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Gauge Hierarchy and Attractive Feeble Long Range Force

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Abstract

We show that in a gauge theory with gauge hierarchy, a long range attractive feeble force can arise in general within the framework of perturbative field theory. The force is the result of the existence of a Higgs boson whose mass and matter couplings are both naturally suppressed by powers of the hierarchical scale ratio. Some simple but realistic examples are used to demonstrate this mechanism.

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A recent reanalysis[1] of Eötvös experiment[2] has reignited the enthusiasm among physicists in looking for possible deviation from Newtonian gravity which may be resulting from a yet unknown long range force. It has become customary to parametrize the corresponding total potential energy of a two–body system as

$$V(r) = V_{\rm Newton} + \Delta V = -\frac{G_N m_1 m_2}{r}(1+\alpha e^{-r/\lambda}) \quad . \tag{1}$$

In general, α depends on material. Experimental constraints come from various sources[3], namely, laboratory measurments, geophysics, planetary measurements. They cover λ from mm to about 10^{15}m. For λ about 1 m, the present constraint is[4]

$$\alpha(\lambda = 1 \text{ m}) \lesssim 0.001 \quad .$$

The bound becomes tighter as the range increases up to 10 km, where we have [4]

$$\alpha(\lambda = 1 \text{ km}) \lesssim 10^{-5} \quad .$$

In quantum field theory, the range λ is related to the mass of the exchange particle, $\lambda = 1/m_\omega$. To produce a long range force one thus requires a very small mass for the particle that mediates the interaction. It is interesting to ask whether within the context of the present theoretical framework one can naturally generate such an interaction.

Many ideas have been put forward in this direction. Some of them predict a repulsive force[5] mediated by vector particles and some predict an attractive force[6] mediated by scalar particles. But none of them seems to be particularly convincing or simple. In this paper we propose a mechanism involving scalar particles which we think is by far the simplest. The mechanism may be realized in grand unified extension of the standard model without any reference to nonperturbative mechanisms like instanton[7] or gravitational mechanisms like compactification[5].

Clearly to produce a very small mass and a very weak interaction one requires a very small parameter. Such a small parameter is already needed in any grand unified theory (GUT) in order to account for the small ratio between the weak scale and the grand unification scale. It is very interesting to see if one can generate an attractive feeble long range force (AFLRF) using this small ratio called gauge hierarchy. Given the gauge hierarchy in a theory we would like to illustrate a mechanism with which one can generate an AFLRF. Note that we have nothing to illuminate about the gauge hierarchy problem. We also do not bother to maintain minimal fine–tuning. Since whatever solution (like supersymmetry) one finally adopts to cure the gauge hierarchy problem, it probably will not require the fine-tuning to be minimal anyway.

The basic idea is not very different from those put forth in the literature before[6] using pseudo-Goldstone bosons (PGB). One way to have a PGB with very small mass and very weak scalar coupling is as follows. First of all, one needs a low energy continuous global symmetry which is not a symmetry of the high energy theory. Since this symmetry is not maintained at the higher energy scale, one cannot simply impose it in the low energy theory. Therefore the symmetry has to be an accidental symmetry of the renormalizable terms of the low energy theory. A well-known example of a symmetry of this type is the baryon number in the Standard Model embedded in any GUT. Secondly, one has to break this symmetry spontaneously in the low energy scale. (This is the step that is complicated to implement on the baryon number symmetry). The resulting Goldstone boson will in general pick up a small mass suppressed by GUT scale. Aided by the CP violation in the low energy theory, the pseudo-Goldstone nature of this particle will eventually lead to a small scalar coupling which is also suppressed by the GUT scale.

