Fermi National Accelerator Laboratory

FERMI–PUB–89/228-T
November 1989
Submitted to Phys. Lett. B

A Naturally Heavy Fourth Generation Neutrino

C. T. Hill
E. A. Paschos[†]

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois, 60510

Abstract

We propose that there exists a fourth generation, all neutrinos have Dirac masses of order the corresponding charged lepton partner, and right–handed neutrinos have Majorana masses of order the weak scale. The see-saw mechanism then implies the light neutrino masses are $O(m^2_{lepton}/M)$, while a fourth generation neutrino is heavy with mass $O(M) \sim O(M_W)$, and would not have been seen in the Z^0 width at LEP. Such a situation appears to be compatible with all known phenomenological and cosmological constraints, and should be readily testable.

[†] On sabbatical leave from The University of Dortmund.

LEP and SLC have ruled out the existence of any massless Standard Model neutrinos beyond those of the three known generations. Does this imply that there are only three generations of quarks and leptons? Obviously not. If, however, there does exist an additional neutrino the following question naturally arises: Why do the first three generations have (nearly) massless neutrinos, while the fourth generation would need to have an unusually heavy neutral member? On the face of it, this would seem an ugly and improbable situation.

We will argue, however, that a very natural mechanism exists which could easily explain such a configuration of neutrino masses. Moreover, it leads readily to experimental tests and should serve as a useful guide for thinking about neutrino masses in general. In short, one needs only to assume that *(i) all neutrinos have Dirac mass terms of the same order as their charged lepton partners within a given generation* and *(ii) all right-handed neutrinos have a Majorana mass term of order the weak scale*. Note that the fourth generation Dirac masses cannot be arbitrarily large, as they are constrained by renormalization group triviality bounds. With both terms present we have a conventional Gell-Mann–Ramond–Slansky–Yanagida [1] see–saw mechanism. The predicted light mass scale of neutrinos will be acceptably small, of order m_{lepton}^2/M. However the fourth generation neutrino, having a charged leptonic partner whose mass is also of order $M_W \sim M$, will be expected to have a Dirac mass also of $\sim M$, and hence a Majorana mass of order M. Thus, it is natural that those neutrinos associated with low mass leptons *have very small masses, while the fourth generation neutrino is expected to be heavy.* For the present purposes it is essential that the Majorana mass scale M, be of order the weak scale; taking M very large will conflict with measured width of the Z^0 by causing the fourth generation neutrino to become too light. On the other hand, taking M very small will cause the light neutrinos to become too heavy. Stated yet another way, a fourth generation is not unnatural provided there exists a Majorana mass term for all the right-handed neutrinos; this mass scale is then bounded from above by experiment in combination with the mass scale of the fourth generation lepton.

We take the full Standard Model Lagrangian L_0 to contain four generations of quarks and leptons. The fourth electroweak leptonic doublet is denoted $\Psi_L^{(4)} = (N,\ L)_L$ and the right–handed charged lepton by L_R. We also supplement the model with four right–handed singlets: $\nu_R^i = (\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}, N_R)$. We assume the leptonic Dirac mass matrices take the form:

$$L_{Dirac} \;=\; \sum_{ij} \left\{ m_{ij}^{(-1)} \overline{\Psi}^i l_R^j (i\sigma_2 H^*/v) + m_{ij}^{(0)} \overline{\Psi}^i \nu_R^j (H/v) \right\} + h.c. \qquad (1)$$

where H is a Higgs doublet, l_R^i the charged right–handed leptons, and the sum runs over the four generations. When H acquires a vacuum expectation value the Dirac masses are generated. The neutrino (as well as the charged lepton) mass matrix is general–complex, and may be written in the form $m_{ij}^{(0)} = U_m m V_m^\dagger$ where U_m and V_m are unitary, m is positive diagonal with eigenvalues m_i. The neutrino Dirac mass eigenvalues will be assumed to be comparable to their charged leptonic partners:

$$m = \mathrm{diag}(\epsilon_1 m_e, \epsilon_2 m_\mu, \epsilon_3 m_\tau, \epsilon_4 m_L) \qquad (2)$$

For concreteness we will assume the range, $0.2 \lesssim \epsilon_i \lesssim 2.0$. Of course, there might be accidental suppressions, which make some $\epsilon_i \lesssim 10^{-1}$, but we do not propose to fine–tune a hierarchy of Dirac masses within a given generation, such as $m_{\nu_e} = \epsilon_1 m_e \leq 10^{-5} m_e$, *etc.*

