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The Static Approximation, Staggered Fermions and f_B

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Abstract

Matrix elements of heavy-light bilinears measured on the lattice are compared to their continuum counterparts. The heavy quark is treated using the static effective field theory, and the light quark is treated as a staggered fermion. The time component of the axial current is the bilinear used to determine f_B. We derive identities which simplify the calculations involving staggered fermions by reducing them to calculations involving naive fermions.

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1. Introduction

Lattice calculations involving direct simulation of the b quark are beyond the reach of current calculational power. However many matrix elements of interest involving the b quark, including that which determines f_B, have the following two properties: (i) the large rest energy of the b quark is not transferred to lighter hadrons, and (ii) the b quark's spatial momentum momentum is small. An effective field theory action valid for matrix elements having these properties is presented in a recent paper by Eichten and Hill [1], which brings together lines developed by Eichten and Feinberg [2], Caswell and Lepage [3], and Politzer and Wise [4]. The remaining scales in the effective field theory are small enough that these matrix elements can be calculated reliably on the lattice.

Using the static approximation for the heavy quark propagator for the lattice determination of f_B and the B meson B parameter was proposed by Eichten [5]. The derivation of the heavy quark propagator on the lattice from an effective field theory action, and the calculation of the perturbative corrections to the matrix element of an arbitrary heavy-light bilinear measured on the lattice with the light quark treated as a Wilson fermion, will be presented in reference [6]. These corrections for the bilinear determining f_B have also been studied by Boucaud, Lin and Pène [7].

In this paper we obtain the perturbative corrections to the matrix element of an arbitrary heavy-light bilinear measured on the lattice with the light quark treated as a staggered fermion. The matching is done in two steps. First the matching between the full theory and the effective theory in the continuum is done, and then the continuum effective theory is matched to the lattice regularized effective theory. Also, rather than directly calculating the perturbative corrections to the Green's functions involving staggered fermions, we will demonstrate that in the continuum limit they can be reduced to Green's functions involving naive fermions, a substantial simplification.

The organization of the paper is as follows: In section two, we will review continuum results for the first step of the matching. In section three, we will give the discretization of the theory, including a brief review of staggered fermions, and propose a discretization of the bilinear. In section four, we will reduce the calculation of the Green's functions involving staggered fermions to Green's functions involving naive fermions. In section five, we conclude by using these results to do the matching between the continuum effective theory and the lattice effective theory and evaluate the result for the bilinear used to determine f_B.

2. The Static Effective Field Theory

The Euclidean static effective field theory action is [1]

$$S_E = \int d^4x\, b^\dagger (i\partial_0 + gA_0) b. \tag{2.1}$$

The two-component field b annihilates heavy quarks and $b^\dagger$ creates them. A fixed momentum $(m, 0)$ has been removed from the momentum of the heavy quark. The most general heavy-light bilinear in the full theory in the continuum is $\bar{b}\Gamma q$. Γ is any Dirac matrix, and q is the light quark field. Parametrize Γ by two-by-two blocks:

$$\Gamma = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}. \tag{2.2}$$

The corresponding operator in the static effective theory is

$$b^\dagger (\alpha\ \beta) q. \tag{2.3}$$

The ratio of the matrix elements of these operators in the effective theory to their counterparts in the full theory was calculated in reference [1]. The calculation was done to order α_S and the operators were renormalized using $\overline{\mathrm{MS}}$. The ratio was determined by comparing the matrix element of the bilinear between an incoming light quark and an outgoing heavy quark. For calculational convenience, the light quark mass was set to zero and the mass shell point was taken to be external momenta equal to zero. The infrared divergences that appear at this point were eliminated by giving the gluon a mass, which is acceptable because all the diagrams are QED-like. The result is

$$1 - \frac{g^2}{12\pi^2}\left(C_1 \ln\frac{m^2}{\mu^2} + C_2\right), \tag{2.4}$$

where $C_1 = 5/2 - H^2/4$ and $C_2 = -4 + 3H^2/4 - HH' - GH/2$. G is defined by $G\Gamma = \gamma_0\Gamma\gamma_0$, H is defined by $H\Gamma = \gamma_\mu\Gamma\gamma_\mu$, and H' is the derivative with respect to d of H in d dimensions. The dependence on the gluon mass dropped out of the ratio, as would the dependence on the external momenta and the light quark mass, had they been included.