This simple idea however is not so easy to implement simply and realistically. The reason is that, in simple models, it is in general difficult to break an accidental symmetry spontaneously. The Higgs boson that is needed to break the symmetry in the low energy will introduce *explicit renormalizable* symmetry breaking terms in the lagrangian and destroy the symmetry. However there is a way to avoid this, as we shall discuss in this article. The mechanism is to have this low energy global continuous symmetry embedded in the high energy local symmetry. It is much easier to recover part of the high energy symmetry than to produce a new one. When the gauge symmetry is broken at high energy a discrete symmetry survives in the low energy theory. When this discrete symmetry is imposed on the renormalizable terms of the low energy theory, a continuous symmetry arises accidentally as a result. If this accidental symmetry were exact, a Goldstone particle would result from the spontaneous breaking of this symmetry. However, the symmetry is exact only in the renormalizable terms of the low energy theory. The full low energy effective theory contains non-renormalizable terms as well, which are damped by inverse powers of the high energy scale. Since in the *high energy* world the continuous symmetry is already broken spontaneously, such terms do not necessarily obey the symmetry. Hence, after the *low energy* spontaneous breaking, we would not obtain a true massless Goldstone boson. Rather, a PGB would arise, with a mass inversely proportional to some power of the high energy scale.

Even a massive PGB does not guarantee an AFLRF. In addition, one has to break the CP symmetry which forbids a pseudo-scalar particle from developing a scalar coupling with fermions. This is not difficult to achieve in general. However, depending on the mechanism of CP violation, it may result in extra suppression factors for the strength of the AFLRF. In particular, one needs this CP violation to be a low energy effect so as to avoid additional powers of suppression due to the gauge hierarchy.

In the following, we will present simple models that realize this scenario. We like to keep the models as simple as possible in order to make the mechanism more transparent. In particular, we shall not use any of the popular gauge groups for GUT. The simplicity of our models should convincingly illustrate that the mechanism can be implemented in any GUT.

The mechanism is to extend the electroweak model to include additional local gauge symmetry which is spontaneously broken at the very high energy scale Λ_G. As the simplest possible high energy symmetry which illustrates the idea, we shall take the extension to be just an additional $U_G(1)$. The interaction at Λ_G scale affects the low energy physics through some light ζ fields which carry $U_G(1)$ charge. We assume that a remnant discrete sysmetry $\mathbf{Z}_n \subset U_G(1)$ remains unbroken below the Λ_G scale,

$$\mathbf{Z}_n : \zeta \rightarrow e^{2\pi m i/n} \zeta \quad , \tag{2}$$

where m and n are mutually prime. For $n \geq 5$, this discrete symmetry guarantees that the low energy *renormalizable* interactions respect an accidental global symmetry $U_\Omega(1)$, $\zeta \rightarrow e^{i\theta}\zeta$.

To generate AFLRF, one still needs low energy CP violation. One can in principle use the Kobayashi-Maskawa(K-M) mechanism in the standard model for this purpose. However, in that case, the mixing will be suppressed by high powers of light fermion masses and some loop suppression factors. This is a result of GIM suppression and the fact that in the K-M mechanism CP violation will disappear if any of the up- or down-quark mass differences are zero. Therefore an alternative mechanism of CP violation is needed to avoid these suppression factors. To implement the CP nonconservation in the Higgs sector, we shall introduce two *light* $SU(2) \times U(1)$ singlets ζ_1 and ζ_2 besides the usual doublet ϕ. For the sake of simplicity, we assume that ζ_1 and ζ_2 have same quantum numbers of the gauge group $U_G(1)$.

Now consider first the renormalizable terms in the Lagrangian. We can arrange that the $U_\Omega(1)$ is spontaneously broken at low energy by $\langle\zeta_i\rangle$. Since two ζ's are identical, one can make linear combinations such that $\langle\zeta_2\rangle = 0$. In the *renormalizable* low energy Lagrangian.

we can use $U_\Omega(1)$ symmetry to rotate a phase away, *i.e.* we can set $Im\,\langle\zeta_1\rangle = 0$. In such convention, the Goldstone boson ω is simply given by $Im\ \zeta_1$. The $Im\,\zeta_2$ will mix with the usual Higgs boson, $Re\phi^0$, via the CP nonconserving interactions

$$C_{ij}(\phi^\dagger\phi)(\zeta_i^\dagger\zeta_j) + D_{klmn}(\zeta_k^\dagger\zeta_l^\dagger\zeta_m\zeta_n) + \text{h.c.} \quad , \tag{3}$$

with complex coefficients C_{ij} and D_{klmn}. Note that after spontaneous symmetry breaking, there is no tadpole term induced for ζ_2 in the convention $\langle\zeta_2\rangle = 0$.