We now assume the existence of a Majorana mass term:

$$L_{Majorana} \;=\; \frac{1}{2} \sum_{ij} \left\{ M_{ij} \bar{\nu}_R^{c\,i} \nu_R^j + h.c. \right\} \qquad (3)$$

Here we adopt the conventions of Bjorken and Drell [2] in which $\Psi^c = C\overline{\Psi}^T = i\gamma^2\Psi^*$ and $C = i\gamma^2\gamma^0$. Note that we then have $\overline{\Psi^c} = \Psi^T C = (\overline{\Psi})^c$; $\left(\frac{1+\gamma^5}{2}\Psi\right)^c = (\Psi_R)^c = (\Psi^c)_L = \left(\frac{1-\gamma^5}{2}\right)\Psi^c$. The following lemma is useful in writing mass matrices $\overline{\Psi}_L^c \chi_R^c = \overline{\chi}_R \Psi_L$, where the anticommutativity of spinor fields is used.

A term such as in eq.(3) is permitted in the Standard Model as ν_R carries no $SU(2) \times U(1)$ quantum numbers. The general form of the mass matrix with both the

Dirac and Majorana terms present is (using the aforementioned reordering lemma):

$$\frac{1}{2}\overline{\begin{pmatrix} \nu_L{}^i \\ \nu_R^c{}^i \end{pmatrix}} \begin{pmatrix} 0 & m_{ij}^{(0)} \\ m_{ji}^{(0)} & M_{ij} \end{pmatrix} \begin{pmatrix} \nu_L^c{}^j \\ \nu_R^j \end{pmatrix} = \frac{1}{2}\overline{\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}} \begin{pmatrix} 0 & m^{(0)} \\ m^{(0)T} & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \tag{4}$$

This is a general–complex symmetric matrix and can be written in the form $W^T D W$ where W is unitary and D positive diagonal. We may perform the unitary transformation:

$$\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \to \begin{pmatrix} U_m^T & 0 \\ 0 & V_m^\dagger \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \tag{5}$$

and the mass term is brought to the form:

$$\frac{1}{2}\overline{\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}} \begin{pmatrix} 0 & m \\ m & (V_m)^T M V_m \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c. \tag{6}$$

(recall that m is positive diagonal as defined above eq.(2)). At this stage a unitary Cabibbo–Kobayashi–Maskawa matrix for the leptonic charged current is generated, and the Z^0 couplings remain diagonal.

We will now consider a special case which allows further detailed discussion, though the general ideas described above go beyond the present special assumptions. We now assume *(i)* $(V_m)^T M V_m$ is approximately proportional to a unit matrix $M_0 I_4$, with *(ii)* small off-diagonal elements of order δM_{ij}. Assumption *(i)* is not unreasonable if one is given that M is initially diagonal, since V is expected to be approximately diagonal; in this case δM will be generated by V. M will be proportional to a unit matrix if a global or discrete right–handed family symmetry is invoked.

Thus, neglecting the corrections of order $\delta M/M_0$ we see that the eigenvalues of the full matrix are:

$$\frac{1}{2}\left|\left(M_0 \pm \sqrt{(M_0^2 + 4m_i^2)}\right)\right| \tag{7}$$

Thus, for $m_i << M_0$ one obtains a large eigenvalue M_0 for the right–handed eigenstate, and m_i^2/M_0 for the left–handed state. This is a standard see-saw mechanism which produces the large suppression for the induced left–handed neutrino masses [1].

Note that the physical low mass state is purely left–handed $\nu_{Li} = \left[\nu^0_{Li} + \frac{m_i}{M_0}(\nu^0_R)^c_i\right]$, while the heavy state is right–handed $\nu_{Ri} = \left[\nu^0_{Ri} - \frac{m_i}{M}(\nu^0_L)^c_i\right]$. This additional mixing spoils the unitarity of the CKM matrix to order m_i/M, which in turn spoils the GIM cancellation and leads to off–diagonal Z^0 couplings of this order.

When the fourth generation Dirac mass, m_4, is of order the Majorana mass scale M_0 the resulting eigenvalues are both $O(M_0)$. Indeed, the Majorana scale can in principle be less than the fourth generation Dirac mass scale, whence both the right and left–handed fourth generation neutrinos are very heavy with masses of $O(m_4)$. Here the mixing between $\nu^0_{L_4}$ and $(\nu^0_{R_4})^c$ is maximal, and *the violation of GIM suppression is maximal.* This may have important consequences for the physics of the heavy neutrinos where box or penguin diagrams are involved that contain the L lepton in a loop.