What remains to be calculated to determine the ratio of the operator in the lattice regularized effective theory to the operator in the full theory in the continuum, is the ratio of the operator in the lattice regularized effective theory to the continuum effective theory. First we describe the lattice regularized theory with the light fermion treated as a staggered fermion.

3. Discretization

A discretization of the effective field theory action for the heavy quark is [6]

$$S_E = ia^3 \sum_n b^\dagger(n)\Big(b(n) - U_0(n-\hat{0})^\dagger b(n-\hat{0})\Big). \tag{3.1}$$

While many discretizations of the action are possible, this one yields the static quark propagator currently in use in lattice calculations [8] and has no doubling problem. Before proposing a discretization of the bilinear (2.3), we briefly review staggered fermions [9] to establish notation and emphasize features that are important in reducing our calculations to perturbative calculations involving naive fermions.

That one might hope to reduce perturbative calculations involving staggered fermions to calculations involving naive fermions is clear from the way staggered fermions are constructed [10]. One begins with the naive fermion action:

$$\frac{i}{2a}a^4 \sum_{n,\mu} \overline{\psi}(n)\,\gamma_\mu\Big(\psi(n+\widehat{\mu}) - \psi(n-\widehat{\mu})\Big). \tag{3.2}$$

The γ_μ are hermitian. We have taken the fermions massless and left out the gauge links. Next "spin diagonalize" the action by the change of variables

$$\psi(n) = \Gamma_n\,\chi(n), \quad \text{where} \quad \Gamma_n = \gamma_1^{n_1}\ldots\gamma_d^{n_d}. \tag{3.3}$$

The field $\overline{\chi}(n)$ is defined as if it were the hermitian conjugate of $\chi(n)$. Since the transformation acts only on spinor indices there is no obstacle to introducing the gauge fields at any point. The resulting action is

$$\frac{i}{2a}a^4 \sum_{n,\mu} c_\mu(n)\,\overline{\chi}(n)\Big(\chi(n+\widehat{\mu}) - \chi(n-\widehat{\mu})\Big). \tag{3.4}$$

The action involves four one-component fermion fields on each site which couple to one another only through the gauge fields. The seeming complexity of staggered fermions arises because the spin-diagonalization results in the position dependent phase $c_\mu(n)$ appearing in the action. Fortunately, the only property of the spin-diagonalized action we need is that it is diagonal in the χ field index and it is identical for each value of the index. Now three of the four χ fields are eliminated. The reduction in the number of fermion fields by a factor of four leaves us with a theory describing four "flavors" instead of sixteen.

To study processes with external fermions, we need one final step: the construction of spinor fields out of the remaining one-component χ field. The field will be a matrix carrying two indices: a spinor index and a "flavor" index. The field is only defined on coarse lattice sites, that is sites for which $n = 2N$:

$$Q(2N) = \frac{1}{C^{3/2}} \sum_\eta \Gamma_\eta\,\chi(2N+\eta). \tag{3.5}$$

$C \equiv 2^{d/2}$ is equal to four in four dimensions, η is a vector with entries of 0 or 1, and Γ_η is defined just like Γ_n. $\overline{Q}$ is defined as if it were the hermitian conjugate of Q. When the gauge fields are introduced, a product of gauge links, $U_\eta(2N)$, can be inserted in the sum so that the field transforms under gauge transformations as a field living at site $2N$. To summarize, the essence of staggered fermions is spin-diagonalization of the naive fermion theory followed by reduction to a single-component fermion field.

An obvious discretization of the bilinear (2.3) to consider is

$$\begin{aligned} J(2N) &= b^\dagger(2N)(\alpha\ \beta)Q(2N) \\ &= \frac{1}{C^{3/2}} \sum\nolimits_\eta b^\dagger(2N)(\alpha\ \beta)\Gamma_\eta\, \chi(2N+\eta). \end{aligned} \tag{3.6}$$

However, this bilinear has two disadvantages. One is that it is not a "zero-distance" bilinear when written in terms of the fundamental fields. When the gauge fields are introduced, gauge links would have to be inserted into the bilinear in order to make it gauge-invariant. This increases the number of diagrams that must be calculated in perturbation theory. Although there are sometimes reasons associated with current conservation for considering extended operators, they do not apply here. The second disadvantage is that the bilinear only depends on the heavy quark field at a single point of the hypercube that is labelled by the coarse lattice site $2N$. Generally an operator that depends more uniformly on the fields at all 16 points of the hypercube will have better numerical behavior.

