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Curved Spacetime One-Loop Gravity in a Physical Gauge

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Abstract

We study the Hamiltonian formulation of perturbative quantum gravity. For a Freedman-Robertson-Walker background, the gravitational constraints can be solved, and the physical degrees of freedom can be isolated explicitly. The resulting action is bounded from below in the Euclidean regime. In the context of wormhole physics this implies that there is no phase of the sum over spheres. We use the zeta-function regularization technique to calculate the so-called scaling behavior of the one-loop partition function about the S^4 saddle point. We find an answer that differs by an integer from the value obtained from the covariant approach using the 'conformal rotation' prescription of Gibbons, Hawking, and Perry. These results indicate that the 'conformal rotation' prescription does not yield the correct partition function for curved backgrounds.

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In their approach to perturbative quantum gravity, Gibbons, Hawking, and Perry [1, 2] (GHP) define the gauge-invariant partition function via a path integral over fluctuations of the Euclidean four-metric about a saddle-point background, with Fadde'ev-Popov measure corresponding to the gauge invariances — the four infinitesimal general coordinate transformations of the metric fluctuations [3],

$$Z = \int [d\phi_{\mu\nu}][d\phi][dV_\lambda][dV_\lambda^*] \exp\left[-I(\phi_{\mu\nu}, \phi, V_\lambda, V_\lambda^*)\right] \tag{1}$$

For one-loop calculations, the Euclidean action I is quadratic in small fluctuations. The metric perturbations $\phi_{\mu\nu}$ are traceless, while the ghosts V_λ and V_λ^* exponentiate the Fadde'ev-Popov determinant, and ϕ is the trace of the metric — the conformal factor.

The action is unbounded from below with respect to variations of the conformal factor, so the path integral does not converge. This is because about any classical background, the kinetic term of ϕ is negative-definite. Gibbons, Hawking, and Perry supply a prescription which makes the path integral converge: rotate the ϕ field at each point in the complex plane, $\phi(x) \to i\phi(x)$, so that the integrand is maximized at the background field saddle point. For non-trivial backgrounds, however, this conformal rotation is not sufficient to guarantee convergence. In particular, for Euclidean de Sitter space (S^4), the six lowest eigenfunctions of the scalar Lichnerowicz operator have unbounded action and must be rotated back. That introduces a phase $i^6 = -1$ in the one-loop partition function. This is the essence of Polchinski's calculation [4] which, if correct, would imply the demise of Coleman's mechanism [5] for the vanishing of the cosmological constant*.

The Euclidean path integral should be given by the integral over physical degrees of freedom with an 'imaginary-time' action. In gauge theories, the covariant, gauge-fixed, second-order Euclidean form correctly imposes the Gauss's law constraints needed to eliminate the unphysical degrees of freedom (so that the physical and covariant path integrals are equivalent), but there is no *a priori* guarantee that this is true. The second-order form must be derived from a first-order Hamiltonian formulation with canonical coordinates and momenta, in the phase space reduced by gauge fixing conditions and primary Hamiltonian constraints [6]. (One may then be able to return to a covariant formulation by adding redundant variables, Lagrange multipliers, and ghosts; in gauge theories, this yields the usual covariant second-order form.)

For perturbation theory about flat space, the GHP Euclidean path integral with the conformal rotation prescription yields the same one-loop partition function as obtained from the path integral over the physical degrees of freedom [7]. What about other backgrounds? In this Letter, we show that the physical degrees of freedom of gravitational perturbation theory can also be isolated for

* As pointed out by Polchinski [4], the ghost path integral is to be understood as yielding an absolute value of a determinant.

backgrounds of Freedman-Robertson-Walker (FRW) form with Minkowski or Euclidean signature. We find, in particular, that the Euclidean path integral for small fluctuations about S^4 converges and hence cannot give rise to the additional phase found by Polchinski using the GHP formalism. Abbott and Deser [8] have also discussed the boundedness of small gravitational fluctuations about Minkowski de Sitter and anti-de Sitter space.