We emphasize that *it is possible that the fourth generation neutrino has a mass less than* $\frac{1}{2}M_Z$ *and be present in the decay of the* Z^0 provided it is not very light. A fourth generation neutrino lighter than $\frac{1}{2}M_Z$ will contribute to the decay width of the Z^0, counting as a fraction of a neutrino channel. This situation arises for two reasons: the threshold suppression *and* the failure of the unitarity of the CKM matrix when ν_L mixes with ν^c_R. Let us denote by θ the left–right mixing angle and we expect $\cos\theta \sim m_4/\sqrt{m^2_{\nu_4} + m^2_4}$. The total number of neutrinos counted in the Z^0 width will be:

$$N_{total} = 3 + \cos^2\theta \left(1 - \frac{4m^2_{\nu_4}}{M^2_Z}\right)^{1/2} \left(1 - \frac{2m^2_{\nu_4}}{M^2_Z}\right) \tag{8}$$

For $m_{\nu_4} = 35$ GeV and $M_0 = 100$ GeV (corresponding to $m_4 \sim 68$ GeV and $\cos^2\theta \sim 0.79$) we see that $N_{total} \approx 3.5$. This is within the experimental bounds reported thus far. Note that without a Majoron such a neutrino is expected to decay as $\nu_4 \to \nu + \gamma$ within $\tau \sim |U|^{-2}10^{-16}$ sec. and would thus contribute visible energy of order $\frac{1}{2}M_Z$. With a Majoron the predominant decay mode is expected to be $\nu_4 \to \nu + \phi$, and is completely invisible. Glashow [4] has argued for the ν_τ what we might be tempted to say about ν_4, *i.e.*, that our ignorance of ν_4 universality would permit $\cos^2\theta$ to be arbitrarily small for any m_4, thus the fourth neutrino might in fact be taken very light. However, it is theoretically implausible that we can take $m_4 \to 0$ without simultaneously taking $\cos^2\theta \to 1$.

One must specify whether the Majorana mass is to be chosen *(i)* by hand (this is unattractive and we reject it), *(ii)* by the condensation of a real scalar field, Φ, without an attendant Nambu–Goldstone mode, (more attractive) or *(iii)* by the condensation of a complex scalar field, Φ with an attendant Nambu–Goldstone mode, ϕ, having decay constant $f_\phi \sim M$ (very attractive). In the latter two cases there is a new coupling constant matrix $g_{ij} = M_{ij}/V$ which may again be specialized to an approximate unit matrix. The Majorana mass terms arise from the Majorana–Yukawa couplings:

$$L_{Majorana} = \frac{1}{2}\sum_{ij}\left\{g_{ij}\bar{\nu}_R^{c\,i}\nu_R^j\Phi + h.c.\right\} \tag{9}$$

when the Majorana–Higgs field, Φ, acquires a vacuum expectation value $V = \langle\Phi\rangle$.

The scale of the fourth generation Dirac mass m_4 must be consistent with the triviality bound for a neutrino. Such bounds are derived by integrating the neutrino Higgs–Yukawa coupling renormalization group equations from low energy scales to a high energy scale Λ. Over the entire intermediate energy range the Standard Model is viewed as the full effective Lagrangian, and all of its couplings must remain finite. The trajectory on which $g_\nu \to \infty$ at Λ defines the triviality bound. In our case the usual renormalization group equations for the neutrino Higgs–Yukawa coupling constants are modified by the presence of the Majorana-Yukawa couplings. Moreover, the new Majorana–Yukawa couplings also satisfy coupled RG equations involving the Higgs–Yukawa coupling constants. We will defer a thorough discussion of this to another place, but it suffices to state that m_4 is expected to be bounded from above by ~ 0.3 TeV. Thus, when the mass for the fourth generation left–handed neutrino is required to satisfy:

$$\frac{m_4^2}{M} \gtrsim \frac{1}{2}M_Z \approx 45 \text{ GeV}, \tag{10}$$

one finds $M \leq 2.0 \times 10^3$ GeV.