We eliminate both these disadvantages by proposing another operator which has the same naive continuum limit:

$$J_\Gamma(2N) = \frac{1}{C^{3/2}} \sum\nolimits_\eta b^\dagger(2N+\eta)(\alpha\ \beta)\Gamma_\eta\, \chi(2N+\eta). \tag{3.7}$$

We have subscripted the bilinear by Γ, the Dirac matrix which parametrizes the corresponding bilinear in the full theory. Note that the bilinear still carries a "flavor" index; if the staggered fermion field is describing four "flavors" of d quark, there are four neutral B mesons, and the index on $J_\Gamma(2N)$ determines which of these the bilinear couples to.

Having established the discretization of the action and the bilinears that we will use, we enumerate the Green's functions that must be calculated to obtain the ratio of the matrix element of the operator in the effective theory in the continuum to its counterpart on the lattice. Since we will use the same mass shell point and method of regulating infrared divergences as in reference [1], there are no new continuum computations to do. The lattice computations required are heavy quark wave function renormalization, the renormalization of the field used to create the light quark, and the Green's function containing the insertion of the discretized bilinear.

Details of the evaluation of the graphs contributing to heavy quark wave function renormalization, figure 1, as well as subtleties associated with linear divergences in the mass renormalization of the heavy quark, will be discussed in reference [6]. The contribution to heavy quark wave function renormalization from these graphs is

$$\frac{g^2}{12\pi^2}\left(-2\log\lambda^2 a^2 + e\right), \tag{3.8}$$

where $e = 24.47$.

The only Green's functions that remain to be obtained are the one determining the renormalization of the field used to create the light quark, and the one with the insertion of the discretized bilinear. These Green's functions both involve the staggered fermion field. We could attempt to directly calculate them. However, the position-dependent phase in the spin-diagonalized action, as well as the the position-dependent matrix, Γ_η, appearing in the bilinear, make this approach somewhat difficult. Instead, we will demonstrate that in the continuum limit these Green's functions can be reduced to Green's functions involving naive fermions.

4. Reduction of Green's Functions

It is the close relation of the staggered fermion theory to the naive fermion theory that makes it possible to relate their respective Green's functions. Sharatchandra, Thun and Weisz [11], in an early perturbative calculation involving staggered fermions, demonstrated a simple relation for the Green's functions they had to calculate, all of which had no external fermions. They introduced a projection formalism rather than explicitly solving for the remaining single component fermion field. They found that if they divided by a factor of four for each internal fermion loop, they could use naive fermion Feynman rules.

The result we obtain for our Green's functions, which do have external staggered fermions and insertions of a bilinear containing a staggered fermion field, is somewhat similar. We do not use the projection formalism, as we need to explicitly build spinor fields creating external fermions out of the one component fermion field. The relationship we derive between the staggered fermion Green's functions and the naive fermion Green's functions only holds in the limit that the lattice spacing goes to zero, which is the limit of interest. The proof is valid to any order in perturbation theory, and is clearly generalizable to a large class of Green's functions.

We give the derivation of the reduction in detail for the Green's function containing the insertion of the discretized bilinear:

$$
\begin{aligned}
G_{KS}(n, 2N', 2N) &= \langle b(n)\, J_\Gamma(2N') S\, \overline{Q}(2N)\rangle \\
&= \frac{1}{C^3} \sum\nolimits_{\eta',\eta} \langle b(n) b^\dagger(2N'+\eta')(\alpha\ \beta)\Gamma_{\eta'}\chi(2N'+\eta') S \overline{\chi}(2N+\eta)\Gamma_\eta^\dagger\rangle.
\end{aligned}
\tag{4.1}
$$

The matrix S is an arbitrary matrix contracted with the "flavor" indices. We will soon see that in the continuum limit only the diagonal elements of S contribute. Notice that the interpolating field we use to create the light quark is the field $\overline{Q}(2N)$ defined as in equation (3.5), without the insertion into the sum of gauge links $U_\eta(2N)$ designed to make it transform as a field that lives at the coarse lattice site $2N$. Of course the choice of the interpolating field for the light quark cannot affect our result for the normalization of the bilinear. If we had included the gauge links in the definition of the field, we would have additional graphs to evaluate, but they would exactly cancel with additional graphs that would then appear in the calculation of the renormalization of the interpolating field.