From the discussion so far, it might seem that the Goldstone boson ω is massless and does not directly couple to fermions, whereas $Im\zeta_2$ couples to fermions in the scalar (not pseudoscalar) form through the mixing with the ordinary neutral Higgs boson. But this is so because we have been talking only of the renormalizable terms in the low-energy effective Lagrangian. In addition, interactions at the Λ_G scale give rise to *non–renormalizable* terms in the effective Lagrangian of the form

$$\sum_{i=0}^{n}\beta_i\zeta_1^{n-i}\zeta_2^i/\Lambda_G^{n-4} + \quad \text{h.c.} \tag{4}$$

Notice that the power n is governed by the discrete symmetry $\mathbf{Z}_n$. The damping by inverse powers of the Λ_G scale comes from dimension counting. Such terms do not obey the $U_\Omega(1)$ symmetry. A tiny mass will be induced for ω,

$$m_\omega^2 \simeq \beta_0\left(\frac{\langle\zeta_1\rangle}{\Lambda_G}\right)^{n-4}\langle\zeta_1\rangle^2 \quad . \tag{5}$$

Also, mixing occurs among $Im\ \zeta_1$ and $Im\ \zeta_2$ such that the pseudo-Goldstone boson ω picks up scalar couplings to fermions,

$$\beta_1\sin\theta_{CPX}\left(\frac{\langle\zeta_1\rangle}{\Lambda_G}\right)^{n-4}\ \frac{m_f}{\langle\phi^0\rangle}\bar{f}f\omega \quad . \tag{6}$$

where $\sin\theta_{CPX}$ parametrizes the mixing between $Im\zeta_2$ and $Re\phi^0$.

The key ingredient of the program is to embed the discrete symmetry $\mathbf{Z}_n$ inside the gauge group $U_G(1)$. Here we give one typical example of $n = 6$. The example contains heavy fields S, z and the light ζ described above. They are all electroweak singlets. The $U_G(1)$ charges of them are,

$$Q_G(S) = 6 \quad , \quad Q_G(z) = 2 \quad , \quad Q_G(\zeta) = 1 \quad . \tag{7}$$

The Higgs potential will contain uninteresting interactions which are invariant under the separate phase changes of each of the scalar fields above. Apart from them, there are the following non-trivial terms :

$$V' = hS^* z^3 + kz^* \zeta^2 + \quad H.c. \quad , \tag{8}$$

with h and k as coupling constants. The vacuum expectation value (VEV) $\langle S \rangle$ at the Λ_G scale breaks the gauge symmetry $U_G(1)$ but preserves the discrete symmetry $\mathbf{Z_6}$,

$$S \rightarrow S, \quad z \rightarrow e^{4\pi i/6} z, \quad \zeta \rightarrow e^{2\pi i/6} \zeta \quad . \tag{9}$$

We shall fine-tune such that $\langle \zeta \rangle \ll \langle S \rangle$. This is just part of the usual fine-tuning that gives rise to the gauge hierachy. It keeps the fields ζ light whereas z and S acquire masses of order Λ_G. The heavy field z will pick up a very small VEV driven by $\langle \zeta \rangle$ as specified by the second term in Eq.(8),

$$\langle z \rangle \sim k \langle \zeta \rangle^2 / \Lambda_G^2 \quad . \tag{10}$$

Interesting effective *non-renormalizable* low energy interactions of the type in Eq.(4) will be induced by the diagram in Fig. 1 with the induced amplitude

$$(k^3 h / \Lambda_G^5) \zeta^6 \quad . \tag{11}$$

After ζ develops VEV, this produces a mass for the ζ field given by

$$m_\omega \sim \left(\frac{k}{\Lambda_G} \right)^{\frac{3}{2}} \frac{\sqrt{h} \langle \zeta \rangle^2}{\Lambda_G} \sim \frac{\sqrt{h} \langle \zeta \rangle^2}{\Lambda_G} \quad , \tag{12}$$

where in the last step, we have used the fact that the dimensional parameter k is naturally of order Λ_G without any additional fine-tuning. One can now obtain an AFLRF if, as shown before, one introduces another copy of ζ in order to produce the scalar coupling with fermions.