One can see that M of order the weak scale is not ruled out phenomenologically. Indeed, Glashow [4] has previously discussed the limits on masses and mixing angles for the three generation case of $M \sim M_W$ and most of this analysis carries over here. As a rough guide to the lower limit on M consider the ratio m_l^2/m_ν for each generation where m_l is the charged lepton mass and $m_{\nu-l}$ is the upper limit on the corresponding neutrino mass [3]. Using $m_{\nu_e} \leq 5$ eV [5], which is consistent with limits on neutrinoless double β–decay, $m_{\nu_\mu} \leq 250$ KeV [6], $m_{\nu_\tau} \leq 35$ MeV [7], one finds:

$$M \gtrsim \frac{m_e^2}{m_{\nu_e}} \gtrsim 50\ GeV; \qquad \frac{m_\mu^2}{m_{\nu_\mu}} \gtrsim 40\ GeV; \qquad \frac{m_\tau^2}{m_{\nu_\tau}} \gtrsim 93\ GeV; \tag{11}$$

Hence, the identification of M with the weak scale is not immediately ruled out, though the constraints from mixing and cosmology are important. In Fig.(1) we show the allowed range of neutrino masses for the ϵ_i bounded by the range $0.2 \leq \epsilon_i \leq 2.0$; here we have taken the Dirac mass of the fourth generation neutrino to range between $50 \leq m_4 \leq 300$ GeV. The experimental upper limits of the light neutrino masses, and the lower limit on the fourth neutrino mass assuming $N_{total} = 3.0$ are also shown. Finally, we remark that the experimental constraints on the neutrino mixing angles should not be viewed as unnatural in a scheme like this. For example the limit $|U_{e\mu}| \lesssim 0.029$ [8] would be consistent with a "Fritzsch Ansatz" for the Dirac mass matrix leading to relations such as $|U_{e\mu}| \sim m_e/m_\mu$.

The usual cosmological problems with massive neutrinos are described in detail in [9]. In summary they consist of: *(i)* the universe cannot be over–closed by long–lived massive neutrinos requiring, $\sum_{long} m_\nu \lesssim 100$ eV; *(ii)* unstable neutrinos with nonelectromagnetic decay modes can have masses up to ~ 4.0 GeV, but then their life–times must satisfy the Lee–Weinberg bound [10]; (iii) dominant electromagnetic decay modes of unstable neutrinos heavier than ~ 100 eV, such as $\nu_\mu \to \nu_e\gamma$, are problematic because they deposit too much entropy into the relic photons and typically demand short lifetimes *e.g.*, $\tau_{\nu_\mu} \lesssim 10^4$ sec [9].

All heavy neutrinos we discussed are unstable and decay to lighter states. Right–

handed neutrinos with masses of order M will have induced electro-magnetic decay modes through operators like $\overline{\nu}_R\sigma_{\mu\nu}\nu_L F^{\mu\nu}$. Their lifetimes are of order $\tau \sim (\alpha G_F^2 M^5)^{(-1)} \sim 10^{-22}$ sec, and present no cosmological problems. Also ν_τ can fulfill the cosmological bounds provided $35 \geq m_{\nu\tau} \geq$(a few) MeV without appealing to a Majoron (since, in general $\nu_\tau \rightarrow \nu_{e,\mu} + 2\gamma$ is not GIM suppressed) [9]. Moreover, we readily expect $m_{\nu e} \leq 10$ eV in this scenario.

The lifetime of ν_μ in our scheme can be sufficiently long, with its decay modes predominantly electromagnetic, thus presenting a problem. There are two escape routes from this difficulty: *(a)* by somewhat, albeit not drastically, tuning the Dirac mass of the ν_μ one can safely accomodate $m_{\nu\mu} << 100$ eV (for example, with $\epsilon_2 = 0.03$ and $M \sim 300$ GeV one has $m_{\nu_\mu} \gtrsim 30$ eV); *(b)* the decay mode $\nu_\mu \rightarrow \nu_e + \phi'$ with ϕ' a familon (Majoron) can readily (probably cannot) deplete the ν_μ. Indeed, a dynamical mechanism as described below which would link the Majorana scale to the electroweak scale (favoring a composite Majoron, and the triviality bounds on couplings would be saturated) favors a large familon coupling constant ~ 1.0 and this assists in depleting ν_μ. One then requires the decay constant near 10^{10} GeV, and thus [9] $m_{\nu_\mu} \gtrsim 17$ KeV appears to be allowed. In Figure (1) we show the cosmological limits as dashed lines with (A) representing the closure density upper limit constraint on the sum of masses of stable neutrinos; (B) is the lower mass limit for a familon decay mode of ν_μ; (C) is the lower mass limit for a Majoron decay mode. These are taken from [9] with the optimistic assumption of strong coupling ($h = 1.0$ in the eq.(10.12) and eq.(10.8) of Harari and Nir [9]; also, we assume the familon decay constant to be $F = 10^{10}$ GeV). We conclude that the heavy ν_μ is not incompatible with the present scheme.