The first step of the reduction is to reintroduce the three other χ fields. Instead of thinking of $\chi(2N'+\eta')\overline{\chi}(2N+\eta)$ as a scalar we think of it as a matrix proportional to the identity and place it in the matrix multiplication between $\Gamma_{\eta'}$ and S. The only effect of this reintroduction on $G_{KS}(n, 2N', 2N)$ is that there are now four fields χ that can propagate in any internal fermi loop. Since the action for each of these fields is identical, this can be compensated for in perturbation theory by dividing by a factor of four for each internal fermion loop.

The second step of the reduction is to undo the spin diagonalization (3.3). This change of variables transforms equation (4.1) to

$$
\begin{aligned}
&G_{KS}(n, 2N', 2N) = \\
&\qquad \frac{1}{C^3} \sum\nolimits_{\eta',\eta} \langle b(n) b^\dagger(2N'+\eta')(\alpha\ \beta)\psi(2N'+\eta')\overline{\psi}(2N+\eta)\rangle \Gamma_\eta S \Gamma_{\eta'}^\dagger,
\end{aligned}
\tag{4.2}
$$

where the action for ψ is the naive fermion action (3.2).

Now Fourier transform (4.2) by setting $N' = 0$, multiplying by $(2a)^4 e^{-ik\cdot 2Na}$ and $a^4 e^{ip\cdot na}$, and summing on N and n. Thus momentum k enters at $2N$ and momentum p leaves at n. $G_{KS}(p, k)$ so defined is doubly periodic in k. We also introduce $G_\Gamma(p, k)$, the corresponding Green's function in the naive fermion theory. The result is

$$
\begin{aligned}
&G_{KS}(p, k) = \\
&\qquad \frac{1}{C^3} \sum\nolimits_{\eta',\eta,\tau} e^{-i(k+\pi_\tau - p)\cdot\eta' a} e^{i(k+\pi_\tau)\cdot\eta a}\, G_\Gamma(p, k+\pi_\tau)\, \Gamma_\eta S \Gamma_{\eta'}^\dagger.
\end{aligned}
\tag{4.3}
$$

Like η' and η, the vector τ has entries of zero or one, while $\pi_\tau \equiv (\pi/a)\tau$. We see there are contributions from the naive fermion Green's function from sixteen points in the Brillouin zone.

We now use the doubling symmetry present in the naive theory to rewrite

$$G_\Gamma(p, k+\pi_\tau) = G_{\Gamma M_\tau}(p,k)M_\tau^\dagger, \quad \text{where} \quad M_\tau \equiv (i\gamma_1\gamma_5)^{\tau_1}\cdots(i\gamma_d\gamma_5)^{\tau_d}. \tag{4.4}$$

At this point it is convenient to parametrize the matrix S as M_σ. We have

$$\begin{aligned} &G_{KS}(p,k) = \\ &\quad \frac{1}{C^3}\sum\nolimits_{\eta',\eta,\tau} \mathrm{e}^{-i(k+\pi_\tau-p)\cdot\eta' a}\mathrm{e}^{i(k+\pi_\tau)\cdot\eta a}\, G_{\Gamma M_\tau}(p,k)M_\tau^\dagger\Gamma_\eta M_\sigma\Gamma_\eta^\dagger. \end{aligned} \tag{4.5}$$

This expression for $G_{KS}(p,k)$ is exact. The only modification of the naive fermion Feynman rules used in calculating $G_{\Gamma M_\tau}(p,k)$ appearing on the right hand side is the division by a factor of four for each fermion loop.

