A constant scalar field gives rise only to a vacuum solution. That is $\Phi''(z_0) \neq 0$ and $p(z)$ has a minimum.

Thus p is negative and has a single minimum at z_0 where $B'(z_0) = 0$.

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FIGURE CAPTIONS

1. General shape of the metric functions $A(z)$ and $B(z)$.
2. General shape of the derivatives $A'(z)$ and $B'(z)$. Note that the asymptotic values are related according to eq. (3.2).
3. The general shape of the effective potential $\exp A(z)$ for a massive test particle. z_T is determined by (3.22).

$A(z), B(z)$

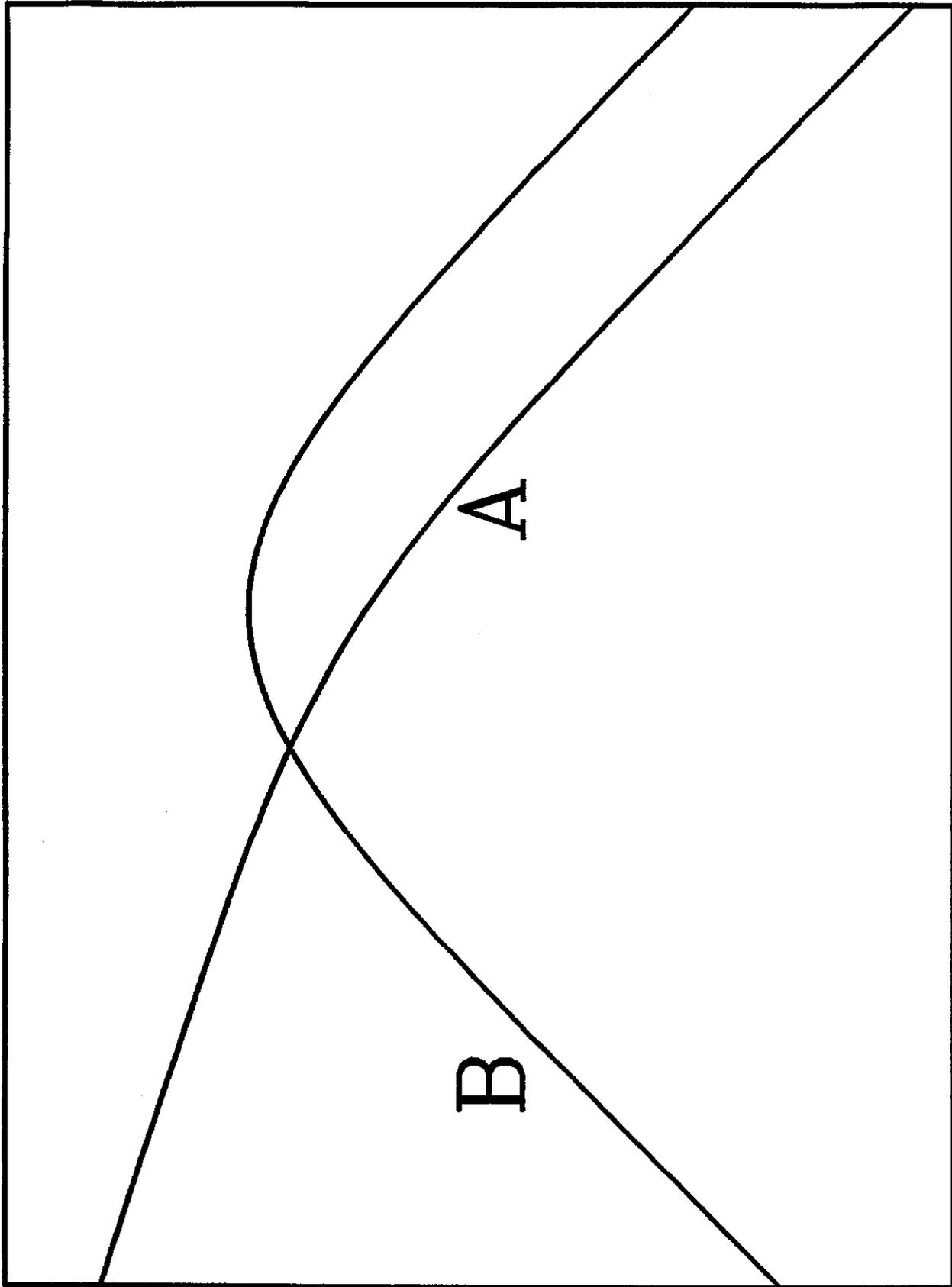


Fig. 1

z

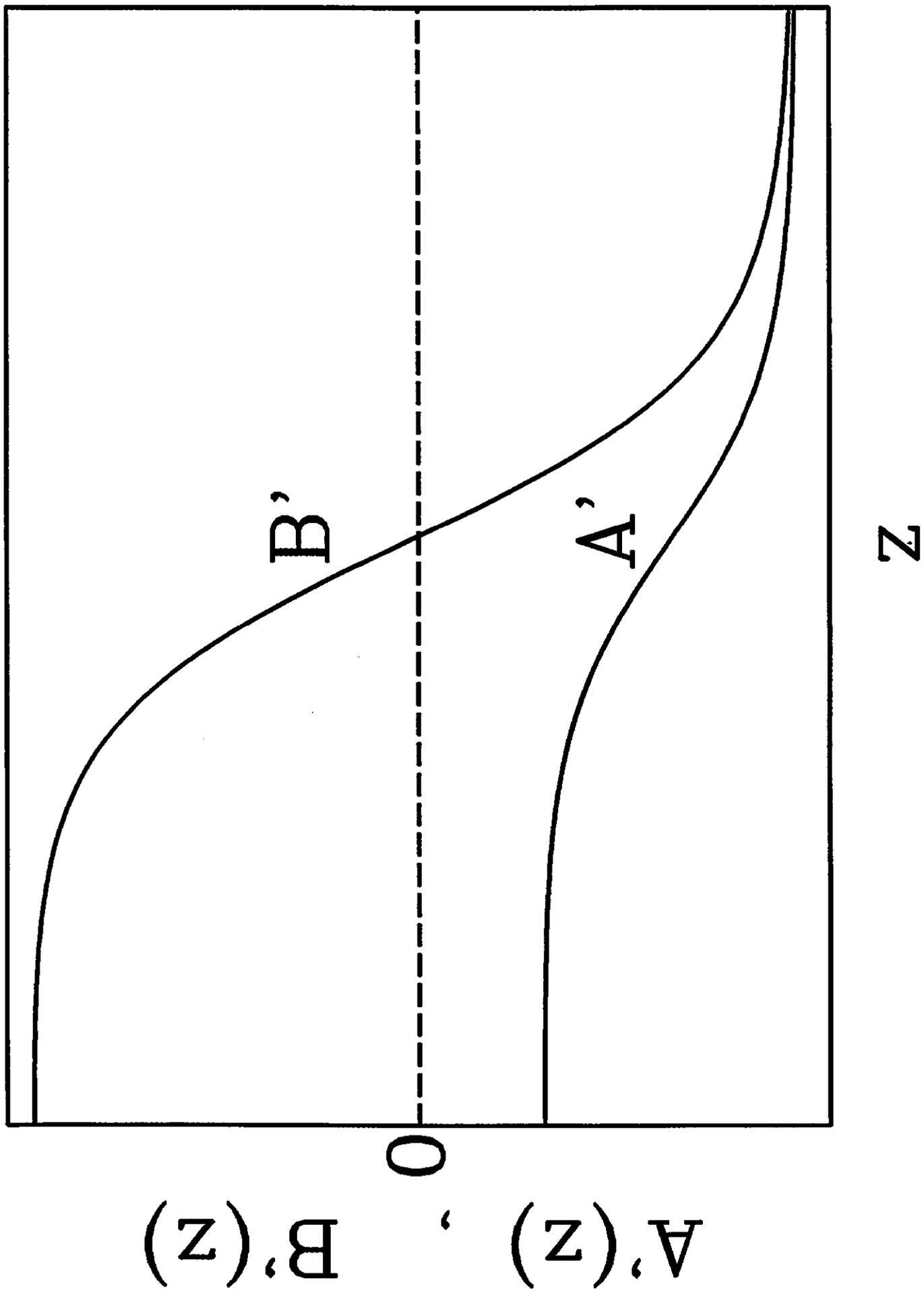


Fig.2

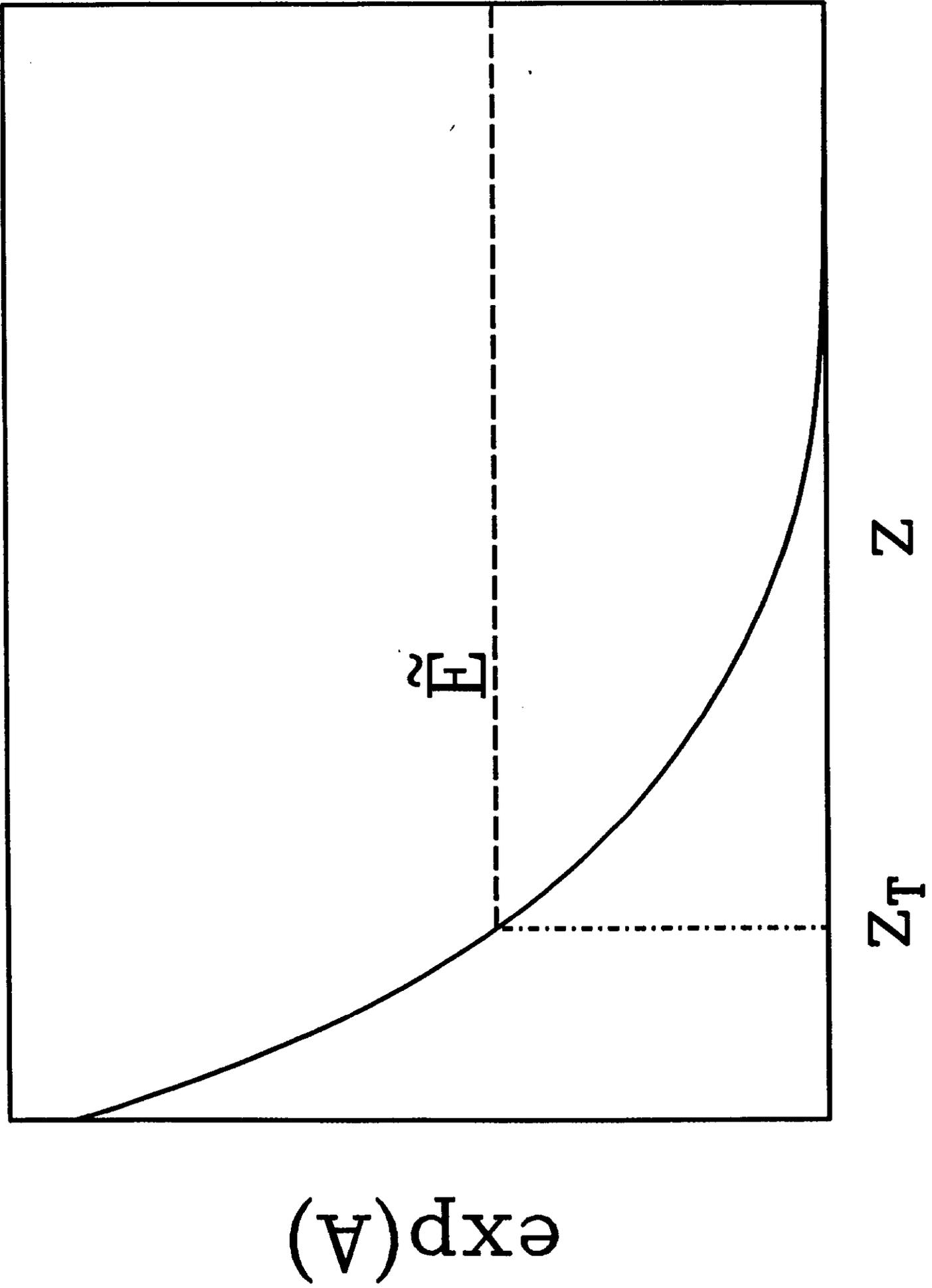


Fig. 3