We begin with the first-order formulation of gravity originally introduced by Palatini [9], in which the metric and connection are independent degrees of freedom. We use the Landau-Lifshitz spacelike conventions, namely the metric has signature $(- + + +)$, and

$$\begin{aligned} R^{\lambda}{}_{\mu\nu\kappa} &= -\partial_{\kappa}\Gamma^{\lambda}_{\mu\nu} + \partial_{\nu}\Gamma^{\lambda}_{\mu\kappa} - \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta} + \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta} \\ R_{\mu\kappa} &= R^{\nu}{}_{\mu\nu\kappa} \end{aligned} \tag{2}$$

Greek indices denote 0–3 and Latin ones, 1–3. The Palatini action is

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{{}^{(4)}g}\,[g^{\mu\nu}R_{\mu\nu}(\Gamma) - 2\Lambda]\ . \tag{3}$$

Variation with respect to the connection yields usual Christoffel relation between the metric and connection (which implies the vanishing of the torsion), and variation with respect to the metric yields the Einstein equation. As pointed out by Arnowitt, Deser, and Misner [10] the connection components $\Gamma^{i}_{\mu\nu}$ and $\Gamma^{0}_{0\mu}$ are nondynamical because there are no variables to which they are canonically conjugate. After eliminating them via equations of motion, one obtains the action (cf. equation (4.1) of ref. [10])

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{{}^{(3)}g}\left[\Pi^{ij}\partial_0\,{}^{(3)}g_{ij} + N\left({}^{(3)}R - 2\Lambda + \frac{1}{2}\left({}^{(3)}g_{ij}\Pi^{ij}\right)^2 - \Pi_{ij}\Pi^{ij}\right) + 2N^{i}\,{}^{(3)}\nabla_j\Pi^{j}{}_{i}\right], \tag{4}$$

where ${}^{(3)}g_{ij} = g_{ij}$ is the metric of the spacelike hypersurface with inverse ${}^{(3)}g^{ij}\,{}^{(3)}g_{jk} = \delta^{i}{}_{k}$ and covariant derivative ${}^{(3)}\nabla_j$;

$$N = 1/\sqrt{-g^{00}}, \qquad N^{i} = {}^{(3)}g^{ij}g_{j0} \tag{5}$$

are the lapse function and shift vector, and

$$\Pi^{ij} = N({}^{(3)}g^{ik}\,{}^{(3)}g^{jl} - {}^{(3)}g^{ij}\,{}^{(3)}g^{kl})\Gamma^{0}_{kl}, \tag{6}$$

so that $\sqrt{{}^{(3)}g}\Pi^{ij}$ is the momentum conjugate to ${}^{(3)}g_{ij}$. The lapse function and shift vector have no canonical momenta and act as Lagrange multipliers at the classical level which impose the gravitational constraint equations.

Quantization proceeds through the construction of the functional integral over field configurations. As emphasized by Fadde'ev and Slavnov [11] in general, and Fradkin and Vilkovisky [12] in the gravitational case, the measure in a path integral is given by $\prod_i [dp_i][dq_i]$ where the q_i are the *physical* degrees of freedom, and the p_i their canonical momenta. However, it has not been possible to solve the full constraint equations in the traditional variables and hence to determine the unconstrained degrees of freedom in the functional measure. The new variables found by Ashtekar [13] may offer a solution to this difficulty, but in the mean time, we will consider only the perturbation theory about a saddle-point of the action (4) defined via the expansions

$$\begin{aligned} g_{ij}^{\text{full}} &= g_{ij} + h_{ij}, \\ \sqrt{{}^{(3)}g^{\text{full}}}\,\Pi^{ij}_{\text{full}} &= \sqrt{{}^{(3)}g}\,\Pi^{ij} + \sqrt{{}^{(3)}g}\,p^{ij}, \\ N_{\text{full}} = N + n, &\quad N^i_{\text{full}} = N^i + n^i, \end{aligned} \tag{7}$$

where the background values are now denoted by g_{ij}, Π^{ij}, N, N^i and the fluctuations by h_{ij}, p^{ij}, n, n_i. We shall raise and lower all indices with the background metric ${}^{(3)}g_{ij}$, and we also define $\Pi = \Pi^i{}_i$, $h = h^i{}_i$, and $p = p^i{}_i$. We will find it convenient to use the reparametrization symmetry implied by the constraints (part of the background field general coordinate invariance) to choose a form of the background configuration where $N^i = 0$ and $N =$ constant. (We shall later set $N = 1$.) Then the background satisfies the classical field equations

$$\begin{aligned} \partial_0\,{}^{(3)}g_{ij} =& 2N(\Pi_{ij} - \frac{1}{2}\,{}^{(3)}g_{ij}\Pi), \\ \partial_0\Pi^{ij} =& N\left(-{}^{(3)}R^{ij} + \frac{1}{2}\,{}^{(3)}g^{ij}\left({}^{(3)}R - 2\Lambda - \frac{1}{2}\Pi^2 + \Pi^{kl}\Pi_{kl}\right) + \frac{3}{2}\Pi\,\Pi^{ij} - 2\Pi^i{}_k\Pi^{kj}\right), \end{aligned} \tag{8}$$

as well as the constraint equations obtained from the action (4) by varying N and N^i.