It is straightforward to modify this procedure for an alternative model using $\mathbf{Z}_5$. In this case, we can use the following charge assignments :

$$Q_G(S) = 5 \quad , \quad Q_G(z) = 1 \quad , \quad Q_G(\zeta) = 2 \quad . \tag{13}$$

The non-trivial terms in the Higgs potential are

$$V' = hS^* \zeta^2 z + \sqrt{2} m \zeta^* z^2 + H.c. \tag{14}$$

As in the previous example, a discrete $\mathbf{Z}_5$ symmetry remains unbroken when S develops a VEV. This ensures a $U_\Omega(1)$ symmetry for the renormalizable terms involving the light field ζ, but allows non-renormalizable ζ^5 couplings with coupling constant of order h^2/Λ_G as illustrated in Fig. 2. In the notation of Eq.(4), $h^2 = \beta_0$. After ζ develops VEV. we thus obtain

$$m_\omega \sim h\langle\zeta\rangle^{3/2}/\Lambda_G^{1/2} \quad , \tag{15}$$

which has the form of Eq.(5).

One may suspect the reliability of these order of magnitude estimates, especially for physics involving Goldstone bosons. We have actually checked these estimates with detailed calculations to be presented elsewhere.[8]

To consider the phenomenological implications, we first consider the $\mathbf{Z}_5$ model. The mass of the PGB is

$$m_\omega \sim \langle\zeta\rangle \sqrt{\beta_0 \frac{\langle\zeta\rangle}{\Lambda_G}} \quad . \tag{16}$$

If, for example, $\langle\zeta\rangle \sim 1\,\mathrm{MeV}$, $\beta_0 = 0.01$ and $\Lambda_G \sim 10^{17}\mathrm{GeV}$, we get

$$m_\omega \sim 10^{-5}\mathrm{eV} \quad ,$$

which corresponds a force of range about a centimeter. It has a strength at a detectable level as a correction to the gravitational force,

$$\alpha \sim \left(\frac{\Lambda_{\mathbf{Planck}}}{\Lambda_G}\right)^2 \left(\frac{\langle\zeta\rangle}{\langle\phi^0\rangle}\right)^2 \left(\frac{m_{\mathbf{quark}}}{m_{\mathbf{nucleon}}}\right)^2 \times \beta_1^2 \sin^2\theta_{CPX} \quad .$$

The present experiments[3] are sensitive to a strength of order 10^{-4} for such a range λ about centimeters,

$$\alpha(\lambda = 1\mathrm{cm}) \lesssim 10^{-4} \quad .$$

The range λ here is very short for the experimental test even though we have squeezed the parameter $\langle\zeta\rangle$ to be unnaturally small.

If we consider the $\mathbf{Z}_6$ model, we can easily obtain a longer range about meters to kilometers. However, in the version that we present above, the strength is also further suppressed by a factor $(\langle\zeta\rangle/\Lambda_G)^2$. In general, for the class of models of arbitrary n discussed above, we have the following relation where the Λ_G dependence has been cancelled out,

$$\alpha \left(\frac{\langle\zeta\rangle}{m_\omega}\right)^4 \sim \left(\frac{\beta_1 \sin\theta_{CPX}}{\beta_2}\right)^2 \left(\frac{\Lambda_{\mathbf{Planck}}}{\langle\phi^0\rangle}\right)^2 \left(\frac{m_{\mathbf{quark}}}{m_{\mathbf{nucleon}}}\right)^2 \quad . \tag{17}$$

Thus we may arrive at a force either of too short range *or* too weak strength for the current experiment to test.

The situation may change as the experimental accuracy has been improved quite quickly in this area. However it will be interesting to have a way of producing a stronger force within the same spirit of naturalness. Before we conclude we like to illustrate such a mechanism. Note that the existence of the non-renormalizable terms serves two purposes: it gives a small mass to the PGB and at the same time provide the mixing between PGB and others Higgs which would have been prohibited had the PGB been an exact GB. The mixing eventually provides the scalar coupling to the fermion. Since the two effects are coming from the same term in our simple model, it results in a correlation between the range and the strengh of the force.