Why should the Majorana mass scale M be of order the weak scale? Recently it has been proposed that the electroweak interactions are broken by condensates of conventional quarks and leptons, most notably the top quark [11]. However, the resulting predicted $m_{top} \sim 230$ GeV is slightly larger than one might wish, given present constraints on $\sin^2\theta_W$, and it has been emphasized that this mechanism could more comfortably apply to a fourth generation [11], [12]. If one extends this idea to assume that a neutrino Majorana mass term is also generated by an analogous dynamical mechanism, with the same input scales, etc., then M is expected to be naturally of order the weak scale. In this context one can analyze the Nambu–Jona-Lasinio model defined by four–fermion interactions such as $G\overline{\nu}^c\nu\overline{\nu}\nu^c$ (continuous global symmetry producing a composite Majoron and a massive Majoron–Higgs) or $G'\overline{\nu}^c\nu\overline{\nu}^c\nu + h.c.$ (discrete symmetry, producing only a composite real scalar Majoron–Higgs).

In conclusion, the model has obvious immediate and near–future tests:

1. A fourth generation has not been ruled out as its neutrino is naturally heavy when $M \sim M_W$ in the see–saw mechanism. We expect a probable scale of ~ 100 GeV for the charged lepton mass and 200 GeV for the quarks [11] [12].

2. The fourth generation left–handed neutrino is expected to be heavy, and four comparably heavy right–handed neutrinos should occur.

3. All light neutrinos are expected to have masses near their upper limits.

4. Neutrino counting on the Z^0 peak can in principle count a fractional number of neutrino species $3.0 \leq N \leq 4.0$ if the fourth generation neutrino has a mass less than $\frac{1}{2}M_Z \sim 45$ GeV.

The precise triviality bounds, renormalization group fixed points and dynamical mechanisms will be discussed elsewhere.

We thank Josh Frieman for useful discussions. One of us (EAP) wishes to thank W. Bardeen and the Fermilab Theory Group for its hospitality, and the Bundesministerium für Forschung und Technologie for partial support.

References

[1] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, North Holland Pub. (1979);
T. Yanagida, in "*Proc. of the Workshop on Unified Theories and Baryon Number in the Universe*," KEK, Japan (1979).

[2] J. D. Bjorken, S. D. Drell, "*Relativistic Quantum Field Theory,*" McGraw–Hill (1964).

[3] For a recent review see R. Peccei, "*The Physics of Neutrinos*," to appear in Proceedings of the Flavor Symposium, Peking University, Beijing, DESY Preprint 89-043, (1989).

[4] S. Glashow, *Phys. Lett.* **187B**, 367 (1987).

[5] See the Particle Properties Data Book, and ref.[4].

[6] R. Abela, *et al.* , *Phys. Lett. B*, 43 (1984).

[7] ARGUS Collaboration, D. Wegener in *Proc. of the XVI INS Symposium on Neutrino Mass and Related Topics*, Tokyo (1988).

[8] L. A. Ahrens, *et al.* , *Phys. Rev.* **D31**, 2732 (1985);
N. Usida, *et al.* , *Phys. Rev. Lett.* **57**, 1694 (1986);

[9] H. Harari, Y. Nir, *Nucl. Phys.* **B292**, 251 (1987).