Under infinitesimal general coordinate transformations $\delta x^\mu = \epsilon^\mu$ acting on the full metric and connection, where the background fields transform as scalars, the fluctuations h_{ij} and p^{ij} have the following transformation properties:

$$\begin{aligned} \delta h_{ij} &= -{}^{(3)}\nabla_i\epsilon_j - {}^{(3)}\nabla_j\epsilon_i + \frac{1}{N^2}\partial_0 g_{ij}\epsilon_0 \\ \delta p^{ij} &= N\left({}^{(3)}g^{ij(3)}\Delta - {}^{(3)}\nabla^i\,{}^{(3)}\nabla^j - \frac{1}{N}\partial_0\Pi^{ij} + \frac{1}{2}\Pi\,\Pi^{ij}\right)\epsilon^0 + {}^{(3)}\nabla_k\left(\Pi^{ik}\epsilon^j + \Pi^{jk}\epsilon^i - \Pi^{ij}\epsilon^k\right), \end{aligned} \tag{9}$$

where ${}^{(3)}\Delta = {}^{(3)}\nabla^i\,{}^{(3)}\nabla_i$ is the three-Laplacian. The background equations of motion and constraints were used to derive these transformation properties. We would like to identify the gauge-invariant degrees of freedom and use these as unconstrained representatives of the gauge orbits generated by the ϵ^μ. This will eliminate four degrees of freedom from the phase space of the h_{ij} and p^{ij} . Four more degrees of freedom are eliminated by the primary constraints that arise from inserting the expansion (7) into the action, truncating to quadratic order, and varying with respect to n and n^i. These are the four perturbative Hamiltonian constraint equations,

$$\begin{aligned} {}^{(3)}\nabla^i\,{}^{(3)}\nabla^j h_{ij} - {}^{(3)}\Delta h + \left(\frac{1}{N}\partial_0\Pi^{ij} - \frac{1}{2}\Pi\,\Pi^{ij}\right)h_{ij} + \Pi\,p - 2\Pi_{ij}p^{ij} = 0, \\ {}^{(3)}\nabla_j p^{ij} + \frac{1}{2}\Pi^{jk}\left({}^{(3)}\nabla_j h^i{}_k + {}^{(3)}\nabla_k h^i{}_j - {}^{(3)}\nabla^i h_{jk}\right) = 0. \end{aligned} \tag{10}$$

We are able isolate the physical degrees of freedom of the reduced phase space for backgrounds which satisfy

$$\Pi^{ij} = \frac{1}{3}{}^{(3)}g^{ij}\Pi. \tag{11}$$

This restriction implies the following functional form for the background momentum and Ricci tensor

$$\Pi^{ij} = \frac{1}{3}{}^{(3)}g^{ij}\Pi(x^0), \quad {}^{(3)}R_{ij} = \frac{1}{3}g_{ij}{}^{(3)}R(x^0). \tag{12}$$

These conditions are satisfied by FRW metrics to which we shall restrict our attention. (Note that we use compact spacelike slices, as these allow for a Euclidean continuation.) The gauge transformations of h_{ij} and p^{ij} given by eqns. (9) simplify to

$$\begin{aligned} \delta h_{ij} &= -\frac{1}{3N}g_{ij}\Pi\,\epsilon_0 - \left({}^{(3)}\nabla_i\epsilon_j + {}^{(3)}\nabla_j\epsilon_i\right), \\ \delta p^{ij} &= N\left[{}^{(3)}g^{ij}\left({}^{(3)}\Delta + \frac{2}{3}\Lambda\right) - {}^{(3)}\nabla^i\,{}^{(3)}\nabla^j\right]\epsilon^0 + \frac{1}{3}\Pi({}^{(3)}\nabla^i\epsilon^j + {}^{(3)}\nabla^j\epsilon^i - {}^{(3)}g^{ij}\,{}^{(3)}\nabla_k\epsilon^k). \end{aligned} \tag{13}$$