What we need is a mechanism to suppress the mass of the PGB without suppressing the mixing which controls the strength. The mass can be interpreted as diagonal mixing. Therefore, in order to achieve stronger strength one needs to suppress the tree level diagonal mixing of the PGB. Then the PGB will pick up mass only through off-diagonal mixing with other scalar bosons through a kind of see-saw mechanism that has become standard practice in neutrino physics. The suppression of diagonal mixing can in general be achieved by additional symmetry or an elaborate choice of Higgs representations. So far we have not found any simple model of this type. We include the idea here in the hope that others may eventually find an interesting, simple realization of the idea.

In light of many recent experiments to search for long range feeble force, it is nice to know that a force of such kind can very well exist within the context of the grand unified gauge theories. We like to re-emphasize that we have not used more finetuning than what is common in grand unified theory. The feebleness and the long range character of the force are direct consequenses of the gauge hierarchy already existing in any grand unified theories. The feebleness of these force within the simplest implementation of the idea (i.e. without the see-saw mechanism mentioned earlier) easily keep it within the limit the recent null experiments. However, we believe the strength that the mechanism predicts is within reach of the experiments in the near future.

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FIGURE CAPTIONS

Fig. 1 The induce vertex ζ^6 in the $\mathbf{Z}_6$ model of Eq.(7).

Fig. 2 The induce vertex ζ^5 in the $\mathbf{Z}_5$ model of Eq.(13).

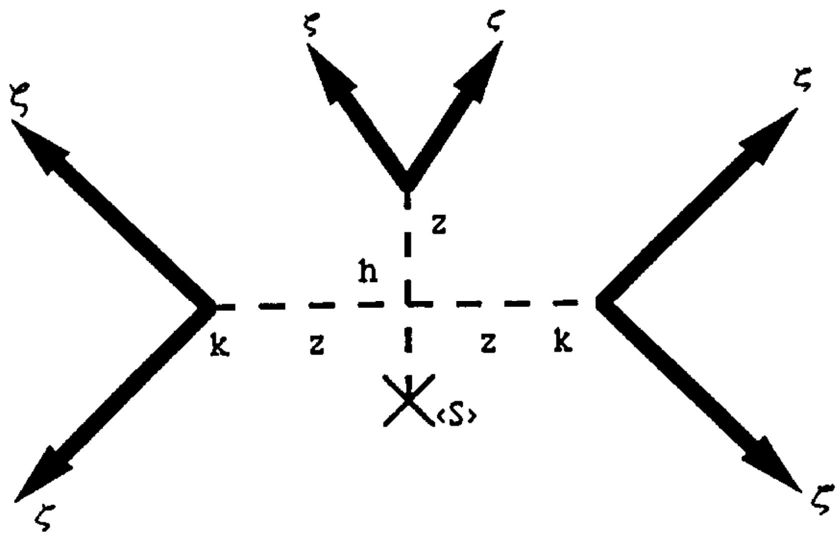

Fig. 1

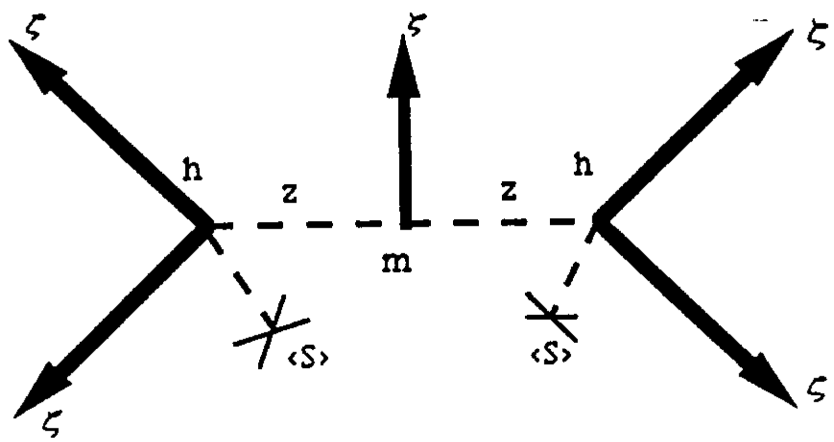

Fig. 2