To make further simplifications, we perform manipulations valid only in the continuum limit. Since the naive fermion Green's function appearing in (4.5) is log divergent by power counting, we can neglect all but the zeroth order piece in $\mathrm{e}^{i(k-p)\cdot\eta' a}$ and $\mathrm{e}^{ik\cdot\eta a}$ in the $a \to 0$ limit. Then apply the following identity:

$$\sum\nolimits_\eta \mathrm{e}^{i\pi_\tau\cdot\eta a}\Gamma_\eta M_\sigma \Gamma_\eta^\dagger = C^2\delta_{\sigma,\tau}M_\sigma. \tag{4.6}$$

The remaining summation on η' is trivial and we obtain,

$$G_{KS}(p,k) = C\delta_{\sigma,0}G_\Gamma(p,k)\big(1+\mathcal{O}(pa,ka)\big). \tag{4.7}$$

We see there is no contribution unless $\sigma = 0$. In terms of the original "flavor" matrix S, the statement is that only the diagonal elements of S contribute in the continuum limit, and their contribution is given by the Green's function in the naive fermion theory, with the rule that for each fermion loop one divides by a factor of four.

The contribution to $G_\Gamma(p,k)$ at $p = k = 0$ from figure 2 is the zeroth order result times

$$\frac{g^2}{12\pi^2}\left(-\ln\lambda^2 a^2 + d\right). \tag{4.8}$$

The evaluation of figure 2 for naive fermions is similar to the evaluation of the vertex correction presented in [6]. The method of isolating $a \to 0$ divergences introduced by Bernard, Soni and Draper [12] was used, and the constant d was evaluated numerically using VEGAS [13]. We find $d = 8.79$. The error in this and other quantities evaluated by Monte Carlo integration are order one in the last decimal place.

Similar manipulations can be applied to the Green's function determining the renormalization of the field creating the light quark:

$$G_{KS}(2N', 2N) = \langle Q(2N')S\overline{Q}(2N)\rangle. \tag{4.9}$$

Because of the chiral symmetry, which is only softly broken even if the fermion has a mass, there are no linear divergences in the naive fermion Green's function and there is no problem neglecting the $\mathcal{O}(ka)$ terms. The result analogous to (4.7) is

$$G_{KS}(k) = C\delta_{\sigma,0}G(k)\Big(1 + \mathcal{O}(ka)\Big). \tag{4.10}$$

Thus the renormalization of the interpolating field we used to create any of the four "flavors" of light quark is obtained directly from the wave function renormalization of a naive fermion.

Calculation of naive fermion wave function renormalization is contained in [14]. We find a contribution of

$$\frac{g^2}{12\pi^2}\Big(\ln \lambda^2 a^2 + f\Big), \tag{4.11}$$

where $f = 6.54$. We now have all the quantities required to compare the lattice and continuum effective theories.

5. Conclusions

The ratio of the matrix element of the discretized operator measured on the lattice to the operator in the continuum effective theory is

$$\begin{aligned} 1 &+ \frac{g^2}{12\pi^2}\Big[-\log \lambda^2 a^2 + d + \frac{1}{2}\Big(-2\log \lambda^2 a^2 + e\Big) + \frac{1}{2}\Big(\log \lambda^2 a^2 + f\Big)\Big] \\ &- \frac{g^2}{12\pi^2}\Big[-\log \frac{\lambda^2}{\mu^2} + D + \frac{1}{2}\Big(-2\log \frac{\lambda^2}{\mu^2} + E\Big) + \frac{1}{2}\Big(\log \frac{\lambda^2}{\mu^2} + F\Big)\Big]. \end{aligned} \tag{5.1}$$

The terms in the first set of brackets are the lattice contributions from the vertex correction (4.8), heavy quark wave function renormalization (3.8), and light quark wave function renormalization (4.11). The terms in the second set of brackets are the corresponding contributions from the continuum graphs. The evaluation of D, E and F is presented in reference [1]. We find $D = 1$, $E = 0$ and $F = 1/2$. The dependence on the gluon mass, λ, drops out of the difference, as would the dependence on the light quark mass and the external momenta, had they been included. Note that unlike the continuum matching, the matching between the lattice effective and continuum effective theories does not depend on Γ. We can simplify (5.1) by taking $\mu = 1/a$. The result is

$$1+\frac{g^2}{12\pi^2}\Big[d+\frac{1}{2}e+\frac{1}{2}f-\Big(D+\frac{1}{2}E+\frac{1}{2}F\Big)\Big]. \tag{5.2}$$

To obtain the ratio of the matrix element of a discretized bilinear measured on a lattice with spacing a to its $\overline{\mathrm{MS}}$ subtracted counterpart in the full theory with $\mu = 1/a$, we take the product of the two ratios, (2.4) and (5.2).