The Hodge-de Rham theorem states that h_{ij} can be decomposed into transverse and longitudinal components:

$$h_{ij} = h^{\mathrm{T}}_{ij} + ({}^{(3)}\nabla_i V_j + {}^{(3)}\nabla_j V_i), \tag{14}$$

where ${}^{(3)}\nabla^i h^{\mathrm{T}}_{ij} = 0$. The longitudinal part of h_{ij} may thus be eliminated by an ϵ_i gauge transformation, and this selects (for example using the differential gauge-fixing condition ${}^{(3)}\nabla^i h_{ij} = 0$) a representative of each gauge orbit of the ϵ^i up to transformations of the form $\epsilon^i = g(x^0)K_i(x^j)$, where the K_i are Killing vectors of the three manifold — ${}^{(3)}\nabla_i K_j + {}^{(3)}\nabla_j K_i = 0$ (these are the analogs of the space-independent residual gauge transformations in flat-space gauge theories). Under the ϵ^0 coordinate transformations,

$$\delta p = N\left(2{}^{(3)}\Delta + 2\Lambda\right)\epsilon^0 \tag{15}$$

For non-flat FRW backgrounds, the operator $(2^{(3)}\Delta + 2N\Lambda)$ has no global zero modes so that the trace p can be eliminated by an ϵ^0 gauge transformation. Thus, on the configuration space reduced by fixing the ϵ_i gauge invariances, the additional gauge fixing condition $p = 0$ selects a unique representative of each gauge orbit of ϵ^0. Those ϵ^i satisfying the spacelike Killing equation are then fixed by the constraint $n_i(x^i = x^i_0, x^0) = 0$ for some point x^i_0 on the three-hypersurface for every value of coordinate time x^0. The perturbative constraint equations (10) reduce to

$$\begin{aligned} \left({}^{(3)}\Delta + \frac{2}{3}\Lambda\right)h &= 0, \\ {}^{(3)}\nabla_j p^{ij} - \frac{1}{6}\Pi\,{}^{(3)}\nabla^i h &= 0. \end{aligned} \tag{16}$$

For FRW backgrounds the first equation is satisfied for non-vanishing h only for isolated x^0; continuity then requires $h = 0$. The second equation then implies that p^{ij} must be transverse — ${}^{(3)}\nabla_i p^{ij} = 0$. Therefore the physical unconstrained degrees of freedom for perturbation theory about FRW backgrounds are the transverse, traceless components of h_{ij}, and the transverse, traceless components of p^{ij}. Referring back to eqn. (13), we see that these components (which we shall denote h_{ij}^{TT} and $p^{\mathrm{TT}\,ij}$ respectively) are invariant under both ϵ_i and ϵ_0 gauge transformations.

In terms of these physical degrees of freedom, the quadratic part of the action is

$$S = \frac{1}{16\pi G}\int d^4\,\sqrt{{}^{(3)}g}\left[\frac{-N}{4}h_{ij}^{\mathrm{TT}\,(3)}\Delta_L^{iljm}h_{lm}^{\mathrm{TT}} + \frac{1}{4N}\left(\partial_0 h^{\mathrm{TT}\,i}{}_j\right)^2 - N(p^{\mathrm{TT}\,i}{}_j + \frac{1}{3}\Pi h^{\mathrm{TT}\,i}{}_j - \frac{1}{2N}\partial_0 h^{\mathrm{TT}\,i}{}_j)^2\right], \tag{17}$$

where the operator

$${}^{(3)}\Delta_L^{iljm} = -{}^{(3)}g^{il\,(3)}g^{jm(3)}\Delta - 2\,{}^{(3)}R^{iljm} \tag{18}$$

is the Lichnerowicz operator for the symmetric traceless-transverse tensor h_{ij}^{TT} . When acting on forms, it corresponds to the Hodge-de Rham Laplacian [14] $\Delta_L = d\delta + \delta d$. On compact manifolds its eigenvalue spectrum is positive definite.