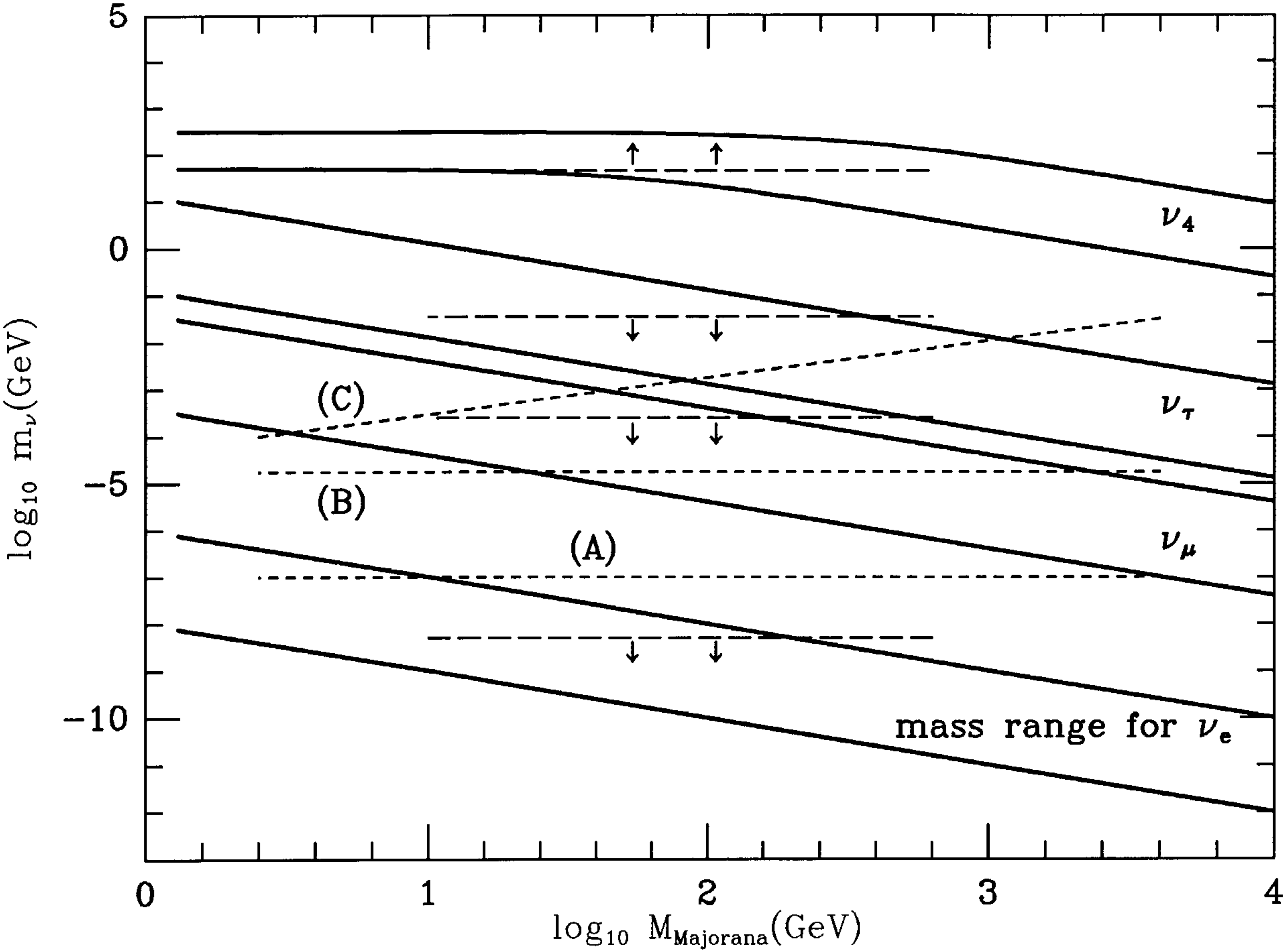
[10] B. W. Lee, S. Weinberg, *Phys. Rev. Lett.* **39**, 165 (1977).

[11] W. A. Bardeen, C. T. Hill, M. Lindner, *Minimal Dynamical Symmetry Breaking of the Standard Model*," FERMI-PUB-89/127-T (1989), and references therein.

[12] W. Marciano, "*Dynamical Symmetry Breaking and the Top Quark Mass*," BNL-43230, (1989).

Figure Caption

Figure (1): Allowed range of neutrino masses for $0.2 \leq \epsilon_i \leq 2.0$ and $50 \leq m_4 \leq 300$ GeV. Experimental upper limits on the light neutrino masses, and the lower limit on the fourth neutrino mass assuming $N_{total} = 3.0$ are indicated by long–dashed lines with double arrows. Cosmological limits are indicated by the dashed lines with labels: (A) closure density constraint for stable neutrinos; (B) lower limit for a familon decay mode $\nu_\mu \to \nu_e + \phi$; (C) lower limit for a Majoron decay mode ([9] with the assumption of strong coupling, $h = 1.0$ and familon decay constant $F = 10^{10}$ GeV).



5
0
−5
−10
$\log_{10}\ m_\nu$(GeV)
0
1
2
3
4
$\log_{10}\ M_{\mathrm{Majorana}}$(GeV)
ν_4
ν_τ
ν_μ
mass range for ν_e
(A)
(B)
(C)