As an example, we compare the ratio of the bilinear used to measure f_B on the lattice to its counterpart in the continuum. If we use the extension of the gamma matrix algebra that γ_5 anticommutes with all the γ_μ, $1 \le \mu \le d$, then for $\Gamma = \gamma_0\gamma_5$, $G = -1$, $H = 2$ and $H' = 1$. So in this case, we have $C_1 = 3/2$ and $C_2 = -2$. Using the continuum value of α_S with $\Lambda_{\overline{MS}} = 200\,\mathrm{MeV}$ for four active quarks, $\mu = 2\,\mathrm{GeV}$, and $m = 5\,\mathrm{GeV}$ we find that expression (2.4) is 0.97. Evaluating expression (5.2) for $\beta \equiv 6/g^2 = 6.0$ gives 1.19. Taking the product of the two ratios, we find that the value of f_B measured on a 2 GeV lattice using the static approximation should be reduced by a factor of 1.16.

Although it is in principle a higher order correction in α_S, the largest source of uncertainty comes from whether to use the lattice or continuum value of α_S in the lattice to continuum effective theory matching since α_S for the lattice is nearly a factor of three smaller than α_S in the continuum even at a matching scale of 2 GeV [11][15]. From experience with operators where the result is known and because the origin of the largest corrections in (5.2) are from the lattice diagrams, it is standard to use the lattice value [16].

We have calculated the relation between the matrix element of an arbitrary lattice-regularized heavy-light bilinear and its continuum counterpart, including the bilinear used to determine f_B, using a method which minimized the difficulty of staggered fermion perturbation theory. The reduction is valid to any order in perturbation theory, should be generalizable to a wide class of Green's functions, and may simplify the calculation of perturbative corrections to other operators [17].

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References

[1] E. Eichten and B. Hill, FERMILAB-PUB-89/184-T, to appear in Phys. Lett. B.

[2] E. Eichten and F. Feinberg, Phys. Rev. D **23** (1981) 2724.

[3] W. E. Caswell and G. P. Lepage, Phys. Lett. **167B** (1986) 437.

[4] H. David Politzer and Mark B. Wise, Phys. Lett. B **208** (1988) 504.

[5] E. Eichten, in *Field Theory on the Lattice,* edited by A. Billoire *et al,* Nucl. Phys. B (Proc. Suppl.) **4** (1988) 170.

[6] E. Eichten and B. Hill, FERMILAB-PUB-89/209-T, in preparation.

[7] Ph. Boucaud, C. L. Lin, and O. Pène, Phys. Rev. D **40** (1989) 1529, see reference [6] for criticism of their lattice results.

[8] Ph. Boucaud, O. Pène, V. J. Hill, C. T. Sachrajda, and G. Martinelli, Phys. Lett. B **220** (1989) 219;
E. Eichten, FERMILAB-CONF-89/211-T.

[9] J. Kogut and L. Susskind, Phys. Rev. D **11** (1975) 395;
T. Banks, J. Kogut, and L. Susskind, Phys. Rev. D **13** (1976) 1043;
L. Susskind, Phys. Rev. D **16** (1977) 3031.

[10] F. Gliozzi, Nucl. Phys. B **204** (1982) 419, for an algebraic construction;
H. Kluberg-Stern, A. Morel, O. Napoly, and B. Petersson, Nucl. Phys. B **220** (1983) 447;
M. F. L. Golterman and J. Smit, Nucl. Phys. B **245** (1984) 61, and references therein.

[11] H. S. Sharatchandra, H. J. Thun and P. Weisz, Nucl. Phys. B **192** (1981) 205.

[12] C. Bernard, A. Soni and T. Draper, Phys. Rev. D **36** (1987) 3224.

[13] G. Peter Lepage, J. Comp. Phys. **27** (1978) 192.

[14] G. Martinelli and Y.-C. Zhang, Phys. Lett. **123B** (1983) 433.

[15] A. Hasenfratz and P. Hasenfratz, Phys. Lett. **93B** (1980) 165;
A. Hasenfratz and P. Hasenfratz, Nucl. Phys. B **193** (1981) 210;
R. Dashen and D. Gross, Phys. Rev. D **23** (1981) 2340.