The Euclidean formulation of the above analysis is obtained by letting $N \to -iN$, which changes the signature of the metric to $(++++)$. In the action, the background momentum also rotates ($\Pi^{ij} \to -i\Pi^{ij}$) because of the implicit dependence on the lapse in eqn. (6). The action transforms as $S[h^{\mathrm{TT}}, p^{\mathrm{TT}}] \to iI[h^{\mathrm{TT}}, p^{\mathrm{TT}}]$, where $I[h^{\mathrm{TT}}, p^{\mathrm{TT}}]$ is the Euclidean action. Note that the canonical momentum p^{TT} *does not rotate.* The functional integral over p^{TT} is a bounded, well defined, real gaussian integral, with saddle point on the imaginary axis. This prescription is equivalent to the usual Wick rotation applied, for instance, to gauge and scalar field theories. That the canonical momentum does not rotate is a generic feature of first-order Euclidean formulations; it is a Euclidean path integral with a 'propagating' (Minkowskian) momentum integral that is well defined. Alternatively, one may approach the Euclidean functional integral by rederiving the Euclidean version of eqn. (17) from the Euclidean formulation of the Palatini action eqn. (3). At the classical level, the canonical momentum would simply be replaced via its equation of motion to obtain the second-order action. The quantum path integral over the momenta, however, is formally divergent, and it requires a rotation to Minkowskian momenta to make it well-defined. There is nothing special about gravity in this respect; it is a general feature of the Hamiltonian formulation of Euclidean field theories when the momentum p conjugate to the coordinate q is defined as $\delta\mathcal{L}_E/\delta(\partial_\tau q)$, where τ is the Euclidean time (because the canonical momentum for the physical theory is really defined by the Minkowski theory).

The second order Euclidean action $I[h^{\rm TT}]$ is thus

$$I = \frac{1}{16\pi G}\int d^4x\, \sqrt{^{(3)}g}\left[\frac{1}{4}h_{ij}^{\rm TT}{}^{(3)}\Delta_L^{iljm}h_{lm}^{\rm TT} + \frac{1}{4}\left(\partial_0 h^{\rm TT}{}^i{}_j\right)^2\right]$$
$$= \frac{1}{16\pi G}\int d^4x\, \sqrt{^{(3)}g}\, {}^{(3)}g^{ik}\, {}^{(3)}g^{jl}\left[\frac{1}{4}h^{\rm TT}{}_{ij}\left(-{}^{(3)}\Delta\right)h_{kl}^{\rm TT} + \frac{1}{72}\Pi^2 h^{\rm TT}{}_{ij}h_{kl}^{\rm TT} + \frac{1}{4}\left(\partial_0 h_{ij}^{\rm TT}\right)\left(\partial_0 h_{kl}^{\rm TT}\right)\right] \tag{19}$$

where the Euclidean-signature lapse has been set to 1. The action is positive-definite (since the spectrum of the Laplacian is negative definite for the FRW backgrounds under consideration), and thus defines a convergent path integral. In terms of canonically normalized fields it is

$$Z = \exp\left(-I_{\rm cl}\right)\int[dh_{ij}^{\rm TT}]\,\exp\left(-\int d^4x\,\sqrt{^{(3)}g}\left[\frac{1}{2}h_{ij}^{\rm TT}{}^{(3)}\Delta_L^{iljm}h_{lm}^{\rm TT} + \frac{1}{2}\left(\partial_0 h^{\rm TT}{}^i{}_j\right)^2\right]\right) \tag{20}$$

No contour rotation is required, and thus no additional phase emerges in the partition function. Fischler, Klebanov, Polchinski, and Susskind [15] have used the claim of a phase in the partition function to discredit the Euclidean path integral approach, but the result here indicates that such a dismissal is premature.

We turn now to the zeta-function computation. Consider the four-sphere S^4 with $N = 1$, $N_i = 0$, three-metric $g_{ij} = f^2(t)\tilde{g}_{ij}(\tilde{x})$ and Ricci tensor ${}^{(3)}\tilde{R}_{ij} = 2\tilde{g}_{ij}(\tilde{x})$, where $\tilde{x}$ and $\tilde{g}_{ij}$ are coordinates and metric on the unit three-sphere $\tilde{S}^3$, and $f(t)$ is the scale factor determined by the background equations of motion and constraints:

$$f(t) = \sin(\omega t), \qquad \omega^2 = \frac{\Lambda}{3}, \qquad t \in [0, \frac{\pi}{\omega}]\,. \tag{21}$$

The Laplacian ${}^{(3)}\Delta$ scales as ${}^{(3)}\Delta = \omega^2 f^{-2}\,{}^{(3)}\tilde{\Delta}$, where the operator ${}^{(3)}\tilde{\Delta}$ is the Laplacian on $\tilde{S}^3$. Eigenvalues and multiplicities of ${}^{(3)}\tilde{\Delta}$ acting on a complete orthonormal set of symmetric transverse traceless rank two tensors on $\tilde{S}^3$ are well-known (see, for instance, Rubin and Ordóñez [16]). We denote the eigenfunctions by $\tilde{Y}_{ij}^{lm}$, where

$$ {}^{(3)}\tilde{\Delta}\tilde{Y}_{ij}^{lm} = -(l(l+2)-2)\tilde{Y}_{ij}^{lm}, \qquad \begin{array}{l} l = 2,\ldots \\ m = 1,\ldots,D_l = 2(l-1)(l+3). \end{array} \tag{22}$$

The action (19) may be diagonalized using the expansion

$$h_{ij}^{\rm TT} = \mu\sum_{n=3}^{\infty}\sum_{l=2}^{n-1}\sum_{m=1}^{D_l}\sin(\omega t)a_{nlm}h_{ij}^{nlm}, \tag{23}$$

where μ is a normalization constant with dimensions of inverse length,

$$h_{ij}^{nlm} = \sqrt{\frac{(2n+1)(n-l)!}{2(n+l)!}}P_n^{l+1}(\cos\omega t)\tilde{Y}_{ij}^{lm}\;, \tag{24}$$

and the $P_n^l(x)$ are associated Legendre polynomials. The eigenfunctions h_{ij}^{nlm} are are orthonormal:

$$\int_0^{\pi/\omega} \omega \sin(\omega t) dt \int d^3\tilde{x} \sqrt{\tilde{g}}\, \tilde{g}^{ii'} \tilde{g}^{jj'} h_{ij}^{nlm} h_{i'j'}^{n'l'm'} = \delta^{nn'} \delta^{ll'} \delta^{mm'} . \tag{25}$$

Completeness follows from the real, elliptic, form of the operator acting on the fields $h_{ij}^{\rm TT}$ in the action (19). In terms of the a_{nlm}, the measure on the space of fields is

$$[dh_{ij}^{\rm TT}] = \prod_{nlm} \frac{da_{nlm}}{\sqrt{2\pi}}. \tag{26}$$

The one loop partition function is formally given by the integration over the coefficients a_{nlm}

$$Z_1 = \prod_n^{\infty} \prod_{l=2}^{n-1} \prod_{m=1}^{D_l} [\frac{\Lambda(n+2)(n-1)}{\mu^2}]^{-\frac{1}{2}} \tag{27}$$

The zeta function technique [17] regularizes the product $Z_1 = \prod_i \lambda_i/\mu^2$ using the generalized zeta function $\zeta(s) = \sum_i \lambda^{-s}$, which converges for $\mathrm{Re}(s) > 2$ for positive eigenvalues λ_i. The infinite product Z_1 is given by

$$Z_1 = e^{\frac{1}{2}\zeta'(0) + \frac{1}{2}\ln(\mu^2/\Lambda)\zeta(0)}, \tag{28}$$

where $\zeta(0)$ and $\zeta'(0)$ are the analytically continued values of the function. In our case we have

$$\zeta(s) = \frac{1}{12}(4)^s \sum_{n=3}^{\infty} \frac{(2n+1)\left((2n+1)^2 - 49\right) + 120}{[(2n+1)^2 - 9]^s} \tag{29}$$

General results useful for evaluation of the $s \to 0$ limit of functions of this type are found in the Appendix of ref. [3]. We find

$$\zeta(0) = -\frac{661}{45} . \tag{30}$$

The value of $\zeta(0)$ is conventionally regarded [1,3,17] as the gauge invariant, on-shell, scaling behavior of the one loop partition function: $\mu \frac{\partial}{\partial \mu} \ln Z_1 = \zeta(0)$, where Z_1 is defined via eqn. (28). The result obtained here differs from that obtained from the covariant approach with the GHP conformal rotation prescription*[3], $-571/45$. The difference is an integer, which is significant because negative and zero modes which occur in the covariant approach contribute integer values to the scaling behavior.

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* A recent reanalysis by Taylor and Venesiano [18] yields $-571/45 + 20 = 329/45$ for the covariant result with the GHP conformal rotation prescription, which also differs from our result.

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