[16] C. T. Sachrajda, in *Lattice 88*, edited by A. S. Kronfeld and P. B. Mackenzie, Nucl. Phys. B (Proc. Suppl.) **9** (1989) 121, for a review.

[17] D. Daniel and S. Sheard, Nucl. Phys. B **302** (1988) 471;
S. Sharpe, Nucl. Phys. B (Proc. Suppl.) **7A** (1989) 255;
S. Sheard, Nucl. Phys. B **314** (1989) 238.

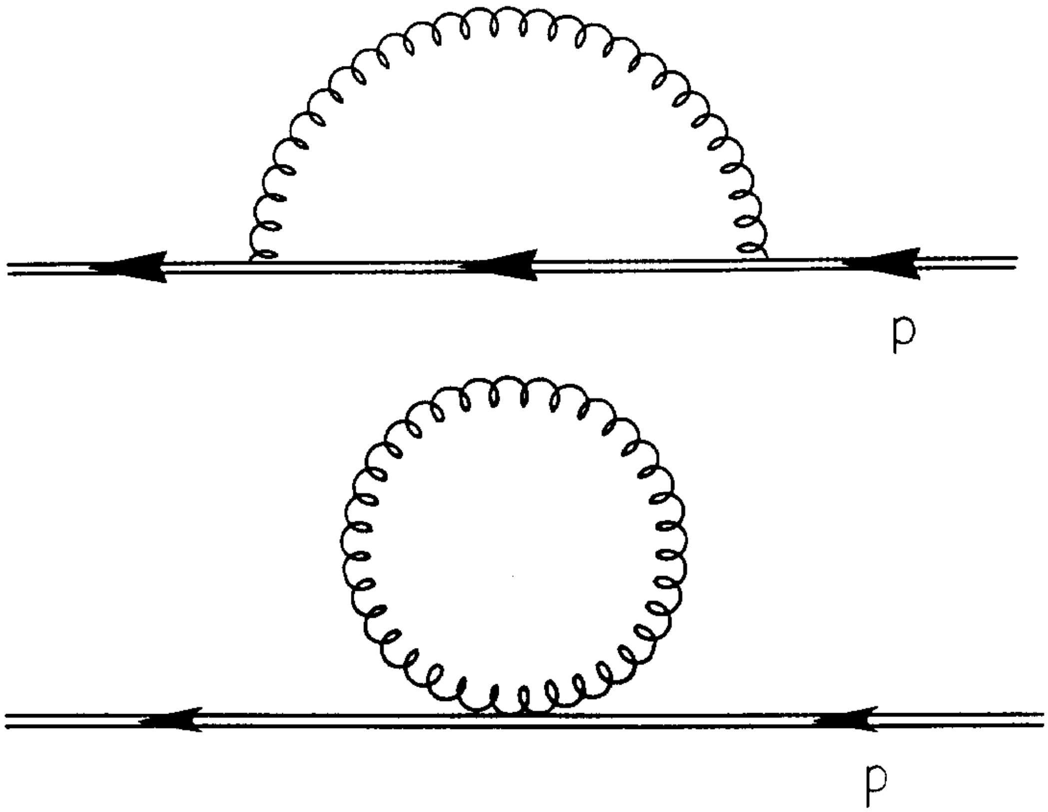

Fig. 1: Contributions to Heavy Quark Wave Function Renormalization

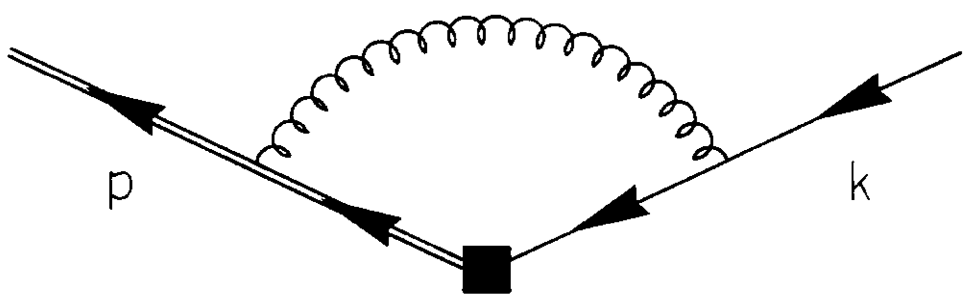

Fig. 2: Contribution to the Green's Function with the Insertion of a Bilinear